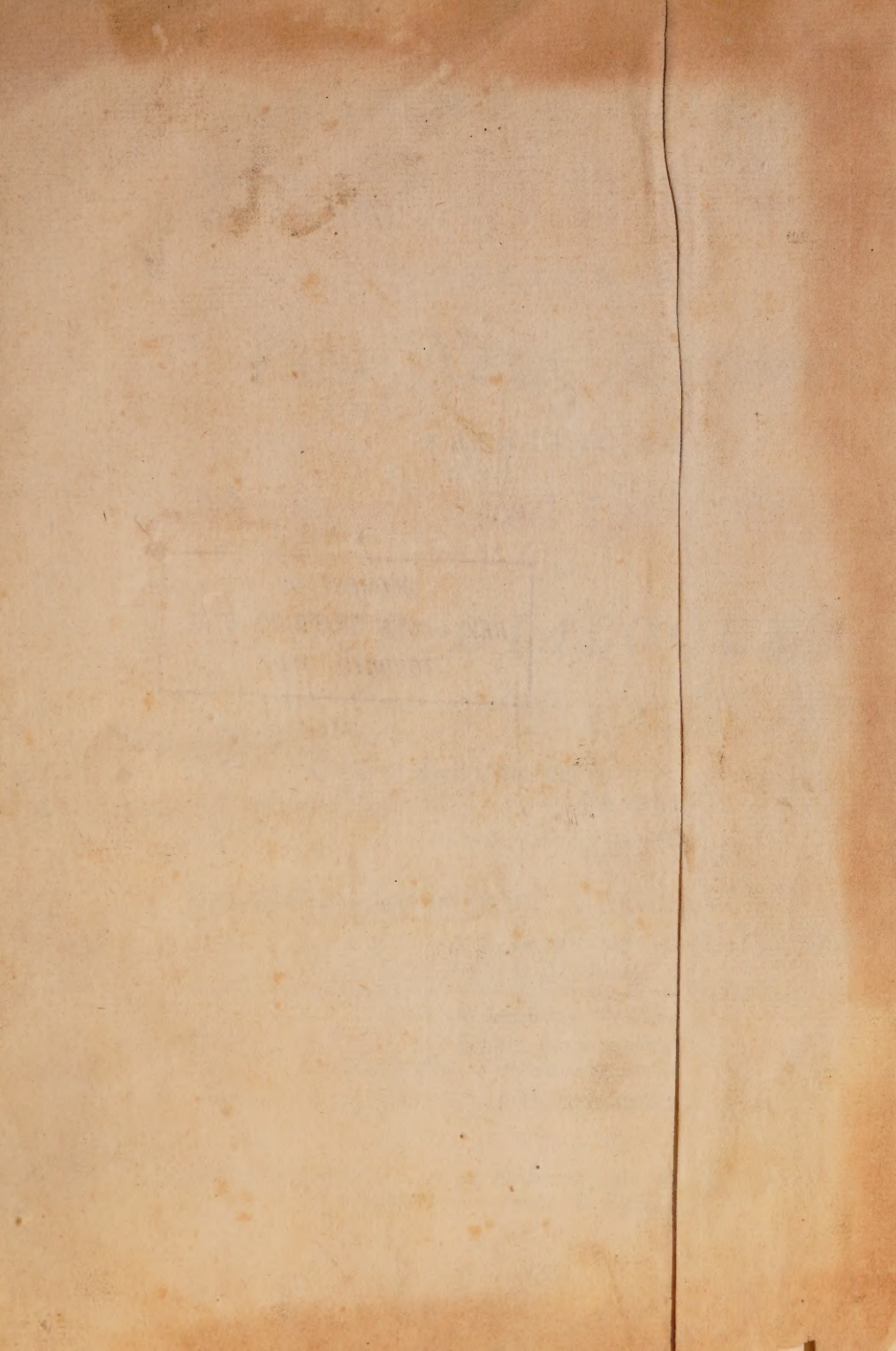


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BEQUEST OF
REV. CANON SCADDING, D. D.
TORONTO, 1901.

*Physic
Mech.*

M A T H E M A T I C A L
D I S C O U R S E S
C O N C E R N I N G

Two New Sciences

R E L A T I N G T O

Mechanicks and Local Motion,

I N

F O U R D I A L O G U E S.

- | | |
|---|--|
| I. Of the Resistance of Solids
against Fraction. | III. Of Local Motion, viz. Equa-
ble, and naturally Accelerate. |
| II. Of the Cause of their Cohe-
rence. | IV. Of Violent Motion, or of
PROJECTS. |

By **GALILEO GALILEI,**

Chief Philosopher and Mathematician to the Grand Duke of TUSCANY.

With an APPENDIX concerning the Center of Gravity of
SOLID BODIES.

Done into *English* from the *Italian*,

By **THO. WESTON**, late Master, and now publish'd by **JOHN WESTON**,
present Master, of the Academy at Greenwich.

L O N D O N :

Printed for J. H O O K E, at the *Flower-de-Luce*, over-against St. Dunstan's
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1687

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To the HONOURABLE

Col. JOHN ARMSTRONG,

Surveyor-General

OF

HIS MAJESTY'S ORDNANCE.

SIR,



T was the laudable Custom of Authors, in ancient Times, to inscribe their Works to truly Illustrious and Worthy Men ; sometimes to Men eminent for their universal Benevolence and publick Merit to Mankind, and sometimes to such as were eminent for their Knowledge in the Author's Subject. The frequent Services you have been, and are constantly call'd to by your PRINCE for your Country's Good,

DEDICATION.

denote you of the *former* Class; and your Experimental, as well as Contemplative Knowledge in GALILEO's Doctrine, ranks you with the *latter*.

On these Accounts, *Sir*, together with a View of fulfilling my late Brother's Intentions, I presume (as, I know, he design'd) to publish this following Translation under your Name. Thus am I faithful to the Deceased, in rendring you all just Acknowledgments for your Approbation of his Undertaking; and am favour'd with an Opportunity of professing my being, with Sincerity and all due Respect,

SIR,

Your most Oblig'd,

and most Obedient

Humble Servant,

J. Weston.



THE
TRANSLATOR'S
P R E F A C E.



THESE Select Pieces of our Excellent Author were, many Years ago, publish'd in an English Dress by Mr. Salusbury, in the second Tome of his Mathematical Collections and Translations; which is now become very scarce: Whereupon a new Translation of them being much desired and promoted; the Countenance I (however unequal to the Attempt) have receiv'd from the Judicious, has been too prevalent for me to decline it.

In Consequence whereof I now make publick a Second Translation (done from the Elzevir Edition of the Italian); and, as it is meerly such, I conceive there need nothing more be added on my Part: But whether it be (as I presume my Friends expect it should be) neater, and more agreeable to the present Taste, and less obscure than the former, I willingly submit to the Decision of the candid Reader.

As

As to our Author, his universally establish'd Fame almost wholly anticipates my saying any thing of him, or his Works. " It is confess'd by all, [says the ingenious " Dr. Pemberton] That GALILEO greatly improved " Philosophy, by shewing, that the Power in Bodies, " which we call Gravity, occasions them to move downwards with a Velocity equably accelerated; and that " when any Body is thrown forwards, the same Power " obliges it to describe in its Motion that Line, which is " called by Geometers a Parabola. [This is the grand " Fundamental in the Art of Gunnery.] When a Body " falls perpendicularly, it is known how long Time it " takes in descending from any Height whatever: And " if it be thrown forwards, we know the real Path, " which it describes; we can determine in what Direction, and with what Degree of Swiftnefs, it must be " projected, in order to its striking against any Object " desired; and we can also ascertain the very Force, " wherewith it will strike. Sir ISAAC NEWTON " [continues the Doctor] has farther taught, That this " Power of Gravitation extends up to the Moon, and " causes that Planet to gravitate towards the Earth: " He has prov'd likewise, that all the Planets gravitate " towards the Sun, and towards one another; and that " their respective Motions follow from this Gravitation. " And as GALILEO has shewn more fully than was " known before, what Effects were produced in the Motion of Bodies by their Gravitation towards the Earth; " so Sir ISAAC NEWTON, by this his Invention, has " much advanced our Knowledge in the Celestial Motions.

For

THE TRANSLATOR'S PREFACE. vii

For an Account of our Author's Discoveries in the Heavens, and Improvements in Astronomy, by Help of the Telescope, who was the first that fitted and applied that Instrument to Celestial Observations, I refer the Reader to the following English Version of the Preface to the Original in Italian; and shall here only remind him of the known hard Fate this great Man, from his superior Knowledge in Astronomical Matters, met with, in that he was, by a Conclave of bigotted Cardinals, condemned for Heresy, for maintaining the Doctrine of the Motion of the Earth, and the Stability of the Sun; it being repugnant (as they ignorantly and maliciously adjudged) to the Authorities of sacred Scripture. A very remarkable Effect of Popish Infallibility!

The only Objection to our Author, as I have heard of, is, that he is too prolix: Nil rectum putat nisi quod ipse facit, may be justly applied to such Objector: All know, that the Advantage, which accrues from the Dialogue Way of Writing, makes more than sufficient Amends for the Prolixity inevitably attending it. Moreover, our Author's Digressions and Reflections are neither trifling nor absurd, but such as are curious, pleasant and useful; as will evidently appear upon a fair Perusal.

T. W.



G A L I L Æ U S
H I S
D I A L O G U E S.

DIALOGUE I.
*Concerning the RESISTANCE of SOLIDS,
against FRACTION.*

SALVIATI, SAGREDO, *and* SIMPLICIUS.

SALV.  HE constant Employments (Gentlemen) in your famous *Arsenal* of *Venice*, and especially those relating to what we call MECHANICS, seem to me to afford, to a speculative *Genius*, a large Field to Philosophize in, there being a great Number of Artificers constantly employ'd in forming Engines and Machines of all Sorts; amongst whom, as well on account of the Observations made by their Predecessors, as of those which their own Care and Industry helps them to, there must certainly be some very skillful, and such as are able to give the just Reasons of Things.

B

SAGR.

SAGR. You are partly in the right, Sir : Being naturally curious, I frequently divert myself by visiting that Place, and talking with the OVERSEERS OR FOREMEN : By conversing with these, I own I have often found great Helps in my Enquiries concerning the Reason of Effects, which were wonderfully abstruse, and, in a manner, incredible ; but yet sometimes I have continued at a great Loss, and, indeed, have despaired of ever understanding how that could follow, which, though I could not see why it should be so, my very Senses proved to me was true. As to what that old Man, not long since, told us, tho' it be, I own, the common Opinion, I can't but think it altogether false, as are many other Conclusions among the Unskilful, who often pretend to account for Things they know little or nothing of.

SALV. You are speaking, I believe, of his last Assertion, when we asked him, Why that *larger* Vessel which was to be launch'd, was arm'd with so much greater *Apparatus* of Stays, Supporters, and other Provisions, than the smaller Sort were ? When his Answer was, That this was by Way of Defence, lest striking against the Ground, it should, by the Weight of its own vast Bulk, break its Back ; An Inconvenience (continued he) *lesser* Vessels are not subject to.

SAGR. I was so, and chiefly of this last Conclusion of his, which I always took for a vulgar Error, *viz.* That in these, and other *Machines*, we must not argue from lesser Things to greater, since many Inventions succeed in
lesser

lesser Machines, which, upon Trial afterwards, will by no Means do in larger. But now all Mechanical Reasonings being founded on *Geometry*, wherein I don't see how the Greatness or Smallness of Circles, Triangles, Cylinders, Cones, or of any other Planes or Solids, can alter the Properties or Passions of those Planes or Solids : I cannot see, if the greater Machine be made in all its Parts similar to, and in the same Proportion with those of the lesser, which is useful and fit for the Purposes it was designed ; I cannot see, I say, why this larger Machine should not be equally exempt from unlucky and destructive Accidents.

SALV. The vulgar Opinion, as to this Affair, is, I own, absolutely false, and its Contrary, *viz. Many Machines may be made more perfectly in large, than in small*, may be advanced with more Truth by half. Thus, for Example, A Clock that shews and strikes the Hours, may be more accurately made of such, or such a Size, than of a less. With much better Reason do others more skilful conclude, who refer the Cause of the Event or Effect of such large Machines, not coming up to that which is collected from the pure and abstracted Demonstrations of *Geometry*, to the Imperfection of *Matter* which is subject to many Changes and Defects. But here I don't know whether I may, without Suspicion of Arrogance, say, That that Recourse to the Imperfection of Matter which may blemish the most pure Mathematical Demonstrations, is not sufficient to excuse that Deviation of Machines in the *Concrete*, from what was conceived of them when consider'd abstractedly, and in *Idea* only.

Nay, I affirm, That tho' we abstract all Imperfection from *Matter*, and suppose it perfect, invariable, and free from all accidental Alteration ; yet, only because it is material, it comes to pass, that the greater Machine, made of the same *Matter*, and in the same Proportion with the lesser, shall answer the lesser in a most exact Symmetry, in all Circumstances, except in Strength, and Power of resisting violent Invasions ; for herein it shall not answer it, but the greater it is, so much, in Proportion, I affirm, it shall be the weaker. And now, because I suppose *Matter* to be unalterable, or always the same, it is manifest, that concerning it, the most simple and pure Mathematical Demonstrations necessarily offer themselves. Therefore, *Sagredo*, fling away the Opinion to which you, and, may be, all others who have studied *Mechanics*, have adhered to, *viz.* That Machines and Compound Engines, made of the same Matter, with the most accurate Observation, of the self-same Proportions between their same Parts, ought to be equally, or, to express myself better, proportionally disposed to resist, and to yield to external Assaults and Impulses : For it may be geometrically demonstrated, that the greater are always, in Proportion, less able to resist than the lesser ; so that, in short, there is a Term necessarily ascribed, not only in all artificial Machines and Structures, but also in natural Ones, which neither Art nor Nature can pass, provided the same Proportions be always observed, with the Identity of Matter.

SAGR. I am perfectly astonish'd ! ——— However, from what you have said, it seems to me necessarily to follow,
That

That it is impossible to make two Machines of the same Matter, similar and unequal, and which shall be able to resist in the same Proportion with each other. And that 'tis also impossible to find two similar Rods, or Staffs, of the same Wood, which shall be alike in Strength, but unequal in Size.

SALV. It is so, Sir; and that you may be further assured that you take my Meaning right, I affirm, That if we take a Staff of Wood, of such Length, and Thickness, that being fix'd fast in a Wall, at right Angles to it, *i. e.* parallel to the *Horizon*, it will just support itself, *i. e.* so that if it was a Hair's Breadth longer, it would break by its own Weight; I say, there cannot possibly be another such a one: I mean thus, Let, for Instance, the Length of the Staff be 100 times the Breadth; then, I say, no other Staff can be made of the same Matter, unequal to it, but similar, *i. e.* whose Length is 100 times its Breadth, that can sustain itself precisely, as the former does, and no more; but all *bigger* than it will break, and those *lesser*, will be able to sustain something more than their own Weight: And what I say of the *State of sustaining itself*, may be understood of every other State. And thus, if a Beam can sustain a Weight of ten Beams, each equal to itself, another similar Beam cannot in like manner sustain a Weight of ten, each equal to itself. Now pray, Gentlemen, observe how Conclusions that are just, although they seem never so improbable at first View, yet, with a little Consideration, sling off the Veil which obscured them, and their Demonstrations appear most pleasant, simple, and easy. Who does not see, that

that a Horse falling from the Height of 3 or 4 Yards, breaks his Bones, whilst a Dog falling from the same Height, or a Cat from a Height 8 or 10 times as great, gets no Hurt ; as neither does a Grasshopper from a Tower, nor an Ant from the Moon ? Little Children take no Harm from their Falls, whilst older People break their Limbs : And as lesser Animals are, in Proportion, more robust and stronger than greater, so also lesser Plants support themselves better than greater. I believe you both know, that an Oak 200 Feet high, cannot support its Branches so spread, as those of another of moderate Height. And that Nature cannot produce a Horse as big as 20 Horses, nor a Giant bigger than 10 Men, unless this be done by a Miracle, or the Proportion of the Members be much altered, and particularly of the Bones, by enlarging them greatly beyond the Measure of common Bones. In like Manner, if we suppose, that greater and lesser artificial Machines can be with equal Ease made and preserv'd, we run into a manifest Error. As thus, for Instance, small Pyramids, Cylinders, and other small Bodies, may be safely mov'd, laid along, and reared upright, without any Danger of breaking them, whilst very large ones break of themselves, or by every little Accident, and that for no other Reason, but on account of their own Weight. And here I can't help telling you of an Accident worth your Notice, as, indeed, are all those Events that occur unexpectedly, especially when the very Means used to prevent an Inconvenience, prove the main Cause of the future ill Event. There was a very great Pillar of Marble laid along, and two Rollers put under it near its Ends : After some Time, a certain Mechanic thought, the better to secure

cure this Pillar from breaking in the midst by its own Weight, to put there a third Roller; this Opinion of his seem'd generally very just, but the following Event prov'd the contrary; for, a few Months after, this Pillar crack'd and broke in two, just over the new Roller.

SIMP. A very wonderful and unexpected Accident indeed! especially if it was caused by this Roller's being placed under its Middle.

SALV. Without doubt this was the Occasion of it, and when once you know the Cause of this Effect, you'll cease wondering. You must know then, upon taking the two Pieces of the Pillar off the Rollers, it was found, that one of these Rollers, on which one of the Ends of the Pillar had rested, was, by Length of Time, grown rotten, and sunk lower, and that in the Middle continuing sound and strong, occasioned one Half of the Pillar to hang in the Air, without any Support at one End; so that its own excessive Weight occasioned that which would not have happened if it had rested only upon the two first Rollers; for then, if either End had given Way, the Middle would likewise have follow'd. And here now, surely, I may assert, That in a small Pillar there had been no Danger of this, though it was of the same Stone, and of a Length answerable to its Thickness, in the very same Proportion as the Length and Thickness of the greater Pillar are to each other.

SAGR. I am now convinced of the *Truth* of the Fact; but don't see the *Reason* why, in the Augmentation of
Matter,

Matter, the Resistance and Strength should not be increas'd at the same Rate: And this seems the more strange to me, because I see, on the contrary, in other Cases, the Power of resisting Fraction increases much more than the Bulk of Matter does; Thus, for Example, let two Nails be drove into a Wall, the one twice as thick as the other, *that* shall bear a Weight, not only double, but triple, nay quadruple of *this*.

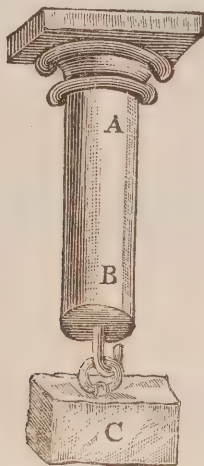
SALV. You may say octuple, and not err from the Truth; but this Effect is not contrary to the former, tho', at first Appearance, it seems so difficult.

SAGR. Therefore, *Salviatus*, remove these Doubts, and explain to us these Difficulties, if it ben't too much Trouble; for I can't but think, that this Matter of *Resistance* will open to us a Field of noble and useful Contemplations. And if you please to let this be the Subject of To-Day's Discourse, you'll much oblige us both.

SALV. With all my Heart; If I can but call to mind what I formerly learnt of our * *Academic*, who has
 * GALILÆUS. made many Discoveries about this Matter; and these all built, according to his usual Manner, on geometrical Demonstrations, insomuch that this of his may be very justly called a *New Science*. For tho' some of his Conclusions have been made by others, and first of all, indeed, by *Aristotle*, yet were they not deduced from their primary and undoubted Principles, by a fair and undeniable Method of Reasoning. And now, because, as I have told you, it is my Design to convince you by Demonstration,
 and

and not content myself with persuading you with Probabilities only ; presupposing that you have already, from others that have treated in a rational Way of *Mechanics*, learn'd enough for our present Purpose ; Before I proceed, it is requisite that we consider what Effect that is which is produced in the Fraction, or Breaking of any Beam, or other Solid, whose Parts are firmly connected, because this is the first Notion whereon the first and simple *Principle* consists, which should be, in our future Reasonings, taken for granted.

The better to explain which, let us suppose a Cylinder or Prism AB of Wood, or some other solid and coherent



Matter, to be firmly fasten'd above in A, and to hang perpendicularly ; to whose other End B, let there be hung the Weight C. Now 'tis manifest, that how great soever the Tenacity and Coherence of the Parts of the Solid be, provided it be not infinite, it may be overcome by the drawing Weight C, whose Gravity, I suppose, may be
C
increased

increas'd as much as we please, and that Solid will, at length, break as a Rope would: And as we know that the Resistance of a Rope is occasion'd by the Multitude of the Strings or Threads of which the Rope is compos'd; so in Wood we find its Fibres or Grain running lengthways, which enable it more powerfully to resist Fraction, than any Rope of the same Thickness. But in a Cylinder of Stone or Metal, the Coherence of its Parts (which seems greater) depends on a Cement different from such Strings, or from what in Wood we call the Grain; and yet by a stronger Force these also are broken.

SIMP. If it be as you say, I conceive well enough, that the Strings or Grain of the Wood, which is as long as the Wood itself, may make it strong, and able to resist almost any Force applied to break it. But how can a Rope compos'd of Hempen Threads, each not above two or three Feet long, be spun out to the Length of, for Instance, 100 Feet, and keep the same Strength? I would likewise know your Opinion concerning the Agglutination or Connection of the Parts of Metals, Stones, or any other Matter wanting those Threads or Ligatures; which, nevertheless, if I am not mistaken, are more tenacious than such as have them.

SALV. To solve these Difficulties you start, we must digress to new Speculations, and these but of little Use to our main Design.

SAGR. But if such Digressions may lead us to the Knowledge of new Truths, where can the Disadvantage be,
†

be, since we are not tied down to a strict and concise Method, but talk to please ourselves, if we do digress sometimes, especially when we are in Danger of losing some Knowledge, which we have a present Opportunity of enquiring into, and which omitted, another, perhaps, may never offer itself? Besides, who knows but by these Digressions we may gain such Hints as are more valuable than the first-intended Conclusions? Therefore I intreat you, that you will satisfy *Simplicius*, and myself, no less desirous than he, to understand what that Cement is which holds the Parts of these Solids so tenaciously join'd, and which can hardly be dissolved. The Knowledge of which, must be a great Help to us to understand how the Parts of these Strings or Ligaments, of which some Solids are composed, cohere.

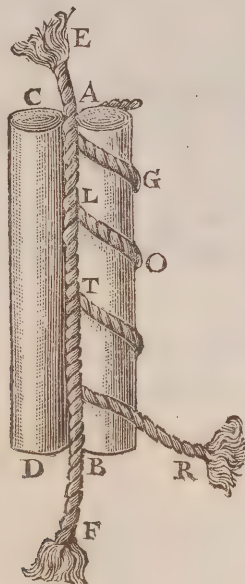
SALV. To please you, I'll do as you would have me. The first Difficulty then, is, How the Threads of a Rope, an hundred Feet long (none of which exceed two or three Feet in Length) can be so closely connected, as to require so great a Force to break them?

Now tell me, *Simplicius*, can you hold one End of one of these Hempen Threads between your Fingers so fast, that I, pulling by the other End should break it sooner than get it from you? Surely you can: When, then, these Threads are not only at the Ends, but are all over grasped, or held very fast, is it not manifestly more difficult to pluck the same from him that holds 'em, than to break them? Now the very same Thing is done in twisting the Rope, *i. e.* the Threads, by these Means,

are so strained, and bound so fast one within another, that pulling the Rope with great Force, its Threads break afunder, but are not parted from one another, or pull'd from their Hold, *i. e.* untwisted, or drawn out to their first Length, or *that* they were of before they were twisted together; as is plainly seen by viewing the Ends of the said Threads in the broken Place, that are very short, which, if the Rope had not been divided by breaking the Threads, but by separating them from one another, they would not be. And now observe how one Truth draws on another: That Thread which, held fast between the Finger and Thumb, did not give Way to, or follow him, who endeavour'd, with all his Might, to draw it from between them, resisteth, because it was staid by a double Compression, since the Finger pressed against the Thumb, as much as this against that: And, without doubt, could only one of these Pressures be retained, the Thread would resist the other Person's Endeavour to pull it away, half as much as it did when it was detained by both Pressures conjointly.

But now, because by lifting off the Thumb, for Instance, its Pressure can't be taken away, but the Pressure of the Finger must also be removed; let's see if there be no other Way to be found to preserve one of these Pressures, the other being moved, that is, to make that Thread, of its own accord, as it were, cling to the Finger, or other Solid, to which it is applied, and that so forcibly, that the more one draws the Thread to get it from the Finger, the faster the Thread presses the Finger, and this is done if the Thread be wound round the Finger. For
the

the better understanding whereof, see the following Figure. Let AB and CD be two Cylinders, and between them, let there be extended a Thread, which, for greater Plainness, we'll represent by the small Cord EF. Now



there's no doubt but these two Cylinders being press'd hard against one another, the Cord EF, pulled by the End F, has a great Force to resist, before it can be drawn from between the two Solids which compress it. But if one of these Cylinders be removed, though the Cord still touches the other, yet shall it not, by such Contact, be hinder'd from slipping away; but if at the Top of the Cylinder at A, the Cord be held but lightly, and then wound about the same in a spiral Manner, as AGLOTR, and if you pull it by the End R, 'tis manifest, that it will begin to
press

press the Cylinder, and if there were more Turnings or Windings about, the harder 'tis pull'd, the closer it will adhere to, or press the Cylinder. And if, by multiplying the Wreaths, the Contact be made longer, and consequently more inseparable, the more difficult it will be to withdraw the Cord, and make it give Way to the Force that pulls it. Now, I say, who does not see that the same Resistance is in the Threads, which, with a thousand and a thousand such Twinings, twist or spin the thick Rope? Nay, the Stress of such Twistings binds so tenaciously, that very strong Bands are made of a few short Rushes, tho' but a little twisted together.

SAGR. Your Discourse has made me cease wondering at two Effects, the Reason whereof, I did not before well understand. One was, to see a Rope turned two or three times at most about the *Axis* of a Crane, adhere so firmly to it, as not to slip away, tho' it be drawn by the immense Force of a vast Weight. Moreover, by turning on the Crane, we see the said *Axis*, by the sole Contact of the Rope begirring it, to draw and raise up vast Stones, while one of little Strength, could hold and stay the other End of the said Rope with his Hands. The other Effect I wonder'd at, was that of a simple, but clever, Instrument, invented by a young Kinsman of mine, by which, and by Help of a Cord, he could let himself down from a Window, without hurting or gawling his Hands, as he, not long before, had done to his great Pain. And that you may the better understand this, take the following Scheme.

Scheme. About a Cylinder, as AB, about two Inches thick, and eight or ten long, he cut a Notch spirally, of one turn and half, and no more, big enough to receive the Cord he design'd to use, and which he put into the Notch at the End A, and brought out at the other B: Afterwards he inclos'd that Cylinder and Cord in a Wooden, or rather a Tin Tube, made with Clasps or Hinges, to open or shut lengthways at Pleasure. Then having made fast the Rope above, grasping of the said Tin Case with both his Hands, he hung by his Arms, whence, such was the Compression of the Cord between the ambient Tube



and the Cylinder, that, at Pleasure, by clinching his Hands closer, he could stop himself, and by a little loosening his Hold, he could let himself down gently as he pleas'd.

SALV. An ingenious Invention truly ; and more fully to explain its Nature, methinks I have a Glimpse of some Discoveries additional to it. But I will not now make any farther Digression about this Matter, since you are so desirous to have my Opinion concerning the Resistance of other Bodies against Fræction, whose Texture is not of Strings and Threads, as is that of *Ropes*, and most Kinds of *Wood*, but the Connexion of whose Parts depends on other Causes ; and which, in my Judgment, may be reduc'd to two Heads : The former of which, is Nature's much-talk'd-of Abhorrence of a *Vacuum* ; for the other (the 'foremention'd one being not sufficient) we must introduce some glutinous Matter that may strongly connect the Particles whereof these Bodies are compos'd.

And first of a *Vacuum* ; where I will shew, by plain Experiments, what, and how great its Virtue is. And, first, we may see when we please, that if two flat Pieces of Marble, Metal, or Glass, exquisitely plain'd and polish'd, be laid one upon another, the one may be easily mov'd horizontally off the other (a plain Proof there's no glutinous Matter holds them together) but if you endeavour to pull them vertically from one another, you'll find so great a Resistance to such Separation, that the upper one will lift the under one up, and carry it, tho' pretty thick and heavy, with it, and hold it up continually. This Experiment, I say, most evidently proves to us how much Nature abhors to admit a *Vacuum*, though for the shortest time imaginable, which would be between them before the Concourse of the Particles of the circumambient Air could take Place, and fill it up. We farther see, that

if
†

if these two Plates are not exactly polish'd, and consequently, their Surfaces applied, don't touch one another exactly all over ; that if you try gently to separate them, there's nothing to hinder you but only the Weight ; but with a sudden Pull the under Plate will rise with the upper one, and immediately fall down again, following the upper one only for that very small Time, which serves for the Expansion of that small Quantity of Air which was between the two Plates, not every where touching one another, and for the Entrance of the other circumambient Air. Now no doubt but the like Resistance which so sensibly discovers itself betwixt the two Plates, is in like manner found between the Parts of any Solid, and, as its concomitant Cause, constitutes their Connection, at least in Part.

SAGR. Hold a little, pray, and let me offer a particular Consideration, just come into my Mind ; 'tis this : We see the lower Plate to follow the upper one, and, by a quick Motion, to be together rais'd with it ; we thence are assur'd, that (contrary to the Opinion of many Philosophers, and may be of *Aristotle* himself) Motion *in Vacuo* would not be instantaneous : For should it be such, the two fore-mention'd Plates, without the least Resistance, would be separated, since the self-same Instant of Time would be sufficient for their Separation, as for the Concurrence of the ambient Air to fill up that *Vacuum*, which might remain between them. From the lower Plate's following the upper one, therefore, we may infer the Motion *in Vacuo* would not be instantaneous. And we may farther gather, that between those two Plates there is some

D

Vacuity,

Vacuity, at least for some very small time, *i. e.* for so long as the Motion of the ambient Air lasts in its Concourse to replenish or fill up the *Vacuum* : For was there no *Vacuity* there, there would be no Need of the Concourse or Motion of the ambient Air. We must therefore conclude, that a *Vacuum* is admitted by Violence, or contrary to Nature, although, in my Opinion, nothing is contrary to Nature, but what is impossible, *i. e.* but what is *Not*.

But here now arises another Difficulty, and 'tis this ; Altho' Experience assures me of the Truth of the Conclusion, yet I do not understand the Cause to which such an Effect may be ascrib'd ; because the Effect of this Separation of the two Planes is prior to the *Vacuum*, which by consequence should succeed it, and because, in my Opinion, the Cause ought, if not in Time, yet in Nature, to precede its Effect, and a positive Effect ought to have a positive Cause : I can't conceive how the Cause of the Coherence or Cohesion of those two Planes, and of their Repugnance to a Separation (which two Effects are already in act) shall be assign'd to a *Vacuum*, which, as yet, is not, but ought to follow ; for of Things not in Being, there can be no Operation, according to an infallible Maxim in Philosophy.

SIMP. But since you grant *Aristotle* this Axiom, sure you can't but grant another every whit as infallible, *viz.* That Nature does not attempt Impossibilities ; on which Axiom, in my Opinion, depends the Solution of our Doubt : Because therefore a *Vacuum* is of itself impossible, thence Nature permits not that to be done, in consequence
of

of which a *Vacuum* must ensue : and such an Action is the Separation of the Plates.

SAGR. Now (admitting what *Simplicius* alledges to be a sufficient Solution of my Doubt) in pursuance to the Discourse we begun with, this Repugnance to a *Vacuum* seems to me a sufficient Cement to hold together the Parts of any Solid of Stone or Metal, or of any other Matter more dense, and consequently more able to resist a Separation. For if-so-be a single Effect hath but one only Cause, as I at present understand and believe, or if many be assign'd, they may be reduc'd to only one ; why should not this Reason taken from a *Vacuum*, be sufficient to account for all Resistances ?

SALV. I will not at this time meddle with this Controversy, viz. Whether a *Vacuum*, without the Help of any other Cement, be sufficient firmly to hold together the scarcely separable Parts of firm Bodies. But I affirm, that the Business of a *Vacuum*, which is of Force, and Conclusive as to the two Plates, is not of itself sufficient firmly to connect the Parts of a solid Cylinder of Marble or Metal, which drawn lengthways with great Force and Strength, are at last separated and divided. But now, if I have found a way of distinguishing this already-known Resistance, depending on a *Vacuum*, from every other that may concur with it, to strengthen the Connection, and can demonstrate to you that it alone is not near sufficient for such an Effect ; will you not allow me to introduce some other ? Help him, *Simplicius*, since he is in Doubt what to answer.

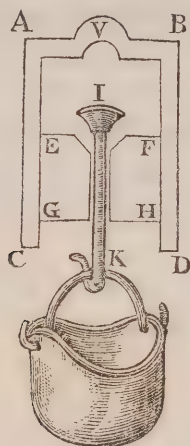
SIMP. *Sagredus's* not giving an Answer, without doubt proceeds from some other Reason, there being no manner of Room for Doubt, concerning so clear and necessary a Consequence.

SAGR. You are right, *Simplicius*, for I was thinking if a Million of Gold from *Spain*, be not sufficient to pay an Army, whether or no it would not be absolutely necessary to provide some farther Means to make up the Deficiency. But go on, *Salviatus*; and supposing I admit your Consequence, shew us how to separate the Operation of a *Vacuum* from all others, and, by estimating it, shew us its Insufficiency to produce the Effect we are talking of.

SALV. Well, I will shew you a Method of separating the Power of a *Vacuum* from all other, and of measuring it: And to do this, we will take some continued Matter, whose Parts resist a Separation on no other Account, but on that of a Repugnance to a *Vacuum*, such as Water is, as has been at large demonstrated in a certain Tract by our Academic; So that whenever a Cylinder of Water drawn, resists a Separation of its Parts, it must be acknowledged, that this can be ascrib'd to no other Reason, but that it may exclude a *Vacuum*. Now to make this Experiment, I have invented a Way, which, by the Help of the following Diagram, may be much better explain'd than it can be by bare Words.

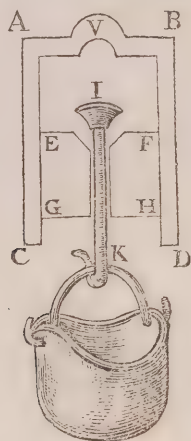
Let

Let C A B D represent a Cylinder of Metal, or of Glass, made hollow and exactly turn'd : into whose Concave put a Cylinder of Wood, fitted most exactly to touch every where the other, which let be E G H F ; which Cylinder may be mov'd upwards or downwards. This I would have bored in the Middle, for an Iron Rod to pass thro', made



with a Hook at one End, as K, and let the other End, I, grow thicker, in Form of a Cone or Top ; wherefore the upper Part of the Hole made for this Rod to pass thro' must be hollow'd conically, so as most exactly to receive the conical End I, of the Rod I K, when drawn down by the End K. But now I would not have the Cylinder of Wood E H, when put into the concave Cylinder A D, touch the uppermost Superficies of the said concave Cylinder, but to continue two or three Fingers Breadth from it ; and the empty Space A F, I would have fill'd with Water, which may be put in by turning the Aperture C D
of.

of the Vessel upwards ; then press down the moving Cylinder or Stopper EH, keeping the conical Part I of the Iron Rod a little Distance from the Hollow that was made in the Stopper to receive it, that the Air may have Room to go out, as it will do upon thrusting down the Stopper by the Hole the Iron Rod goes thro', provided



it be made a little wider than the Rod IK is. The Air being let out, and the Iron Rod pull'd to stop close the Hollow of the Stopper by its conick Part I; then turn the Mouth of the Vessel downwards again, and hang to the Hook K a Bucket, which fill with Sand, or with such a Quantity of some other weighty Matter, 'till at length the upper Surface of the Stopper EF be separated from the lower Surface of the Water, to which it was connected by no other Cause but the Repugnance to a *Vacuum* : And then if you weigh the Stopper with the Iron Rod, the Bucket, and all that was in it, you will have the Quantity

Quantity of the Force of the *Vacuum*. And if to a Cylinder of Marble, or Crystal, of the Thickness of the Cylinder of Water, you fasten such a Weight, that, together with the Weight either of that Marble or Crystal itself, it may equal the Weight of all the fore-mention'd Things: If then the Cylinder breaks, we may most certainly affirm, that the sole Cause of the Connection of the Parts of the Marble or Crystal, is that fetch'd from a *Vacuum*. But if this Cylinder does not break, but to break it, there's requir'd a Weight four times as great as the former, we may most certainly conclude, that the Resistance of a *Vacuum* is one Part of 5; and that there is another Resistance to Fraction 4 times as powerful, still to be accounted for.

SIMP. I can't deny but the Invention is an ingenious one; but yet I can't but think it subject to many Difficulties, which make me in doubt about it; for who can assure us that the Air can't penetrate between the Glass Cylinder ABCD and the wooden one EFGH, tho' stop'd as close as you can with Tow, or some other plyant Matter? and I question whether the Cone (I) will stop the Hollow made to receive it sufficiently close, altho' you wax it, or use any other Cement: And again, why may not the Particles of Water distend and rarefy themselves? and why may not the Air, or Exhalations, or some other more subtile Matter, penetrate thro' the Pores of the Wood, or even Glass itself?

SALV. These Difficulties *Simplicius* starts, are not improper. And himself indeed in some measure helps us to a Remedy, as to what relates to the Penetration of the
Air

Air thro' the Wood, or between the Wood and the Glas. But I observe farther, that allowing the Difficulties urg'd to take Place, yet we at the same time may gain some new Knowledge : Because, if Water be of that Nature as to admit of Extension (tho' by Force) as Air does, we shall see the Stopper to descend. And if in the upper Part of the Glas we make a little jetting out, as V, in case any Air or other more subtile and more spirituous Matter should penetrate thro' the Pores of the Glas or Wood, it would be seen to resemble (the Water giving Way) in that Eminence V : Which Things, since they are not discover'd, we may rest satisfied that our Experiment is made with due Caution ; and thence conclude safely that the Water is not capable of Extension, nor the Glas penetrable by any Matter, tho' never so subtile.

SAGR. Well, this Discourse has taught me the Reason of an Effect that I have for a long time wonder'd at, and never till now could understand : 'Tis this ; I have observed a Cistern, wherein, to draw out the Water from it, there was made a Pump, by the Help whereof, I thought, but vainly enough, may be, might be raised the same or a greater Quantity of Water with less Labour, than by the ordinary Buckets ; this Pump had its Sucker and Valve on high, so that the Water was raised by Attraction and not by Impulse, as it is in those Pumps which work below. This drew the Water in great Plenty, whilst the Water was at a certain determinate Height in the Cistern ; below which if the Water subsided, the Pump was useless. The first Time I observed this, I thought the Engine had been broke ; I sent for a Workman to mend it, who
told

told me, that this Defect was owing to nothing but to the Water's being too low, and thence not suffering itself to be raised to such a Height; and he added, that neither Pump nor any other Machine which raises Water by Attraction, could ever raise the Water a Hair's Breadth more than 18 Cubits; and let the Pump be big or little, that this is the utmost limited Height. Now although I knew that a Rope, a wooden Staff, and an iron Rod might be so lengthen'd, that lifting them upright, they would break by their own Weight; yet I was so silly, as never till now to consider, that the same Thing would much more easily happen in a String or Thread of Water; and what else is that which is attracted within the Pump, but a Cylinder of Water, which, being more and more prolong'd, at length arrives at that Term, beyond which if it be elevated, it, like a Rope, is broke by its own excessive Weight?

SALV. 'Tis exactly as you say; and since the same Height of 18 Cubits, is the prefix'd Term to which any Quantity of Water can sustain itself, be the Pump broad or narrow, or even no thicker than a Straw; whenever we weigh the Water contain'd in 18 Cubits of Pipe, be it broad or narrow, we shall find the Value of the Resistance of a *Vacuum* in Cylinders, of any solid Matter whatsoever, of the same Thickness with the Bore of the proposed Pipes. Now from what has been said, may be shewn how easily to determine to what Length we may extend Cylinders, of any Thickness, made of Metal, Stone, Wood, or Glass, beyond which if they are extended, they cannot sustain themselves upright, but will break, being too much oppress'd with their own Weight:

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Take,

Take, for Example, a Brass Wire of any Thickness and Length, and fixing one of its Ends on high, hang more and more Weight continually to the other End, 'till at last it break; and let the greatest Weight it can sustain, be, for Example, 50 Pounds; it is manifest that 50 Pounds of Brass more than its own Weight, which we will suppose an 8th of an Ounce, drawn out into a Wire of the same Thickness with the former, would be the utmost Length this Wire would be able to sustain itself at: Then measure how long the Wire was which brake, and let it be, for Instance, one Cubit, which, since it weigh'd the 8th of an Ounce, and was able to sustain itself and 50 Pounds over, which make 4800 8ths of an Ounce; we say, that all Brass Wires, be their Thickness what it will, will be able to sustain themselves at the Length of 4801 Cubits, and no more: Wherefore, since a Brass Wire of the Length of 4801 Cubits can sustain itself, the Resistance we find depending on a *Vacuum* in respect of the other that remains, is as much as is equivalent to the Weight of a Rope of Water 18 Cubits long, and of the same Thickness with the said Brass Wire. Now since Brass is 9 times heavier than Water, the Power any Brass Wire has to resist Friction depending on a *Vacuum*, is equivalent to the Weight of two Cubits of the same Wire. And arguing and working on after this Manner, we may find the utmost Length of Threads or Wires made of any solid Matter whatsoever, at which they can sustain themselves, as also what Share a *Vacuum* has in their Resistance against Friction.

SAGR. It

SAGR. It remains now that you explain to us in what consists the Remainder of that Resistance against Fraction, or of that Tenacity; *i. e.* What that Cement is which connecteth the Parts of a Solid; for I can't conceive what it can be that can't be burnt or consum'd in a very hot Furnace in a Month's Time, no not in 10 nor 100 Months: For if Gold, Silver, or Glass continue so long melted or liquified, take them but out of the Furnace, and the Parts, as they cool, reunite, and are conjoin'd as before; so that the same Difficulty arises in relation to this Cement, as did in relation to the Connection of the Parts of the Glass, *i. e.* I can't imagine what it should be that so closely connects its Parts together.

SALV. I myself have labour'd under, and entertain'd the self-same Difficulties; but since the Exclusion of a *Vacuum*, without all doubt, is that which forbids the Separation of the two Plates, as also of the two great Pieces of the Pillar of Marble or Brass, without great Force: Since this is so, I say, I can't see why it should not in like Manner take Place, and be the Cause of the Coherence of the lesser Parts, nay, of the most minute Particles of the same Matter: And since that of one Effect, there is but one only true and most potent Cause, if I can find no other Cement, why may I not enquire whether that which is deduc'd from a *Vacuum*, which really exists, be not sufficient?

SIMP. But since you have already demonstrated the Resistance of the greater *Vacuum*, in the Separation of the

two great Parts of a Solid, to be but very small in Comparison of that which holds together its least Particles, why don't you rest assured that this is of a far different Nature from that ?

SALV. To this *Sagredus* answers ; Altho' a Million of Gold suffices not to pay a whole Army, that yet each single Soldier may be paid with Monies collected from the general Tax or Subsidy of Shillings and Pence. And who knows for certain whether other the smallest *Vacuums* may not operate in like Manner amongst the most minute Particles, whereby these Parts are connected and held together ? I'll tell you what's just come in my Head, tho' not as an absolute Truth, but as a Sort of an indigested Conjecture, submitting it to be more exactly weigh'd and consider'd of : Take what you like of it, and judge of the rest as you please. When I have sometimes consider'd how Fire, penetrating between the minutest Particles of this or that most solid Metal, at length separates and divides them ; and how afterwards, the Fire being taken away, they are again as firmly connected as they were before, without losing any thing of their Quantity, if in Gold, and but very little in other Metals, tho' they continue a long time melted ; I have thought that this might happen from the most subtile Particles of Fire, penetrating thro' the narrow Pores of the Metals, (thro' which, by Reason of their Smallness, neither the least Particles of Air, nor of other Fluids can enter) and filling up the most minute interposed Vacuities, might free the smallest Particles of the same, from that Violence wherewith the same Vacuities attract them, and hinder their Separation : And thus,

thus, being able to move freely their Mass, may become fluid, and so continue as long as the Fire continues betwixt them : but this departing, and leaving the former Vacuities, their former Attraction may return, and consequently the Cohesion of their Parts. And as to what *Simplicius* alledges, it may be thus, I think, answer'd ; Altho' those Vacuities were most minute, and consequently each of them easy to be overcome, that nevertheless their innumerable Multitude (if I may so speak) multiplies the Resistances innumeraibly : And we have an evident Proof what, and how great the Power is, that results from the Conjunction of an immense Number of the weakest Moments, when we see the Weight of many thousands of Pounds held by very great Cables, to yield, to be overcome and lifted by the Assault of the innumerable Atoms or Particles of Water ; which (either driven by the South Wind, or else by being distended into a very thin Mist, move to and fro in the Air) insinuate themselves between the Threads of the hardest twisted Cables : Nor can the immense Force of the Weight which hangs to it, hinder their Entrance ; so that perforating betwixt the narrow Passages, they swell the Ropes, and by Consequence shorten them, whereupon the weighty Mass is forcibly rais'd.

SAGR. There's no Doubt but any Resistance, provided it be not infinite, may be overcome by a Multitude of the minutest Powers ; insomuch that even a Number of Ants, may be supposed able to carry on Shore a Ship Load of Corn ; for we see by daily Examples, that one Ant can easily carry a single Grain : and it's plain that
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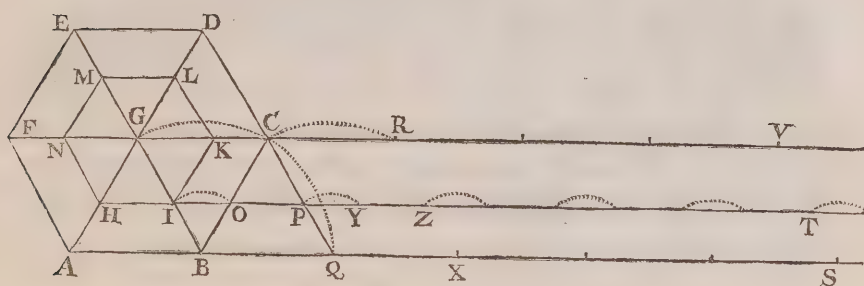
the Grains in the Ship are not infinite, but make up only some certain limited Number, than which another Number may be taken 4 or 6 times bigger, and a Number of Ants may be supposed equal to this last Number, and these, if they all set to work together, will be able to carry both Ship and Lading. It's true, indeed, the Number must be great, as, in my Judgment, those Vacuities are, which hold together the smallest Particles of Metals.

SALV. What if these Vacuities should be required to be infinite in Number? But may be this you think impossible.

SAGR. Not if the Mass of Metal was infinite: But otherwise —

SALV. Otherwise, what? But now we are fallen upon Paradoxes, let's see whether we can any way demonstrate, that infinite Vacuities may possibly be in a finite Extension; and by endeavouring this, tho' it fail, yet we shall, at least, obtain a Solution of a most wonderful Problem, proposed by *Aristotle* amongst those he himself calls Admirable; I mean, amongst his Mechanical Questions, the Solution whereof may possibly be as plain and conclusive, as that he himself gives us, and different from that which the learned Monsig. *di Guevara* with great Subtilty discusseth. But I must first give you a Proposition not touch'd by others, on which depends the Solution of the Question, which afterwards, if I am not mistaken, will draw after it other new and admirable Notions; for your better understanding whereof, take this Scheme: Conceive
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an equilateral and equiangled Polygon, of any Number of Sides, to be describ'd about this Centre G ; for Example, an Hexagon, as $ABCDEF$; similar to which, and concentric with the same, describe another lesser Hexagon, as $HIKLMN$; and let one Side AB of the greater be indeterminately produc'd towards S , and let the correspondent Side HI of the less be produc'd the same Way, by drawing

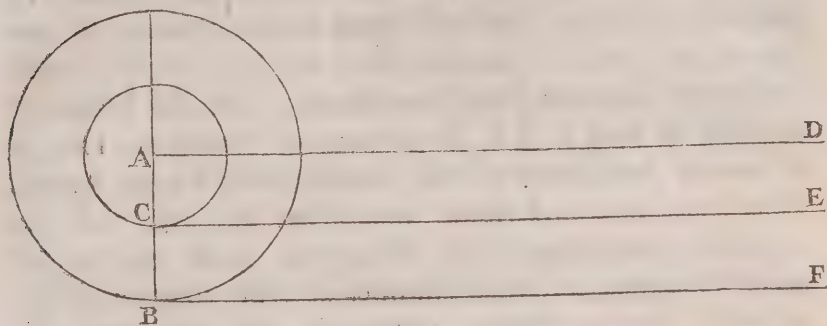


the Line HT parallel to AS , to which, thro' the Centre, draw the Line GV parallel. This done, suppose the greater Polygon to turn about upon the Line AS , and to carry with it the other lesser Polygon; it is manifest, that the Point B , the End of the Side AB remaining fix'd in the Beginning of the Revolution, the Angle A must be rais'd, and, on the contrary, the Point C will descend, describing the Arch CQ ; so that the Side BC falls upon the Line BQ equal itself. But in this Conversion, the Angle (I) of the lesser Polygon will be lifted up above the Line IT , because the Line IB is oblique to AS ; nor will the Point (I) return to the Parallel IT , till the Point C comes to Q : And then the Point (I) falls on O , having describ'd IO without the Line HT ; and the Side IK falls

at

Parts of the Parallel HT, not touch'd by the lesser Polygon in its Revolution. And lastly, the Centre G hath never touch'd the Parallel GV, except in six Points: From all which you may understand how the Space run over by the lesser Polygon, is almost equal to that run over by the greater; I mean if you take the Line HT, together with the Spaces subtending the five Arches; *i. e.* the Line HT is nearly as long as AS, for it wants of it only the Length of one of the Chords of the 'fore-mention'd skipt Arches. Now you must understand that what has been said and explain'd concerning these Hexagons, holds true in all other Polygons, of what Number of Sides soever they be, provided they are similar, concentric, and conjoyn'd; so that the greater revolving, the other, how much less soever it be, be supposed to revolve with it: You must understand, I say, that the Lines describ'd by them are very nearly equal, if, into that run over by the Perimeter of the lesser Polygon, you take the Intervals under the little Arches, not touch'd by any Part of that Perimeter. Imagine now, a greater Polygon, of a thousand Sides, suppose, to revolve, and consequently to measure out a right Line, equal to its Perimeter; the other lesser one will, at the same Time, pass over a Line very nearly equal to the former; but a discontinu'd one, being compos'd of a thousand small Parts, equal to its thousand Sides, together with the interpos'd thousand void Spaces, for such we may call them, in respect to the thousand little Lines touch'd by the Sides of the Polygon:

What I have hitherto said carries no Difficulty with it, nor admits of any Doubt. But now, if about any Centre,



as suppose the Point A, two concentric and conjoin'd Circles be describ'd, and from the Points C and B be drawn the Tangents CE and BF, and from the Centre A the Parallel AD, and the greater Circle be imagin'd to revolve upon the Line BF, (which, as are the other two CE and AD, is suppos'd equal to its Circumference) pray tell me, when the greater Circle shall have perform'd one entire Revolution, what the lesser Circle, and what the Centre shall have done? The Centre shall doubtless have run over, and touch'd the whole Line AD, and the Circumference of the lesser Circle, shall, with its Touches, have measured the whole Line CE, after the same Manner as the above-mention'd Polygons did, differing only in this, that the Line HT was not touch'd, in all its Parts, by the Perimeter of the lesser Polygon, but there were as many Parts left untouch'd, as there were touch'd by the Sides, occasion'd by the interpos'd skipt Spaces : but here in the

Circles,

Circles, the Circumference of the lesser Circle never departs from the Line CE, so as to leave any of its Parts un-touch'd ; nor can there be any Touch made by the Circumference, which is not made in that right Line. Now the great Question is, how is it possible that the lesser Circle should, without Skips, in one Revolution, run over a Line so much longer than its own Circumference ?

SAGR. I was considering whether it mayn't be said, that as only the Centre of the Circle is drawn along the Line AD, since its whole Contact is nothing but a Point, so likewise each Point of the lesser Circumference, drawn by the Motion of the greater, might not only touch the Line CE in one single Point, but might move over some small Part of the Line CE.

SALV. This can't be, on a double Account ; first, Because there's no more Reason why one of those like Touches (as, for Instance, C) should move or glide along any Part of the Line CE, and the others not : and tho' it was so, yet since those Touches (being Points) are infinite, those Tracts or Glidings would be also infinite, and being so they would make an infinite Line ; but, on the contrary, the Line CE here made, is finite. The second Reason is this ; Since the greater Circle, in its Revolution, continually changes its Contact, the lesser Circle must needs do the like, there being no other Point but B, from which a right Line can be drawn to the Centre A, to pass thro' the Point C ; so that when the greater Circumference changes Contact, the lesser also does so too : nor can any Point of the less touch more than one Point of its right

Line CE ; because in the Conversion of the Polygons, no Point of the Perimeter of the lesser falls on more than one Point of that Line, which was traced by its Perimeter ; as may be easily understood, considering that the Line IK is parallel to BC ; whence, so long as BC keeps from falling upon BQ, the right Line IK will be above IP, nor will it fall upon it, till the very Moment the Line BC coincides with BQ : at which Time the whole Line IK also coincides with OP, and then immediately it again rises above it.

SAGR. This is really very intricate ; nor can I think of any Solution of it ; therefore pray explain it to us.

SALV. To do this, I should have Recourse to the Consideration of the fore-mention'd Polygons, the Effect of which is intelligible, and already known : Thus, as in the Polygons of an hundred thousand Sides, the Lines run thro' and measur'd by the Perimeter of the greater Polygon, that is, by its hundred thousand Sides directly and continually extended, is equal, or very nearly so, to that which is measur'd by the hundred thousand Sides of the lesser, together with the interjacent hundred thousand void Spaces : So I would say in the Circles, (which are Polygons of an infinite Number of Sides) that the Line run over by the infinite Number of Sides of the greater Circle, contiguous to one another, is equal to the Line run over by the infinite Number of Sides of the lesser Circle, together with as many interposed Vacuities betwixt them. And as the Sides, making up the Circumferences of these Circles, are not quantitative, but infinite, so also
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the interposed Vacuities are not quantitative, but infinite ; *i. e.* the former Line is made up of infinite Points conjoin'd, the latter of infinite Points discontinu'd or disjoint'd. And here I would have it observ'd, how any Line resolv'd and divided into quantitative Parts, and consequently of a finite Number, can't possibly be dispos'd into a greater Extension than that which it possess'd whilst the Parts were continued and connected, without the Interposition of a like Number of void Spaces ; but imagining it to be resolv'd into Parts not quantitative, *i. e.* into its infinite Indivisibles, we may conceive it immensely produc'd, not with the Interposition of quantitative, but of infinite indivisible Vacuities. And what we have said of simple Lines, may be understood as said also of Surfaces and of Solids, considering them compos'd of infinite non-quantitative Particles. These, indeed, if we suppose them divided into quantitative Parts, it's most certain we cannot dispose them farther, or into larger Spaces than what the Solid occupied before such Division, except there be an Interposition of void Spaces ; void, I say, at least of the Matter of the Solid. But if we imagine an extreme and ultimate Resolution to be made, that is, the Body to be divided into its infinite non-quantitative first-compounding Parts or Principles, we may easily conceive such compounding Parts to be extended unto, or spread thro' an immense Space, without the Interposition of quantitative void Spaces, but only of infinite non-quantitative Vacuities. And thus, *v. g.* a very small Ball of Gold may be drawn out to a vast Expansion, without admitting any quantitative void Spaces, provided the Gold be supposed compos'd of its infinite Indivisibles.

SIMP. Me-

SIMP. Methinks what you say has a direct Tendency to the disseminated Vacuities of a *certain antient Philosopher*.

SALV. But you add not, *who denied a Divine Providence*; as, on such another Occasion, a certain Adversary to our Academic subjoin'd, tho' very little to the Purpose.

SIMP. I see very well, and not without Indignation, the Envy of such a malicious Opponent: But I'll say no more, not only on the Score of good Manners, but because I know how disagreeable such Tenets are to the well-temper'd and well-dispos'd Mind of a Person so pious, religious, very orthodox, and holy as you, Sir, are. But to return to our Purpose: I find many Difficulties to arise about your last Discourse, and how to get clear of them I know not. And the first of them is this: If the Circumferences of two Circles be equal to the two right Lines CE and BF, this continued, and that interrupted by the Interposition of infinite void Points, how then can the right Line AD, describ'd by the Centre, which is only a Point, be said to be equal to the same, and how can it contain infinite Points? Again, the composing a Line of Points, a Divisible of Indivisibles, a Quantitative of Non-quantitatives, are all too great Difficulties for me to get over; and the very Necessity of admitting a *Vacuum*, so thoroughly confuted by *Aristotle*, is no less puzzling than the rest.

SALV. These

SALV. These you have mention'd, and some others I could name, are, indeed, Difficulties : But let us remember, that we are amongst Infinites and Indivisibles, the former incomprehensible by our finite Understandings, on Account of their Greatness, and the latter on Account of their Minuteness : but yet we see that human Reasoning will find out some Way, tho' it be a round-about one, of considering them : wherefore, I will take the Liberty to give you a Conceit of my own, which, if nothing be necessarily concluded therefrom, yet, on Account of its Novelty, it will at least excite Admiration in us. But maybe so long a Digression may appear to you unseasonable and impertinent.

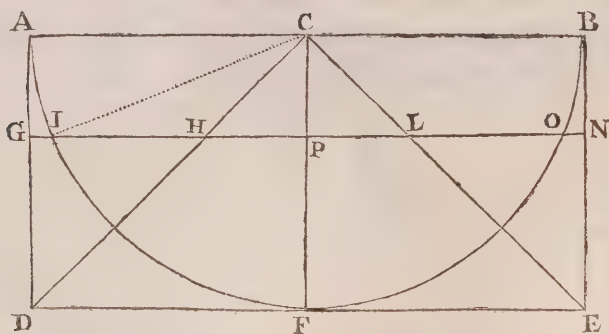
SAGR. Pray let us reap the Benefit and Privilege arising from talking freely with our living Friends, especially concerning Things arbitrary, and of our own chusing, which is vastly different from reading dead Authors, who start a thousand Difficulties, and solve not one of them. Make us therefore Partakers of those Considerations, which the Course of our Conferences suggest to you, since we want not Time, being disengaged from other urgent Business, to continue and discuss the Thing you've just hinted to us ; and pray endeavour in a particular Manner to clear those Doubts which *Simplicius* has offer'd.

SALV. Be it so then, since you'll have it : And to begin with the first of them, *viz.* how it's possible to imagine one single Point equal to a Line : Now as to this, since I can do no more at present, I will endeavour to explain
one

one Improbability, or at least to soften it, by another like it, or by a greater ; as sometimes a Wonder is qualify'd or lessen'd by a Miracle. And this I will do, by shewing you, that if there be two equal Surfaces, and at the same time two equal Bodies plac'd upon these Surfaces, as their Bases ; and these be imagin'd continually to move, so that both Surfaces and Bodies be equally diminish'd in the same Time, and their Remainders be always equal between themselves : at length, I say, these Surfaces, as also the Solids, will terminate this their perpetual Equality, *viz.* one of the Solids with one of the Surfaces, in a very long Line, and the other Solid with the other Surface, in a single Point, *i. e.* these two in a single Point, and the other in Infinity.

SAGR. An admirable Proposition this ! pray explain and demonstrate it to us.

SALV. I must draw you a Figure then, because the Demonstration is purely Geometrical. Conceive a Semicircle



AFB, whose Centre is C ; about this Semicircle describe a right-angled Parallelogram ADEB, and from the Centre
to

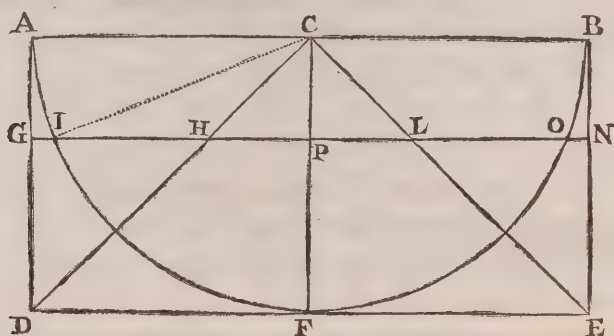
to the Points D, E, draw the right Lines CD and CE, as also the Radius CF, perpendicular to AB or DE, imagine this Radius CF to be immoveable, and about it this whole Figure to turn round : Then it's manifest, that by the Revolution of the Rectangle ADEB, a Cylinder is form'd, by that of the Semicircle AFB a Hemisphere ; and by that of the Triangle CDE a Cone. These presuppos'd, imagine the Hemisphere to be taken away, in such manner that there remains the Cone, and what is left of the Cylinder, which, because it is of the Figure of a Dish, we'll hereafter call *the Dish* : and this we'll prove in the first Place to be equal to the Cone : And then a Plane being drawn parallel to the Circle, which is the Base of the Dish, whose Diameter is the Line DE, and whose Centre is F ; then I will demonstrate that such a Plane passing by, *v. g.* the Line HG, by cutting the Dish in the Points G, I, O, N, and the Cone in the Points H, L, will cut off a Part of the Cone CHL always equal to a Part of the Dish, whose Profile is represented to us by the Δ 's GAI and BON ; and we will further prove, that the Base of the same Cone (that is, the Circle whose Diameter is HL) is equal to that circular Superficies, which is the Base of Part of the Dish, which is, as I may say, a Rim as Broad as GI. (Note here, by the way, what Mathematical Definitions are, *viz.* nought else but an Imposition of Names, or, if you will, Abbreviations of Speech, ordained and introduced to prevent that tedious Prolixity which I and you meet with, merely for not having before agreed to call, *v. g.* this circular Superficies, a *circular Rim*, and that very acute Solid of the Dish, a *round Razor*.) But now call them by what Name you please,

the Diminution of these two Solids, they, to the very last, keep their Equality to each other, I may, I think, assert, that the highest and ultimate Terms of such Diminutions are equal, and not one infinitely bigger than the other : And if so, then the Circumference of an immense Circle may be said to be equal to a single Point ; and this that happens in the two Solids, happens also in the two Surfaces, the Bases of these Solids, *i. e.* these also in their common Diminution always preserving an Equality at the Instant of their ultimate Diminution, end, the one (*i. e.* what we before call'd the Rim) in the Circumference of a Circle, and the other (*i. e.* the Base of the Cone) in a single Point ; And why may not these (*viz.* the Circumference of a Circle and a single Point) be call'd equal, since they are the last Remainders, Tracts, Appearances, or Footsteps left by equal Magnitudes ? And pray note farther, that were the Vessels as large as the Celestial Hemispheres, their uppermost Rims and the Extremities of the contain'd Cones always preserving an Equality between each other, would, after the very same manner, determine the former in Circumferences equal to those of the greatest Circles of the Celestial Orbs, and the latter in single Points ; whence methinks the Circumferences of Circles, however unequal among themselves, may be said to be equal to one another, and each of them equal to a simple Point.

SAGR. This Speculation seems to me so curious, that I would not contradict it, methinks, tho' I could ; for I take it for a kind of Sacrilege to deface so fine a Structure, or to trace, pursue, or examine it with any pedantick Remarks. But, however, for entire Satisfaction, pray give us

the Demonstration, which you say is Geometrical, of the continual Equality of these two Solids, as also of the two Surfaces, their Bases, which I fancy must be extremely subtile ; the Philosophical Contemplation depending on such a Conclusion, being so vastly subtile and pleasant.

SALV. The Demonstration is short and easy : Let us resume the foregoing Figure, in which the $\angle$ IPC being right, the Square of the Semidiameter IC is



equal to the two Squares of the Sides IP, PC. But the Semidiameter IC is equal to AC, and this to GP ; and CP is also equal to PH : therefore the Square of the Line GP is equal to the two Squares of IP and PH ; and its Quadruple to the Quadruple of these ; *i. e.* the Square of the Diameter GN is equal to the two Squares of IO and of HL. And because Circles are one to another as the Square of their Diameters, the Circle, whose Diameter is GN, will be equal to the two Circles, whose Diameters are IO, HL ; take away the common Circle, whose Diameter is IO, and the Remainder of the Circle, whose Diameter

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is GN, or which before we call'd the Rim, will be equal to the Circle, whose Diameter is HL : so much for the first Part. And as for the latter, we'll omit its Demonstration, as well because the *Archimedes* of our Age, Signeur *Lucas Valerius*, who made use of it upon another Occasion, has given it us in the 12th Proposition of his 2d Book *De Centro Gravitatis Solidorum*, as because in our Case it sufficeth to have seen that the 'foremention'd Surfaces are constantly equal, and that they continually diminishing equally, at length terminate one in a single Point, and the other in the Circumference of the greatest Circle ; for this was the Consequence which rais'd our Wonder.

SAGR. The Demonstration is as ingenious as the Reflection grounded upon it is wonderful. But now pray, if any thing particular occurs to you concerning the other Difficulty started by *Simplicius*, let's hear it. But I hardly believe this possible, it being a Controversy already so canvass'd.

SALV. I will give you my particular Sentiments of that too ; but first let me repeat what I just now told you, namely, that Infinity, as also Indivisibility, from their very Nature, are Things incomprehensible to us : What must they be then when join'd together ? And certainly if we would compose a Line of Indivisible Points, you must make them Infinite ; and thus we must at the same time apprehend both Infinite and Indivisible. Many Things on this Occasion have often come into my Mind, some of which, and those indeed of the greatest Moment,
I can't

I can't perhaps recollect on a sudden ; but in the Progress of our Discourse it may happen, as you both, but especially *Simplicius*, shall find Occasion of starting Objections and Difficulties, that they may come into my Head, which otherwise might possibly be quite lost : Wherefore, with my usual Freedom, give me leave to produce wild Notions, and my own Conjectures, for such may we justly call them, in Comparison of supernatural Doctrines, the only true and certain Determiners of our Controversies, and unerring Guides in our obscure and doubtful Paths, or rather Labyrinths. Amongst the chief Instances usually produced against such as compose a continued Quantity of Indivisibles, this is one, That one Indivisible added to another Indivisible, can't produce a Thing Divisible : For if this could be, it would follow, that that very Indivisible must be Divisible : wherefore, if two Indivisibles, as, *e. g.* two Points conjoin'd, should make a Quantity that should be a divisible Line, much more any Line would become such which is composed of 3, 5, 7, or any other odd Number of Points : Whence, since these Lines may be cut into two equal Parts, the middle indivisible Point of each will become divisible. As to these and other Objections of this kind, we may satisfy the Proposer of them, by telling him, that not only not two Indivisibles, but not 10, nor 100, nor 1000, can compose a divisible Quantity ; but being Infinite they may.

SIMP. Here already arises a Doubt which I think is not to be resolv'd ; and that is this : Since 'tis plain that one Line is given greater than another, and since both contain infinite Points, we must surely necessarily infer, that we

have

have found in the same Species something greater than Infinite, since the Infinity of Points of the greater Line exceeds the Infinity of Points of the lesser. But now, to assign an Infinite greater than an Infinite, is what I can't possibly conceive.

SALV. These are some of those Difficulties which arise from Discourses which our finite Understanding makes about Infinites, by ascribing to them Attributes which we give to Things finite and terminate, which I think most improper, because those Attributes of Majority, Minority, and Equality, agree not with Infinities, of which we can't say that one is greater than, less than, or equal to another. For Proof whereof I have something come into my Head, which (that I may be the better understood) I will propose by way of Interrogatories to *Simplicius*, who started the Difficulty. To begin then: I suppose you know which are square Numbers, and which not?

SIMP. I know very well that a square Number is that which arises from the Multiplication of any Number into itself: thus 4 and 9 are square Numbers, that arising from 2, and this from 3, multiplied by themselves.

SALV. Very well; And you also know, that as the Products are call'd Squares, the Factors are call'd Roots: And that the other Numbers, which proceed not from Numbers multiplied into themselves, are not Squares. Whence taking in all Numbers, both Squares and Not Squares, if I should should say, that the Not Squares are more than the Squares, should not I be in the right?

SIMP. Most

SIMP. Most certainly.

SALV. If I go on with you then, and ask you, How many squar'd Numbers there are? you may truly answer, That there are as many as are their proper Roots, since every Square has its own Root, and every Root its own Square, and since no Square has more than one Root, nor any Root more than one Square.

SIMP. Very true.

SALV. But now, if I should ask how many Roots there are, you can't deny but there are as many as there are Numbers, since there's no Number but what's the Root to some Square. And this being granted, we may likewise affirm, that there are as many square Numbers, as there are Numbers; for there are as many Squares as there are Roots, and as many Roots as Numbers. And yet in the Beginning of this, we said, there were many more Numbers than Squares, the greater Part of Numbers being not Squares: And tho' the Number of Squares decreases in a greater Proportion, as we go on to bigger Numbers, for count to an Hundred you'll find 10 Squares, *viz.* 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, which is the same as to say the 10th Part are Squares; in Ten thousand only the 100th Part are Squares; in a Million only the 1000th: And yet in an infinite Number, if we can but comprehend it, we may say the Squares are as many as all the Numbers taken together.

SAGR. What

SAGR. What must be determin'd then in this Case?

SALV. I see no other way, but by saying that all Numbers are infinite; Squares are Infinite, their Roots Infinite, and that the Number of Squares is not less than the Number of Numbers, nor this less than that: and then by concluding that the Attributes or Terms of Equality, Majority, and Minority, have no Place in Infinites, but are confin'd to terminate Quantities. And therefore when *Simplicius* proposes many unequal Lines, and asks me how it can be that there are not more Points in the greater than in the less? I answer, That in them there are neither more nor less, nor just so many, but that the Points in each are infinite: Or certainly, if I had answer'd how that in one Line there are as many Points as there are squar'd Numbers; in a greater, that there are as many as there are Numbers; and in the least, that there are as many Points as there are cub'd Numbers; might not I, think you, have satisfy'd him by assigning more to one than to another, and yet to every one an infinite Number? Thus much to the first Difficulty.

SAGR. Hold a little, pray, and give me leave to add a Thought of my own, just come into my Head; 'tis this; That, granting what you have said, methinks 'tis not only improper to say that one Infinite is greater than another Infinite, but also that an Infinite is greater than a Finite; for if an infinite Number be greater, v. g. than a Million, then it would follow, that, going on from a Million to other Numbers, and so to others continually greater, we

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at length should attain to Infinity : but now the contrary of this appears, for to how much greater Numbers we proceed, so far we recede from Infinite, because by how much the greater the Numbers are taken, so much the fewer will the Squares be comprehended in them ; but in an infinite Number there can't be fewer than there are Numbers, as just now concluded ; and therefore the approaching to Numbers bigger and bigger, is going back from infinite Number.

SALV. And so from your ingenious Discourse we may conclude, that the Attributes of greater, less, or equal, have no Place, not only among Infinites, but also among Infinites and Finites. I pass on now to another Consideration ; and that's this : Since it now is certain that a Line, and every continu'd Quantity, may be divided into Parts always farther divisible, I don't see how we can avoid granting it to be compos'd of infinite Indivisibles ; because a Division and Subdivision, that may be prosecuted perpetually, supposes that the Parts are infinite, since otherwise the Subdivision would at length be terminated. And since the Parts are infinite, we must conclude them non-quantitative, since infinite Quantitatives make an infinite Extension : And thus now we have a continued Quantity compos'd of infinite Indivisibles.

SIMP. But if we may continually prosecute the Division into quantitative Parts, why are we oblig'd, in that respect, to introduce Non-quantitatives ?

SALV. The

SALV. The very Possibility of continually prosecuting the Division into quantitative Parts, introduces the Necessity of a Composition of infinite Non-quantitatives. Wherefore to come closer to you, pray tell me directly whether the quantitative Parts of a continu'd Quantity are, in your Opinion, Finite or Infinite ?

SIMP. They are both Infinite and Finite ; Infinite in Power, and Finite in Act. They are Infinite in Power, *i. e.* before Division ; but Finite in Act, *i. e.* after they are divided ; because the Parts are not actually conceiv'd to be in the Whole, 'till it is divided, or at least till its Parts are continually mark'd, for otherwise they are said to be in Power.

SALV. So that a Line, *v. g.* 20 Foot long, can't be said to contain actually 20 Lines of one Foot a-piece, 'till it is divided into 20 equal Parts : But before Division, 'tis said to contain these Parts only in Power. Be it so as you would have it ; and now pray tell me whether this actual Division into such Parts being made, whether, I say, that first Whole is encreased or diminished, or whether it continueth of the same Bigness.

SIMP. It is neither encreased nor diminished.

SALV. So think I too : Therefore the quantitative Part in a continued Quantity, whether in Act or Power, neither augment nor diminish its Quantity : But 'tis manifest, that the quantitative Parts actually contain'd in their

Whole, if they be infinite, make it an infinite Magnitude; and therefore quantitative Parts, tho' infinite only in Power, can't be contain'd but in an infinite Magnitude: And therefore in a finite Magnitude, infinite quantitative Parts cannot be contain'd either in Act or in Power.

SAGR. How then can it be true, that a continu'd Quantity may be continually divided into Parts still capable of new Divisions?

SALV. That Distinction of Act and Powers seems to make that possible one Way, which another Way would be impossible: But I will endeavour to reconcile these Matters by some other Method. To the Question then, *viz.* Whether the quantitative Parts in a terminated continued Quantity, are finite or infinite? And to this I answer directly, and quite contrary to *Simplicius's* Answer, That they are neither finite nor infinite.

SIMP. I never should have thought of such an Answer truly, having never so much as dreamt that there was any mean Term between Finite and Infinite: So then that Division or Distinction which supposes any thing to be either finite or infinite, is lame and deficient.

SALV. In my Opinion it is; and if we talk of discrete Quantities, there seems to me to be a third mean Term between finite Ones and infinite, and this is agreeable to every assign'd Number. Thus, if in the present Case, it be ask'd, Whether the Parts of a continued Quantity be finite or infinite, the properest Answer is, That they are
neither

neither finite nor infinite, but such as may be agreeable, or may answer to any, or to every assign'd Number ; that they may do which, 'tis necessary, that those Parts be not comprehended within any limited Number, because then they would not answer a bigger Number. Again, 'tis required that they be not infinite, because no assign'd Number is infinite. And thus, at the Pleasure of the Demander, in a propos'd Line, we may assign it an hundred quantitative Parts, or a thousand, or a hundred thousand, or any greater Number of such Parts, provided they are not infinite. I grant therefore to the Philosophers, that a continu'd Quantity contains as many quantitative Parts, as they please, and that either in Act or in Power, as they please : But I add, that in like manner, as in a Line of 10 Yards, there are contain'd 10 Lines of one Yard apiece, and 30 Lines of a Foot apiece, and 360 Lines of an Inch apiece ; so it contains infinite Points, call it in Act or in Power, as you like best : And in this Particular I refer myself to your Opinion and Judgment, *Simplicius*.

SIMP. Why, I can't but commend your Discourse ; but yet I much fear that your concluding that Points are contain'd in a Line as quantitative Parts, comes up to the Hypothesis of indivisible Points. And that you can't so easily divide a propos'd Line into infinite Points, as those Philosophers can into 10 Yards, or 30 Feet : Nay, I don't think it possible to effect such a Division ; wherefore this will be one of those Powers which can never be reduc'd to Act.

SALV. Tho' a thing can't be done without Pains and Diligence, or under a long Time, yet this renders it not impossible. You can't easily, I dare say, divide a Line into 1000 Parts; and if you are to divide it into 937, or some other great prime Number, you'll still find it more difficult. But now, if I should dispatch that Division, which you believe impossible, in as short a time as another would take to divide it into 40, you'll the easier admit it, I hope, in our future Discourse.

SIMP. I can't say but I am pleas'd with the Method you talk of; and to your Question I reply, That I should think it more than sufficiently easy, if a Line could be as readily divided into Points, as into a thousand Parts.

SALV. I'll tell you a Thing that perhaps will startle you, at least whilst you presuppose that you would, or are able to, resolve a Line into its Infinites, by the Method others observe in dividing it into 40, 60, or 100 Parts; *i. e.* by dividing it first into 2, and then into 4, &c. For he that by such a Process proposes to divide this Line into its infinite Points, would be greatly deceiv'd; for by this Progress, tho' he goes on to Eternity, he would never arrive at the Division of all its quantitative Parts; and he is so far from ever arriving at the wanted Term of Indivisibility by these Means, that, on the contrary, he goes farther and farther from it: And whilst by continually repeating the Division, and by multiplying the Multitude of the Parts, he thinks he is approaching to Infinity; I, for my Part, think that he recedes farther and farther from it; and

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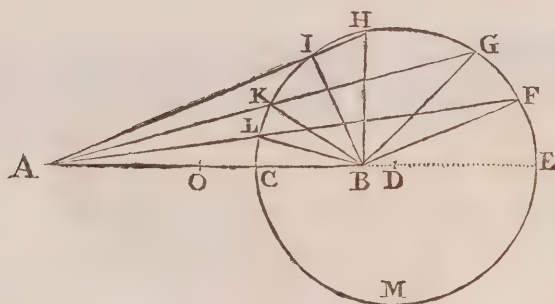
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my Reason is this : In the Discourse we had just now, we concluded, That in an infinite Number there was of Necessity as many Squares, or as many Cubes, as there are Numbers in all, because both those and these are as many as their Roots are, and their Roots are as many as there are Numbers in all. After this we saw, that the greater the Numbers were taken, the fewer Squares, and still the fewer Cubes, were found in them ; and thence 'tis manifest, the greater Number you arrive at, the farther you recede from infinite Number : whence it follows, That by going Backwards, (since the 'foresaid Progression moves us farther and farther from the desired Term) if any Number may be said to be infinite, it is an Unit ; and, indeed, in the Unit there are all the Conditions and necessary Requisites of an infinite Number ; I mean thus, it contains in it as many Squares, or as many Cubes, as there are Numbers.

SIMP. I don't understand this.

SALV. There's no Difficulty at all in it, for the Unit is a Square, a Cube, a squar'd Square, and every other Power : Nor is there any Particular whatsoever essential to the Square, or to the Cube, which is not found in the Unit : Thus, *v. g.* 'tis a Property of two square Numbers to have between them a mean Proportional : Take any square Number for one Term, and the Unit for the other, and you'll always find a mean Proportional between them. Let two square Numbers be 9 and 4 ; then between 9 and 1 the mean Proportional is 3 ; and between 4 and 1 it is 2 ; and between 9 and 4 it is 6. And
again,

again, 'tis a Property of Cubes to have necessarily between them 2 mean Proportionals. Suppose two Cubes be 8 and 27, then the Means between them are 12 and 18; and between 1 and 8 the Means are 2 and 4; as between 1 and 27 they are 3 and 9. Hence we conclude, that there is no infinite Number but the Unit. These now are some of those Wonders which surmount our Comprehension, and which should teach us how exceedingly they err, who, reasoning about Infinites, use the same Attributes as they do about Finites, which, in their Nature, are quite different from each other. Upon which I will not conceal from you a wonderful Accident, just come in my Mind, which will explain to you the infinite Difference, nay, the Contrariety of Nature that a terminate Quantity would incur, by passing to an infinite. Draw the right Line AB of any Length, and take in it any Point C, which may divide it into two unequal Parts; then, I say, if two Lines, keeping the same Proportion

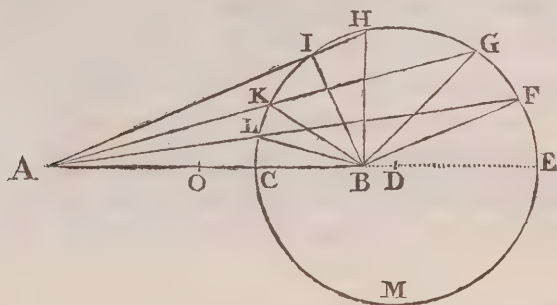


to each other, as the Parts AC and BC have, are drawn from the Extremities A and B, to meet one another, the
Points

Points of their Interfection shall fall in the Circumference of one and the same Circle : As, *e. g.* if from the Points A and B, the right Lines AL and BL are drawn, which have the same Proportion to each other, as the Parts AC and BC have, and meet in the Point L : And if other two right Lines be drawn, keeping in like manner the same Proportion, and meet in K. And so on, if others as AI and BI, AH and BH, AG and BG, AF and BF, AE and BE ; I say, that all the Points of their Intersections L, K, I, H, G, F, E, fall in the Circumference of one and the same Circle ; so that if we conceive the Point C to be so mov'd continually, that the Lines drawn from it to the fix'd Terms A and B, retain always the same Proportion as there is between the first Parts AC and CB ; then that Point C shall, by such Motion, describe the Circumference of a Circle, as I will presently demonstrate to you. And the Circle thus describ'd will be greater and greater *ad infinitum*, as the Point C is taken nearer and nearer to the middle Point O ; and less and less as the same Point C is taken nearer and nearer to the Point B, the Extremity of the said first-given-Line : So that from the infinite Points which may be taken in the right Line OB, (if they are suppos'd to move according to the fore-mention'd Law) there may be describ'd Circles of any Magnitude ; some less than the Pupil of the Eye of the least Insect, and others bigger than a great Circle of the *Primum Mobile*.

Now, since if any of the Points between the Terms O and B, be moved as aforesaid, by every one of them a Circle will be describ'd, and those immense ones, describ'd by the Points very near the Point O ; thence, if

the Point O itself be in like manner mov'd, *i. e.* so that the Lines from it to the Terms A and B, may always keep the same Proportion to each other, as the first Lines AO and OB have ; what Sort of a Line will it describe ? I say, that this Point O will also describe, by such Motion, the Circumference of a Circle, and of a Circle the greatest of all, and therefore of an infinite one : Nay, indeed, it rather describes a right Line perpendicular to AB from the Point O, and infinitely produc'd, so as never to return into itself as the others did ; as thus, the Point C, by its



limited Motion, having describ'd the upper Semi-circumference CHE, went on and describ'd the lower one EMC, returning into itself in C : But now the Point O moving to describe, as the other Points of the Line AB do, (since also the Points taken in the other Part OA describe their Circles, and since those describe the greatest which are nearest to the Point O) its Circle ; and that the greatest of all, and consequently infinite, it never can return to its first Term ; and thence, in a Word, it describes an infinite right Line for the Circumference of an infinite Circle.

Pray

Pray consider now what Difference there is between a finite Circle and an infinite, which changeth its very Being in such Sort as to lose its very Existence, nay, its very Power of existing, since we most evidently see that there can't possibly be an infinite Circle: whence we may fairly conclude, that much less can there be given in Nature an infinite Sphere, or any other Body, nor can there be an infinite Superficies made. What shall we say then to this Metamorphosis in the Transition from Finite to Infinite? And why should we meet with greater Opposition when, seeking an Infinite in Numbers, we conclude it to be in the Unit? And again, whilst we breaking any Solid to Pieces, and endeavouring to reduce it into the smallest Dust, when it is resolv'd into Particles admitting no farther Division; why may we not say, that it is return'd into one *Continuum*, but perhaps fluid, as Water is, or Mercury, or melted Metal? And do we not see Stones liquified into Glass, as also Glass itself, with a vast Heat, to become as fluid as Water?

SAGR. Must we then believe Fluids to be so call'd, because they are resolv'd into their first infinite, indivisible, composing Parts?

SALV. I don't know how better to account for many sensible Appearances, of which this is one. When I take any hard Body, for Example, a Stone, or Piece of Metal, and, with a Hammer, or very fine File, divide it, as much as possible, into the finest and most impalpable Powder, 'tis manifest that its most minute Particles, altho' they are singly so small as to escape our Sight, yet are they quantitative,

titative, figured, and numerable : And hence, if you heap them up, they continue in that Heap, or if you make a Hollow or Pit in that Heap, such Pit continues, the Particles heap'd about it not falling in so as to fill it up ; and if you shake or stir these Particles, yet as soon as the external moving Cause quits them, their Motion ceases : And the like Effects are seen in all the Aggregates bigger and bigger of small Bodies, let them have what Figure they will, nay, tho' they are Spherical, as we see in Heaps of any small round Seeds, as (Millet) Wheat, or any other Matter. But now none of these Accidents are met with in Water ; for if Water be rais'd on an Heap, it instantly returns to a Level, unless it be upheld by any Vessel or other external Stay : make this hollow, it presently runs back to fill up the Cavity ; and being mov'd it keeps a long while in Motion, and extends its Waves a vast Way.

From all which, I think, we may very reasonably conclude, that the most minute Particles of Water of which it seems to be resolv'd, (since it has less Consistence than any the finest Powder, nay, since it has no Consistence at all) are vastly different from the most minute Quantitatives and Divisibles ; and what can they be then but indivisible ? Again, I can't but think that its most exquisite Transparency affords sufficient Grounds for this Inference ; for if we take the most diaphanous Crystal that is, and beat and pound it to Powder, it will lose its Transparency, and the more, the finer 'tis made ; but now stamp or pound Water as much as you will, yet it retains its Transparency. Gold and Silver, tho' reduced by *Aqua-Fortis* into a finer Powder than any File can make, yet they

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are but Powder, they acquire no Fluidity, nor can they be liquified, unless the Fire or the Sun-beams resolve them, as I believe, into their first and highest infinite and indivisible composing Parts.

SAGR. What you say of the Sun I have often observ'd with Admiration : For Instance ; I have seen a concave *Speculum*, of a Foot Diameter, melt Lead in an Instant ; whence I can't but think, that if the *Speculum* be large and well-polish'd, and of a parabolick Figure, it would in a very short Time melt any other Metal, since I have seen the 'fore-mention'd one, which was but small, not very well polish'd, and a spherical Concave, with such Force melt Lead, and fire any other combustible Matter : Which Effects, I must own, make the Wonders reported of the *Specula* or Burning-Glasses of *Archimedes* credible to me.

SALV. As to the Effects of the *Specula* of *Archimedes*, all the Miracles met with concerning them in others, are render'd credible to me, by reading *Archimedes's* own Book, which I have with infinite Amazement perus'd and studied : And, indeed, if he had left me in any Doubt, yet that Treatise which not long since Father *Buonaventura Cavalieri* hath publish'd of Burning-Glasses, entitled, *Lo Specchio Ustorio*, which I have read with Admiration, would have answer'd all Difficulties.

SAGR. I formerly knew the Author, and have seen and read the Treatise you speak of with Pleasure and great Admiration ; and indeed it confirm'd me in the Opinion which

I before had conceiv'd of him, *viz.* That he was likely to become one of the greatest Mathematicians of the Age. But to return to the admirable Effects of the Sun-beams in melting Metals : Must we think so fierce and violent an Operation is perform'd without Motion ; or must we think it is perform'd by Motion, and that extremely swift ?

SALV. We see other Burnings, and Dissolutions, or Meltings, to be made by Motion, and that the swiftest too. Observe the Operations of Lightening, of Gun-powder in Mines, and in Pieces of Ordnance ; and lastly, we see how, by quickning with Bellows the Flame of Coals, mixt with gross and impure Vapours, its Force is increased in melting of Metals : Wherefore I can't conceive how the Action of Light, tho' most pure, can be without Motion, and that the swiftest.

SAGR. But what, and how great may we judge this Operation of Light ? Is it instantaneous, or is it like other Motions, some time in doing ? Is it not possible, by some Experiment, to settle this Matter ?

SIMP. Daily Experience shews the Expansion of Light to be instantaneous : For when a Cannon is discharged a great Distance from us, we see the Flash instantly, tho' the Report reaches us not till some time after.

SAGR. Nay, *Simplicius*, nothing can be gather'd from this common Experiment but this, *viz.* Sound is longer in reaching our Ear, than Light the Eye. It does not assure me that the Accession of Light is instantaneous ; it

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may move very swiftly, and yet be some time in reaching us, for any thing your Experiment proves to the contrary. And another like Observation proves no more than this of yours ; I mean, when we are told that as soon as the Sun reaches the Horizon, we see his Light ; for pray who can assure me, that he does not reach the Horizon sooner than we see him ?

SALV. The weak Conclusions of these, and other Observations of the like Nature, occasion'd me to think of some other Way, whereby I might be sure whether the Illumination, *i. e.* whether the Expansion of Light was really instantaneous or no ; since the Motion of Sound assures us that of a Light is extreme swift. And the Experiment I hit upon was this : Let two Persons take each of them a Light in a Lanthorn, so made that the Light may be hid from each other, by clapping the Hand before it, or by some other Means, and again discover'd at Pleasure : then let them stand at a little Distance, with the Lights against one another, and let them practise the Discovery and Occultation of their Lights, from the Sight of each other ; so that when one sees the other's Light, he immediately disclose his : In doing this mutually to each other several times, they will be so ready, that, without sensible Variation, the Moment the one opens his Light, the other shall answer him with a Discovery of the Light he holds : Having practis'd this sufficiently at a small Distance, let these two Persons, with their two Lights, go from each other the Distance of two or three Miles ; and repeating the same Experiment by Night, let them most carefully observe whether the mutual Discoveries and Occultations

cultations answer one another, according to the same Tenor which they did at that first small Distance : If they do, we may certainly enough conclude, that the Expansion of Light is instantaneous : But if it should require Time at the Distance of three Miles, which, accounting the Going of one Light, and the Return of the other, make six, that Stay must be sufficiently observable. And if this Experiment be made at a greater Distance ; for Instance, at the Distance of eight or ten Miles, each Observator may make use of a Telescope fixt in that Place, where by Night he is to make the Experiment ; and then, tho' the Lights are so small as not to be discern'd at that Distance by the bare Eye, yet, by Help of these Telescopes, they may be easily seen.

SAGR. The Experiment seems to me capable of being made as exactly as the Invention of it is ingenious : But pray tell me what you have concluded from it.

SALV. Really I have tried it only at a small Distance, less than a Mile ; whence I could not safely gather, whether the Appearance of the opposite Light was instantaneous or no ; but this I am sure of, if it was not instantaneous, it was exceedingly swift, and, I may say, momentary. I would liken it, methinks, to that Motion we see a Flash of Lightning has among the Clouds eight or ten Miles off, the Beginning of which Flash, and, I may say, the Source and Rise of it, we distinguish in a particular Place among those Clouds ; which Rise its wide Expansion immediately succeeds amongst the adjacent Places : which seems to me a strong Argument, that it is some small Time in doing :
For

For had the Illumination been made all at once, and not gradually, I can't think we could be able to distinguish its Rise, and, I may say, Centre of its flashing Rays, and Extremities of their Dilatation. But we are plunging ourselves insensibly into such an Ocean of *Vacuum*s, Infinites, Indivisibles, and instantaneous Motions, that a thousand Discourses will hardly be able to set us on Shore.

SAGR. Things indeed far exceeding the narrow Bounds of our Understanding. Whilst, pray mind, an Infinite sought amongst Numbers determines in the Unit : From Indivisibles always arises a Divisible : A *Vacuum* seems to reside no where, unless indivisibly mix'd with the *Plenum*. And, in short, the Nature of Things commonly understood by us, may be so changed, that the very Circumference of a Circle may become an infinite right Line : Which is the Proposition, if I remember aright, which you, *Salviatus*, promis'd to give us a Geometrical Demonstration of. If you please, therefore, pray give it us without any farther Digression.

SALV. I will do it : but first, by way of Preparation, I must demonstrate this Proposition.

P R O P O S I T I O N.

Any right Line being given, cut into two unequal Parts in any Proportion, to describe a Circle, that two right Lines being drawn from the Ends of the given right Line, to any Point of the Circumference of that Circle, may keep the same Proportion, as the Parts of the given Line

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have

dicular, which produc'd, let meet AB indefinitely produc'd in F. Then I say first, that FE and FC are equal; for drawing EC, in the two Triangles DEC, BEC, the two Sides of the one DE, EC, are = two Sides of the other BE and EC, since DE and EB are Tangents to the Circle DB: and the Bases DC and CB are also equal; wherefore the two Angles DEC and BEC will be equal. And since $\angle BCE$ wants of a right Angle the Quantity of the Angle CEB, and $\angle CEF$ wants of a right Angle the Quantity of $\angle CED$, and these Defects are equal to each other; therefore $\angle$'s FCE and FEC will be also equal, and so consequently are the Sides FE and FC: Wherefore if upon F as a Center with the Radius FE, a Circle be describ'd, it will pass thro' the Point C. Let this Circle be describ'd, and let it be CEG; then, I say, this is the Circle sought, that any two Lines drawn from the Terms A and B, and meeting each other in any Point of the Circumference whatsoever, will keep the same Proportion, as the two Parts AC and BC have to each other, which before did meet in the Point C. This is manifest of the two Lines, which meet in the Point E, *i. e.* of AE and BE; for in $\triangle AEB$, $\angle AEB$ is bisected by the Line CE, and thence 'tis $AC:CB::AE:BE$: For draw BN parallel to CE, cutting AE in N, then $\angle AEC$ being = $CEB = EBN = ENB$, the Line EN will be = EB; but now it is $AC:CB::AE:EN$, *g^o* $AC:CB::AE:EB$. *Q. E. D.*

I'll demonstrate the same of the two Lines AG and BG meeting each other in the Point G, because (by Reason the Triangles AFE and EFB are similar) it is $AF:FE::FE:FB$, *i. e.* $AF:FC::FC:FB$, it will be by Division $AF-FC:FC=FG::FC-FB:FB$, *i. e.* $AC:CF$

meet in any one Point, either within or without the Circle CEG. For if it be possible let two such Lines meet in L, without the Circle, and let them be AL, BL; produce LB to the Circumference in M; join MF: If then it be $AL : BL :: AC : BC$, *i. e.* $:: MF : FB$, we shall have two Triangles ALB, MFB, which have the Sides Proportional about the two Angles ALB and MFB, with $\angle$'s at the Vertex or Point B equal, and the two remaining ones FMB and LAB less than right ones, (because the right Angle at M has for its Base the whole Diameter CG, and not only Part of it BF: And the other at A is acute, because the Line AL, homologous to AC, is greater than BL, which is homologous to the Part BC) and therefore the Δ 's AB L and MB F are similar; and g^o 'tis $AB : BL :: MB : BF$, whence $\square ABF$ will be $= \square MBL$. But $\square ABF$ was prov'd $= \square CBG$, g^o $\square MBL$ is $= \square CBG$, which is impossible: therefore their meeting without the Circle can't possibly happen. In like manner it may be prov'd, that such Meeting can't happen within the Circle; and therefore all these Meetings must fall in the Circumference of the Circle.

But it is time to return, and answer *Simplicius's* Request, by shewing him that to resolve any Line into its infinite Points, is not only not impossible, but as easy as to divide it into its quantitative Parts; premising this, which I suppose, *Simplicius*, you will grant me, *viz.* you will not, I hope, require me to separate the Points one from another, and shew you them one by one distinctly upon this Paper; for I should think it sufficient, that without actually separating four or six Parts of any Line, you would shew
me

me the Divisions mark'd, or at most shew them inclin'd one to another, making of them into a Square or Hexagon : If I do this, I persuade myself you'll allow that they are sufficiently and actually distinguish'd.

SIMP. I allow it.

SALV. Very well : Since then a Line made to incline at Angles, framing thereby sometimes a Square, sometimes an Octagon, and sometimes a Polygon of forty, of an hundred, or of a thousand Sides, be a Mutation sufficient to reduce into Act these 4, 8, 40, 100, and 1000 Parts, which before, in your Opinion, were potentially in the said right Line at first : If I make of that right Line a Polygon of infinite Sides ; *i. e.* if I bend it into the Circumference of a Circle, won't you allow me to say, I have reduc'd all those infinite Parts into Act, which you, whilst it was strait, said were potentially contain'd therein ? Nor can it be denied that such a Resolution into its infinite Points is made, any more than it can, that that Line is divided into four Parts, by forming it into a Square, or into a thousand, by forming thereof a thousand-sided Figure ; since in such Resolution there wants no one Condition found in the Polygon of a thousand or of an hundred thousand Sides : Which Polygon, if apply'd to a right Line, touches it with one of its Sides, *i. e.* with one of its hundred thousand Parts : The Circle, which is a Polygon of infinite Sides, toucheth the same right Line with one of its Sides, which is one single Point different from all its Collaterals, and therefore it is divided and distinct from them, no less than the Side of a Polygon is from all its

Conterminals :

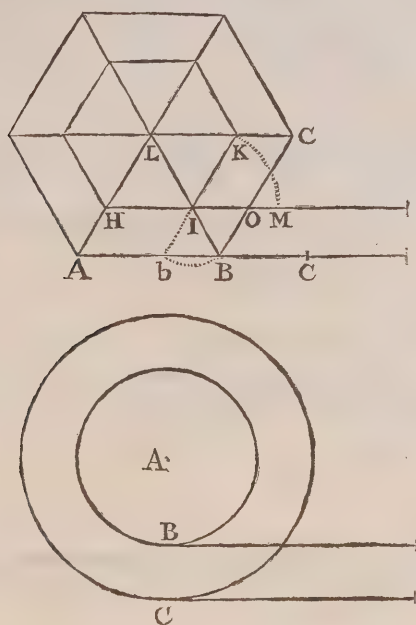
Conterminals : And as the Polygon, turn'd round upon any Plane, describes, with the successive Touches of its Sides, a right Line = its own Perimeter ; so a Circle, mov'd round upon such a Plane, describes, by its infinite successive Touches, a right Line = its Circumference. Now I know not, *Simplicius*, whether the Peripateticks, (to whom I freely grant, as a thing most certain, that a continu'd Quantity is divisible into Parts always divisible, so that continually repeating such Division and Subdivision, we shall never come to an End ; whether these, I say) will grant me, that one of those Divisions is the Ultimate, as indeed it is not, since there always remains another ; but the last and highest to be that which resolves it into infinite Indivisibles, to which, I own, we can never attain by dividing and subdividing it successively into a greater and greater Number of Parts : But making use of the Method I propose of distinguishing and resolving it by one only Tract, (an Artifice which ought not to be denied me) I should think they should be satisfy'd, and admit that a *Continuum* is composed of Particles absolutely indivisible : And especially when by Help of this Method, more than by any other, we can disengage or rid ourselves of many intricate Labyrinths, such as are, besides that already mention'd concerning the Coherence of the Parts of Solids, the comprehending the Business of Rarefaction and Condensation, without the Inconvenience of being forc'd to admit of void Spaces, and, on their Account, the Penetration of Bodies ; which are two Inconveniencies, which, in my Opinion, may be shunn'd by admitting the afore-said Composition of Indivisibles.

SIMP. I know not what the Peripateticks would say, since the Considerations you have offer'd, I believe, would appear to them mostly new, and as such they ought to examine them : And then, may be, they would find your Answers and Solutions sufficient to explain those Difficulties, which I have neither Time nor Ingenuity enough to solve. Wherefore, suspending that for the present, I would willingly hear how the Introduction of these Indivisibles facilitates the Conception of Condensation and Rarefaction, by which we may avoid, at the same time, a *Vacuum*, and the Penetration of Bodies.

SAGR. I also greatly desire to understand this, which at present is so obscure to me, but upon this Condition, that I may not be denied the Hearing the Reasons *Aristotle* brings to confute a *Vacuum*, and then those you have to offer to prove the being of one.

SALV. I will do both : And as to the first ; it's necessary that as on Account of Rarefaction, we make use of the Line describ'd by the lesser Circle, which is greater than the Circumference of that Circle, whilst it was mov'd by the greater's revolving about ; so also to our understanding of Condensation, we'll shew how the greater, if mov'd by the lesser Circle's turning about, describes a right Line less than its Circumference : For the clearer Explication of which, let us consider first what happens in the Polygons. About the common Centre L, describe two similar Hexagons, as ABCA and HIKH, with the parallel Lines HOM and ABC, upon which they are to make
their

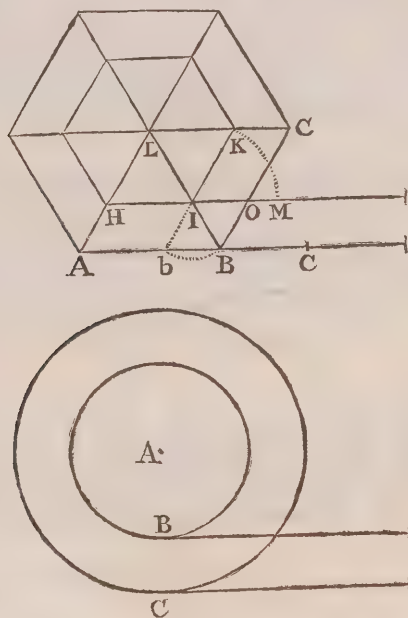
their Revolutions; and then the Angle (I) of the lesser Polygon remaining fix'd, turn about the said Polygon, till its Side IK fall on the Parallel HM, by which Motion the Point K will describe the Arch KM, and the Side IK will coincide with the Part IM; now let's see what the Side BC of the greater Polygon does in the mean time. Because the Revolution is made about the Point (I), the End B of the Line IB turning backward, will describe



the Arch Bb , under the Parallel CA ; so that when the Side IK coincides with IM , the Side BC shall fall upon b_c , advancing forwards only the Length B_c , and retiring back, the Line subtending the Arch bB , and which falls upon the Line BA : And if we imagine the Revolution of the

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lesser

leffer Polygon to be continued after the same manner, this leffer Polygon will, in turning once round, describe on its Parallel HM , a Line = its Perimeter; but the greater shall describe a Line less than its Perimeter, by the Quantity of so many times the Line Bb as the Polygon has Sides, wanting one: And that Line is very nearly = that which is describ'd by the leffer Polygon, exceeding it only by the Quantity of Bb . Here then, without any Trouble, you



see why the greater Polygon (carried by the leffer) measures not out with its Sides a longer Line than the leffer moves over, *viz.* because one Part of each Side of the greater Polygon, falls upon its precedent contiguous Side.

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But now if we consider two Circles describ'd about the Centre A, standing upon their Parallels, so that the lesser touches its Line in the Point B, and the greater its in the Point C : Here now, so soon as the less begins to revolve, the Point B will not remain, as before, for some time fix'd ; whence the Line BC continually carries backward the Point C : As it happen'd in the Polygons, where the Point (I) remaining fix'd, till the Side KI falls upon the Line IM, the Line IB carried back the End B of the Line CB to *b*, whence the Side BC fell on *b c*, the Part *b B* coinciding with the Line BA, and advancing forwards by the Part *B c* only = IM, *i. e.* to one Side of the lesser Polygon, by which Coincidencies or Applications which constitute the Excesses of the greater's Sides above the Sides of the lesser, the Advances forwards, which are = the Sides of the lesser Polygon, do compose in one whole Revolution a right Line = that trac'd and measur'd out by the Perimeter of the lesser Polygon. But now, I say, if we come to reason after this manner of the Effects of Circles, we must own, that whereas the Sides of every Polygon whatsoever, are of some limited Number, the Sides of a Circle are infinite ; those are quantitative and divisible, these non-quantitative and indivisible : Again, the Terms of the Sides of a Polygon, in its Revolution, continue fix'd for some Time, *i. e.* for such a Part of the Time of one whole Revolution, as that Side is of the whole Perimeter : But in Circles, the Stay of the Terms of their infinite Sides are momentary, since a Moment is such a Part of quantitative Time, as a Point is of a Line which contains infinite Points. The Retrogressions of the Sides of the greater Polygon are not of the whole Side,

but only of its Excess above the Side of the lesser Polygon, so that the Spaces advanc'd forwards, are only such as are the Sides of the lesser. In Circles, the Point or Side C, in the instantaneous Rest of B, recedeth but as much as is its Excess above the Side B, advancing forwards as much as is the Quantity of the same B. And thus, at length, the infinite indivisible Sides of their greater Circle, by their infinite indivisible Retrogressions, made in the infinite instantaneous Stays of the infinite Terms of the infinite Sides of the lesser Circle, and by their infinite Progresses or Advances, which are = the infinite Sides of the said lesser Circle, they compose and measure out a Line = that which the lesser Circle describes, containing in itself infinite Applications or Supra-positions non-quantitative, which, without any Penetration of quantitative Parts, make a Constipation and Condensation ; which cannot be conceiv'd to be in a Line divided into quantitative Parts, such as is the Perimeter of any Polygon, which being distended at length into a right Line, cannot be contracted to a lesser Length, unless the Sides be applied to, or fall upon, and so penetrate one another : In this Constipation of Parts not quantitative, but infinite, without the Penetration of quantitative Parts, and in the former Distraction of infinite Indivisibles, with the Interposition of indivisible Vacuities before explain'd, consists, as I believe, the most that can be said of the Condensation and Rarefaction of Bodies, without the Necessity of introducing the Penetration of Bodies, or quantitative void Spaces. If there be any thing herein that pleases you, much good may it do you ; if not, account it and my Discourse vain, and seek some other Explanation more satisfactory to you. Let

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me only remind you of this one Thing, that we are talking of Infinites and Indivisibles.

SAGR. I must confess this Invention of yours is very subtile, and to me wholly new and strange ; but whether or no Nature proceeds in this Order, I'm not able to say : But, however, until I meet with an Explanation more satisfactory, I'll content myself with this. But may be *Simplicius* may offer something (which I have not yet met with) farther to explain the Explication given us by the Philosophers on this abstruse Subject ; for, indeed, what I have hitherto read about Condensation, seems to me so dense, and that of Rarefaction so rare and subtile, that my weak Sight can neither comprehend this, nor penetrate that.

SIMP. I am full of Confusion, and find great Difficulties which Path ever I take, and more particularly in this new one : for according to this Rule, an Ounce of Gold may be rarefied and drawn out into a Mass bigger than the whole Earth ; and the whole Earth may be condensed and contracted to a less Bulk than a Nut, which I neither believe, nor think that you yourself do ; and the Reasonings and Demonstrations which you have hitherto brought, as they are abstracted and purely Mathematical, I don't believe that, when they are applied to Things Physical and Natural, these Rules will take Place.

SALV. It is not in my Power, nor, I believe, do you desire me, to make that Visible which is Invisible : But to appeal to our Senses ; do we not see an immense Extension

tenſion of Parts in Gold, ſince you have mention'd it ? I can't tell whether you have ever ſeen the Method Artificers uſe in drawing out Gold Wire, the Outſide of which is only in Reality Gold, for the inward Matter is Silver ; and the Way of ſtretching or diſgroſſing it is this : They take a cylindric Ingot, or Rod of Silver, about half a Yard long, ſuppoſe, and about three or four Inches thick ; and this they gild with Leaf-Gold, which you know is ſo thin, that the leaſt Breath of Air will blow it about ; and of theſe Leaves they lay on eight or ten thick, and no more. So ſoon as this Rod is gilt, they begin to draw with a vaſt Strength, and force it thro' the Hole of the drawing Plate, and then thro' ſeveral other Holes ſucceſſively narrower, till at length, after many ſuch Tranſitions, they bring it to the Fineneſs of an Hair, or finer, its Surface or Outſide continuing ſtill gilt over. Now I leave you to conſider to what a Fineneſs and Diſtenſion the Subſtance of the Gold is reduc'd.

SIMP. I do not ſee that from this Operation it follows, that the Subſtance of the Gold is ſo leſſen'd, as to effect thoſe Wonders which you ſpeak of : Firſt, becauſe the firſt gilding was with ten Leaves of Gold, which make a conſiderable Thickneſs : Secondly, becauſe, altho' in the Extension and leſſening that Silver, it increaſeth in Length, it yet decreases as much in Thickneſs ; by which Compensation the Superficies cannot be ſo enlarged, as to require, in covering the Silver with the Gold, the ſaid Gold to be reduc'd any thinner than the firſt Leaves were.

SALV. You're

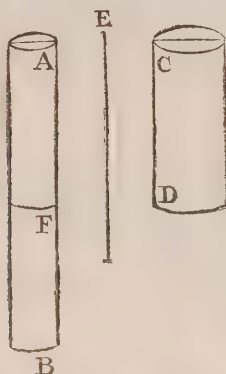
SALV. You're much deceived, *Simplicius*, for the Increase of the Surface is subduple the Extension or Prolongation, as I could demonstrate geometrically to you.

SAGR. I beseech you, both in Behalf of myself, and of *Simplicius*, to favour us with that Demonstration, if you think we are capable of understanding it.

SALV. I will see whether I can, thus upon a sudden, recal it to Mind——To begin then : 'Tis already manifest that the first thick Cylinder of Silver, and the Wire drawn out to so great a Length, are two equal Cylinders, for in each there's the same Silver : Now, if I shew you what Proportion the Surfaces of equal Cylinders have to one another, the Thing is done.

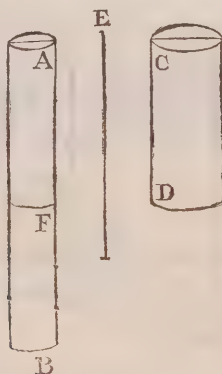
P R O P O S I T I O N .

The Surfaces of equal Cylinders, their Bases omitted, are to one another in a subduple Proportion of their



Lengths : Let there be two equal Cylinders, whose Heights are AB and CD, between which let the Line E be a
mean

mean Proportional : Then, I say, the Surface of the Cylinder AB is to the Surface of the Cylinder CD, the Bases of both being omitted, as AB to E, which is subduple of the Proportion AB has to CD : Let the Cylinder AB be cut in F, and let the Height AF be = CD : Now because that in equal Cylinders the Bases and Heights are reciprocal ; therefore the Circle the Base of the Cylinder CD, is to the Circle, the Base of the Cylinder AB, reciprocally as the Height BA to the Height CD : And because Circles are to each other as the Squares of their Diameters, the said Squares shall have the same Proportion as BA to CD : but as BA to CD, so the Square of BA to the Square of E ; wherefore those four Squares are proportional, and therefore their Sides will be proportional : And as the Line AB to E, so the Diameter of the Circle C, to the Diameter of the Circle A : but now as the Diameters are, so are the Circumferences, and as are the Circumferences, so also are the Surfaces of the Cylinders of



equal Height ; therefore as the Line AB to E, so the Surface of the Cylinder CD to the Surface of the Cylinder
AF :

AF : Therefore because the Height AF is to the Height AB, as the Surface AF to the Surface AB, and as the Height AB to the Line E, so the Surface CD to the Surface AF : Thence by Perturbation of Proportion, as the Height AF to E, so the Surface CD to the Surface AB ; and by Inversion, as the Surface of the Cylinder AB to the Surface of the Cylinder CD, so the Line E to the Height AF, *i. e.* CD, *i. e.* as the Line AB to the Line E, which is a Ratio subduple of that which AB has to CD. *Q. E. D.*

Now if we apply what we have just now demonstrated, to our Purpose, presupposing that that Cylinder of Silver, which was gilt, when it was not more than half a Yard long, and four or five Inches thick, when lessen'd to the Fineness of an Hair, acquires the Length of Twenty-thousand Yards, for Instance, we shall find its Surface to become two hundred times greater than it was at first ; and consequently those ten Leaves of Gold at first laid on, being distended on a Surface two hundred times bigger, assure us that the Gold which covereth the Surface of so many Yards of Wire, retains no more than the twentieth Part of the Thickness of an ordinary Leaf of Gold.

Pray consider now what this Thinness must be, and whether it is possible to conceive it done, without a wonderful Distension of its Parts ; and whether this seems to you an Experiment that tends to a Composition of infinite Indivisibles, in Physical Matters : Tho' to assist in this there are not wanting other stronger and more concluding Accidents.

SAGR. The Demonstration seems to me so ingenious, that, tho' it should not be of Force enough to prove what was propos'd, and that on whose Account it was brought, (and yet in my Opinion it carries great Force with it) yet I think the Time well spent, which was employ'd in hearing it.

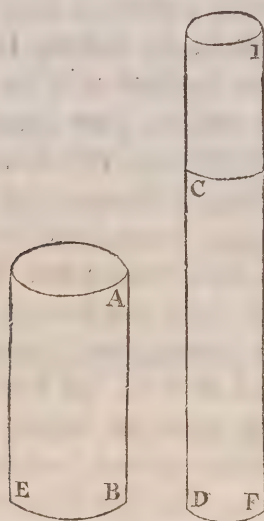
SALV. Since I see you are so well pleas'd with Geometrical Demonstrations, which bring with them certain Profit; I will give you the Fellow of this, which will answer a very curious Question often ask'd. ———

In the former we saw what happens in Cylinders that are equal, but of different Heights or Lengths: It will not be unpleasant to hear what happens to Cylinders equal as to Surface, but of different Heights; note, I always mean those Surfaces only which surround the Cylinders, *i. e.* not comprehending the Bases upper or lower: I say then, That

PROPOSITION,

Upright Cylinders, whose Surfaces, omitting the Bases, are equal, are in reciprocal Proportion to their Heights: Let the Surfaces of the two Cylinders AE and CF be equal, but let the Height of the one CD be greater than the Height of the other AC: I say the Cylinder AE has the same Proportion to the Cylinder CF, as the Height CD has to the Height AB: For since the Surface CF is = the Surface AE, the Cylinder CF will be less than the Cylinder AE: For if it was equal to it, its Surface by the
last

last Proposition would be greater than the Surface AE; and much more so, if the Cylinder CF were greater than AE: Let the Cylinder ID be supposed = AE; then by the preceding Proposition, the Surface of the Cylinder ID will be to the Surface of the other AE, as the Height IF to a mean Proportional between IF and AB: But since the Surface AE is given = CF; and the Surface ID has the same Proportion to the Surface CF, as the Height IF



has to CD; therefore CD will be a mean Proportional between IF and AB: Moreover, since the Cylinder ID is equal to the Cylinder AE, each of them will have the same Proportion to the Cylinder CF; But ID is to CF as the Height IF to the Height CD; therefore the Cylinder AE has the same Proportion to the Cylinder CF, as the Line IF has to CD, *i. e.* as CD has to AB. *Q. E. D.*

Hence now an Accident, amazing to the Vulgar, is easily accounted for, *viz.* how it comes to pass, if of the same Piece of Cloth or Sacking, longer one Way than the other, a Corn Sack be made, as the usual Manner is, with a round Board for a Bottom, that it will hold more, if the Breadth of the Cloth be made the Depth of the Sack, (the Length encompassing the round boarded Bottom) than if the Sack be made up the other Way of the Sacking : Thus, for Example, suppose the Sacking was twelve Foot long, and six Foot wide ; then, I say, it will hold more, if it be made into a Sack, by sewing the two Ends together, by which Means the Sack will be six Foot deep, encompassing a Bottom twelve Foot round, than it will by sewing together the two Sides, which will make it twelve Foot deep, encompassing a Bottom six Foot round : But farther, we do not only learn from the foregoing Proposition, that the Sack holds more that Way than this, but we likewise are taught how much more ; as thus, that the less Height it has, the more it will hold ; and the greater Height, the less : And thus in the Measures aforesaid, the Sacking being as long again as broad, it will hold but half the Quantity when sewed lengthways, as it will if sewed the other Way. In like Manner, if there be a Piece of Matt 25 Foot long, and seven broad, this sewed together lengthways, will hold only seven Measures, such as had it been sewn together the other Way, that is, endways, it would have held 25.

SAGR. And thus, with great Pleasure, we are continually discovering new and useful Knowledge. But as to the thing just now mention'd, I verily believe, that among
such

such who are wholly ignorant of Geometry, there would not be found one in twenty who, at first Thinking, would not be pretty positive that those Bodies which are contain'd under equal Surfaces, are in all Respects equal : As they run into the same Errors, speaking of Surfaces, whilst they are determining, as it often happens, the Magnitudes or Largeness of different Cities, they think they have done the Business, if they do but know how far they are about, not considering that one Circuit may be equal to another, and yet the Place, *i. e.* the Area contain'd by this, may be much greater than that comprehended by the other, as it happens, not only in irregular Surfaces, but also in regular ones, amongst which, those of more Sides are always more capacious than those of fewer ; so that the Circle, as being a Polygon of infinite Sides, is more capacious than any other Polygons of equal Perimeter : A Demonstration whereof I remember I saw, with singular Delight, when I studied the Sphere of *Sacroboſſo*, with a very learned Commentary upon the same.

SALV. It's true ; and I lighting upon the very same Place, it gave me an Occasion of investigating how it may, with one sole Demonstration, be concluded, that the Circle is the greatest of all the regular Isoperimetral Figures, and of the rest, those of more Sides are greater than those that have fewer.

SAGR. I am a great Lover of such select and uncommon Demonstrations ; therefore, pray Sir, (tho' I'm afraid I'm too troublesome) make me a Partaker therein.

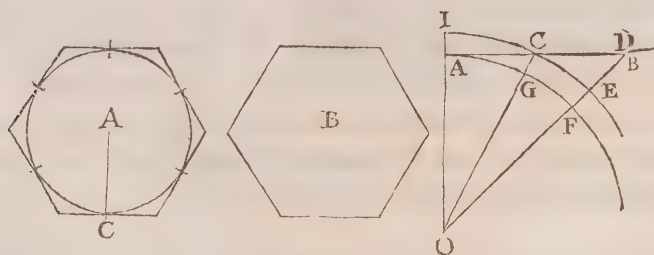
SALV. I'll

SALV. I'll do't in a very few Words, by demonstrating the following Theorem.

P R O P O S I T I O N .

The Circle is a mean Proportional between any two Polygons similar to each other, one of which is circumscribed about it, and the other Isoperimetral to it ; so that it is less than all those circumscribed, and on the other hand, greater than all the Isoperimetral.

Again, of the circumscribed ; those that have most Angles, are less than those that have fewer ; and, on the other Side, of the Isoperimetral, those which have the most Angles are greatest : Of the two similar Polygons A and B, let A be circumscribed about the Circle, and the other B, let be Isoperimetral to it ; then, I say, the Circle is a mean Proportional between them : For (having drawn the Semidiameter AC) the Circle being equal to a right-angled



Triangle, one of whose Sides, containing the right Angle, is = the Semidiameter AC, and the other = its Circumference : And, in like manner, the Polygon A being = a
right-angled

right-angled Triangle, which hath one Side containing the right Angle = the same right Line AC, and the other = the Perimeter of the same Polygon A; 'tis manifest that the circumscrib'd Polygon A, has the same Proportion to the Circle, as its Perimeter has to the Circumference of the Circle, that is, to the Perimeter of the Polygon B, which is suppos'd = to the said Circumference: But the Polygon A to the Polygon B (since they are similar Figures) has a duplicate Ratio of that which the Perimeter of the Polygon A has to the Perimeter B; therefore the Circle A is a mean Proportional between the two Polygons A and B. And the Polygon A being greater than the Circle A, 'tis manifest that the same Circle A is greater than the Polygon B, its Isoperimetral; and consequently the greatest of all regular Polygons that are Isoperimetral to it.

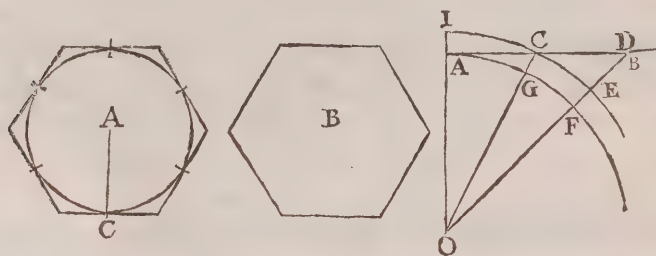
Q. E. D.

Now to the other Part of the Proposition, which is to prove that of the Polygons circumscrib'd about the same Circle, that which has the fewer Sides, is bigger than that which has more; And on the contrary, of Isoperimetral Polygons, that which has more Sides is greater than that which has fewer. And these I prove thus;

To the Circle whose Centre is O, and Semidiameter OA, draw the Tangent AB, in which, for Example's Sake, suppose AD to be half the Side of a circumscrib'd Pentagon, and AC half the Side of an Heptagon; and draw the right Lines OGC and OFD; and on the Centre O, with the Radius OC, describe the Arch ECI; Now because the Triangle DOC is greater than the Sector EOC, and the Sector COI greater than the Triangle COA, the Triangle DOC shall have a greater Proportion to the

Triangle

Triangle COA, than the Sector EOC to the Sector COI, *i. e.* than the Sector FOG to the Sector GOA : And by Composition and Permutation, than the Triangle COA to the Sector GOA ; and ten Triangles DOA shall have greater Proportion to ten Sectors FOA, than fourteen Triangles COA to fourteen Sectors GOA, *i. e.* the circumscrib'd Pentagon shall have greater Proportion to the Circle, than the Heptagon hath to the same Circle : therefore the Pentagon is greater than the Heptagon.



Suppose now an Heptagon and a Pentagon Isoperimetral to the same Circle, I say the Heptagon is bigger than the Pentagon : For the said Circle being a mean Proportional between the circumscrib'd Pentagon and its Isoperimetral Pentagon ; and likewise a mean Proportional between the circumscrib'd Heptagon and its Isoperimetral Heptagon ; since it has been prov'd, that the circumscrib'd Pentagon is greater than the circumscrib'd Heptagon, the said Pentagon shall have a greater Proportion to the Circle, than the Heptagon ; *i. e.* the Circle will have a greater Proportion to its Isoperimetral Pentagon, than to its Isoperimetral Heptagon : And therefore the Pentagon is less than the Isoperimetral Heptagon. *Q. E. D.*

SAGR. A

SAGR. A most elegant and curious Demonstration ! But how came we to change our Course, and plunge ourselves thus into Geometry, when we were about considering, the Difficulties started by *Simplicius*, which are of great Moment ? And in particular, that relating to Condensation, is, in my Opinion, very abstruse.

SALV. If Condensation and Rarefaction are opposite Motions, where there is found an immense Rarefaction, it can't be denied that there is also an extraordinary Condensation : But immense Rarefaction, and, what increases the Wonder, as it were momentary, we see daily : For what a boundless Rarefaction is that of a little Quantity of Gunpowder, resolv'd into a vast Mass of Fire ? And again, what is (I could almost say) the indeterminate Expansion of its Light ? If now that Fire and this Light should reunite together, which is not impossible, since at first they lay in that little Room, what a Condensation would this be ? Upon considering, you'll find a thousand such Rarefactions, which, indeed, are much more observable than Condensations, since dense Matters are more tractable and more subject to our Senses : thus we can handle Wood, and see it resolv'd into Fire and Light ; when, on the contrary, we cannot see Fire and Light condense to make Wood : So we find Fruits, Flowers, and a thousand other solid Matters, to resolve in a great measure into Odours ; when, on the contrary, we never can observe those odoriferous Effluvia to concur and make those fragrant Solids. But when sensible Observation is wanting, Reason must supply its Place, by whose Help we shall be sufficiently

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able to apprehend the Motion of the Rarefaction and Resolution of Solids ; as also to the Condensation of the finest and most rare Substances. But we not only enquire how the Condensation and Rarefaction of Bodies, which are subject to Rarefaction and Condensation, can be effected, but also how these can be conceived to be done, without introducing a *Vacuum*, and the Penetration of Bodies ; which does not hinder but that in Nature there may be Matter which admits no such Accident, and consequently allows no Room for those Things which you call inconvenient and impossible : And to be plain, *Simplicius*, I have taken a great deal of Pains for your Sake, and that of such other Philosophers, who think as you do, in considering how Condensation and Rarefaction may be conceiv'd to be effected, without admitting the Penetration of Bodies, or introducing void Spaces ; which two Effects you deny, and reject, as it were, with Horror ; and which if you would grant me, I should not so resolutely oppose you as I do : Wherefore, either allow those Inconveniencies, or acquiesce with my Speculations, or find out others yourself more to the Purpose.

SAGR. As to the Penetration of Bodies, I, with the Peripatetick Philosophers, altogether deny it : But as to what relates to a *Vacuum*, I own I should gladly hear *Aristotle's* Demonstration, that opposes it, thoroughly examin'd, as also what you, *Salviatus*, can object to it. *Simplicius* will favour me punctually to recite the Philosopher's Demonstration ; And I doubt not, *Salviatus*, but you will give us your Answer to it.

SIMP. *Aristotle*,

SIMP. *Aristotle*, as near as I can remember, inveighs against certain of the Antients, who introduc'd a *Vacuum*, as necessary to Motion, pretending that this could not be effected without that : *Aristotle*, opposing these Persons, demonstrates, that, on the contrary, the effecting of Motion (as we see) necessarily destroys the Supposition of a *Vacuum*. The Method he took was this : He supposes two Things ; the one is touching Moveables of different Gravity, mov'd in the same *Medium* ; the other is, concerning the same Moveable, mov'd in different *Mediums*. As to the former ; He supposes Moveables of different Gravity, to be mov'd in the same *Medium* with unequal Velocities, which bear to each other the same Ratio as their Gravities : As for Example, Let one Moveable be ten times heavier than another, it shall move ten times more swiftly : In his other Supposition he assumes, that the Velocities of the same Moveable in different *Mediums*, do keep a reciprocal Proportion to that which the Thickness or Density of those *Mediums* have : So that supposing, *v. g.* that the Density of Water be ten times as great as that of Air, he will have the Velocity in the Air to be ten times greater than in Water. And from this second Supposition he deduces his Demonstration in this Manner : Since the Tenuity of a *Vacuum* infinitely surpasses the Corporiety, tho' never so subtile, of any replete *Medium*, every Moveable that in the full *Medium* runs thro' a certain Space in a certain Time in a *Vacuum*, ought to run thro' the same Space in an Instant : But for a Motion to be made in an Instant is impossible ; and therefore to introduce a *Vacuum* on Account of Motion is impossible.

SALV. The Argument, you see, is *ad Hominem*, i. e. against such as would have a *Vacuum* necessary to Motion : But now if I should admit, that *in Vacuo* there would be no Motion, yet the Position of a *Vacuum* taken absolutely, and without any Relation to Motion, is not thereby destroy'd ; but to tell you what the Antients, may be, would answer, that thus you may the better see how far *Aristotle's* Demonstration is conclusive : 'Tis my Opinion, that both his Assumptions may be contradicted and denied. As to the former of them : I very much doubt whether *Aristotle* ever found, by Experience, how true it is, that two Stones, one ten times heavier than the other, let fall at the same Instant from an Height, e. g. of 100 Yards, differ so in Velocity, that when the greater comes to the Ground, the lesser has not fellen above ten Yards.

SIMP. His Words seem to intimate that he has try'd it ; for he says, *we see* that the heavier : now from these Words of his *we see*, we may conclude, that he had actually made the Experiment.

SAGR. Hold, *Simplicius*, for I myself have prov'd by Experience, that if a common Bullet, weighing 100, 200, or more Pounds, let fall from the same Height of 200 Yards, and at the same Instant, with a Musket Bullet that weighs but half a Pound ; at the Arrival of the Cannon Bullet at the Ground, the other will not want a Hand's-breadth of it.

SALV. But

SALV. But without Experiments, it may be by short and evident Demonstration clearly prov'd, that it is not true that a Moveable heavier than another, moves or falls more swiftly than that other, the Moveables being suppos'd of the same Matter, and therefore such as *Aristotle* speaks of : For pray tell me, *Simplicius*, whether you admit that every heavy Body, in its Fall, has by Nature one determin'd Velocity, which can't be either accelerated or diminished without some Force or Impediment ?

SIMP. Without doubt the same Moveable, in the same *Medium*, hath a certain establish'd, and, by Nature, determin'd Velocity ; which cannot be increased without it suffer some new *Impetus* ; nor diminished, unless some Obstacle retard it.

SALV. If therefore we take two Moveables, whose Velocities are unequal, 'tis manifest, that if we join the slower with the swifter, this would be something retarded by the slower, and that something accelerated by the swifter ; don't you agree with me in this ?

SIMP. This seems to me an undoubted Consequence.

SALV. But now, if this be so, it will likewise be true, that if the greater Stone, moving, *e. g.* with eight Degrees of Velocity, and the lesser with four, be joined together, that their compound Motion ought to be with a Velocity less than eight Degrees : But those two Stones joined together, make a Stone greater than that which before mov'd
with

with eight Degrees of Velocity ; therefore this greater Stone moves with less Velocity than the lesser, contrary to your Hypothesis. You see, therefore, how from your Supposition, that the heavier Body is moved swifter than the lighter ; I, on the contrary, conclude it to move slower.

SIMP. I find myself in a Snare, for certainly the lesser Stone joined to the greater, must add Weight to it ; and Weight being added to it, I don't see why its Velocity should not be increased ; at least, certainly, it ought not to be diminished.

SALV. Here now you run into another Error, *Simplicius*, for that lesser Stone does not increase the Weight of the greater.

SIMP. No ! I own this quite exceeds my Apprehension.

SALV. Your Surprise will be over, so soon as I shall have shewn you what's equivocal here, which occasions your Doubt. Know then, that we must distinguish betwixt heavy Bodies *in Motion*, and the same *at Rest*. A Stone put into a Ballance, certainly acquires, not only a greater Weight by laying another Stone upon it, but even by the Addition of a little Wool or Flax, it will in Weight acquire an Increase, as much as is the Weight of the Wool or Flax added ; But yet if you should let fall freely the Stone with the Flax tied about it from on high, do you think that in that Motion the Flax presses that Stone with

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its Weight, so as to accelerate its Motion ; or do you think it in a manner sustaining the Stone, will retard it? Thus, for Example, we feel indeed a Weight upon our Shoulders, so long as we oppose the Motion the Weight which lies upon our Shoulders would have, was it not so opposed: But if we also should descend or fall with the same Velocity, where-with that heavy Body would naturally descend ; how do you think this Weight would press, gravitate, or bear upon us then? Is not this the same thing as a Man's striving to wound another with a Lance, who runs before him with as much or more Speed than the other pursues with? You may conclude then, that in the free and natural Fall, the lesser Stone doth not gravitate upon the greater, and consequently doth not add to its Weight, as it does when they are at Rest.

SIMP. But what if the greater be placed upon the lesser?

SALV. It would increase the Weight of this, if its Motion were swifter: But it hath been already concluded, that in case the lesser be more slow, it would in Part retard the Velocity of the greater, so that both together would move less swiftly, being greater than the other; which is contrary to your Hypothesis: Therefore we conclude, that greater and lesser Moveables, being of the same specifick Gravity, move with equal Velocity.

SIMP. Your Discourse is, I own, very ingenious; yet methinks 'tis hard to conceive that a single Bird-shot should fall with the same Velocity as a Cannon-Bullet.

SALV. You

SALV. You might have said, that a Grain of Sand should be carried with the same Celerity as a Mill-stone : But, *Simplicius*, I would not have you do as some others usually do, I mean by diverting the Discourse from the principal Design, to fasten upon some one Thing I have said, that may want a Hair's-breadth, perchance, of the Truth, and under that Hair hide the Fault of another Man as big as a Cable. *Aristotle* says, an Iron Ball of an hundred Pounds Weight, falling from an Height of an hundred Yards, comes to the Ground before another of a Pound Weight has descended one Yard : And I say, that they'll both come to the Ground at the same Time. You find that the bigger out-strips or gets before the other by two Inches ; that is to say, when the greater reaches the Ground, the lesser wants two Inches of it : But now you would, under these two Inches, hide *Aristotle's* ninety-nine Yards ; and only mentioning my very small Error, would pass over his great one in Silence.

Aristotle affirms, That Moveables of different Gravity move in the same *Medium* (as far as concerns Gravity) with Velocities proportional to their Weights, and exemplifies it by Moveables, wherein the pure and absolute Effect of Weight may be consider'd, omitting the other Considerations, as well of the Figures, as of the least Moments, which Things suffer great Alteration from the *Medium*, by which the simple Effect of Gravity alone is alter'd : Whence we see Gold, that is heavier than any other Matter, beat out into a very thin Leaf, to fly to and again thro' the Air ; and Stones beaten to very fine Powder to do the like : But if you would defend the universal Proposition,

sition, 'tis requisite that you shew the Proportion of the Velocities to be observ'd in all grave Bodies, and that the Motion of a Stone of twenty Pounds Weight, is ten times swifter than that of a Stone of two Pounds : Which I tell you is false, since falling from the Height of 50 or 100 Yards, they both come to the Ground at once.

SIMP. Perhaps from very great Heights of Millions of Yards that might follow, which is not seen to happen from these lesser Heights.

SALV. If this was *Aristotle's* Meaning, he must be guilty of another Error ; nay, thus you'll make him tell a Lye : For there being no such perpendicular Height to be come at on Earth, *Aristotle* could not make the Experiment, which he would persuade you he had, by saying that such an Effect was *seen*.

SIMP. However *Aristotle* certainly made no use of *this*, but of *that other* Principle, which I don't believe is subject to these Difficulties.

SALV. You think so : But I say that's as false as this : And I admire you don't of yourself perceive the Fallacy ; and see, that tho' it was true, that the same Moveable in *Mediums* of different Subtilty and Rarety, and therefore of a different Resistance, as are, *e. g.* Water and Air, is mov'd with a greater Velocity in Air than in Water, according to the Proportion of the Air's Rarety to the Rarety of the Water, yet it does not follow, that all Moveables which descend in Air, do descend in like manner in Water ;

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which is so false, that very many Bodies descend in Air, that in Water are so far from descending, that, on the contrary, they always ascend.

SIMP. I don't see the Necessity of your Consequence: And I say farther, that *Aristotle* speaks of such heavy Bodies as descend in both *Mediums*, and not of such as descend in Air, and ascend in Water.

SALV. To defend this Philosopher, you produce such Pleas as he himself certainly never would, lest he should aggravate his first Mistake: Pray tell me whether the Body and Substance of the Water, or of whatever it be which retards the Motion, hath any Proportion to the Crassitude or Body of the Air that less retards it; and if it has, assign some Proportion at Pleasure.

SIMP. It has: Let us suppose that Proportion to be decuple, and thence that the Velocity of any heavy Body that descends in both those Elements, will be ten times slower in Water than in Air.

SALV. I'll take one of those heavy Bodies that descend in Air, but not in Water; as for Instance, a Ball of Wood; Now I desire you to assign it what Velocity you please, whilst it descends thro' the Air.

SIMP. Let's suppose it to move with twenty Degrees of Velocity.

SALV. Very

SALV. Very well : Now 'tis manifest that that Velocity to some other lesser may have the same Proportion, that the Solidity of the Water has to the Solidity of the Air ; and this lesser Velocity will be of two Degrees only : So that exactly and directly, according to *Aristotle's* Hypothesis, it should be concluded, that the Ball of Wood, which in the Air, ten times more yielding than Water, descends, mov'd with twenty Degrees of Velocity, in the Water it should descend with two, and not to ascend from the Bottom to the Surface of the Water, as we see it does : unless you'll say, that the ascending of the Wood in the Water to the Top, is the same with its sinking towards the Bottom with two Degrees of Velocity ; which I don't believe you'll venture to say : But seeing that this Ball of Wood tends not to the Bottom, I believe you'll grant me that some other Ball of other Matter, different from Wood, may be found to descend in Water with two Degrees of Velocity.

SIMP. Questionless there may ; but it must be made of Matter considerably heavier than Wood.

SALV. That's what I desir'd to know : But now this second Ball, which in the Water descends with two Degrees of Velocity, with what Velocity will it descend in the Air ? You must answer (if you'll follow *Aristotle's* Rule) that it will descend with twenty Degrees ; but you yourself have assigned twenty Degrees of Velocity to the Ball of Wood, therefore both this and the other, tho' much heavier, will move in the Air with equal Velocities :

But how will the Philosopher reconcile this Conclusion with that other of his, which asserts, that Moveables of different Gravities move in the same *Medium* with different Velocities ; and indeed as different as their different Gravities ?

But omitting any farther deep Speculations ; how comes it to pass that you have not observ'd Accidents so very frequent and evident, and have not consider'd two Bodies, one of which will move in Water 100 times more swift than the other ; but in the Air, that that which is so much swifter in Water, will not out-go the other one hundredth Part in the Whole ? Thus, *e. g.* a Piece of Marble in Form of an Egg, will sink an hundred times swifter in Water than an Hen's Egg will ; but in the Air, from the Height of twenty Yards, it will not out-fall it four Inches : And, in a Word, that heavy Body which is three Hours in falling to the Bottom of ten Fathom Water, will fall the same Depth in the Air in one or two Seconds ; and such another (as for Instance a Ball of Lead) will pass thro' the same Space in half the Time : And here, *Simplicius*, I'm sure you'll allow there's no Room left for any Distinction or Reply : We conclude therefore, that that Argument concludes nothing against a *Vacuum* ; But if it should, it would only destroy Spaces considerably great, such as neither I, nor, as I take it, those Antients ever supposed to be naturally allow'd or given, tho' perhaps with Violence they may be effected ; as seems to be gather'd from several Experiments, too long to be insisted on here.

SAGR. Seeing

SAGR. Seeing *Simplicius* is silent, I'll take leave to speak : Since you have with sufficient Plainness demonstrated that it is not true, that Moveables unequally heavy, move in the same *Medium* with Velocities proportional to their Gravities, but with equal Velocities ; the heavy Bodies being supposed of the same Matter, or of the same specific Gravity, but not (I suppose) of different specific Gravities, (for I don't think that you pretended to prove to us, that a Ball of Cork moves with like Velocity with a Ball of Lead) : Moreover, since you have also demonstrated plainly that it is not true that the same Moveables in *Mediums* of different Resistances, observe the same Proportion in their Velocities and Tardities, as their Resistances have ; you'd greatly oblige me to tell me what those Proportions are, which are observ'd in both Cases.

SALV. The Questions are very ingenious, and I have many times thought of them : I will relate to you the Contemplations I have had concerning them, and from thence what Conclusion I am come to about them.

After I had assured myself that it was not true, that the same Moveables in *Mediums* of different Resistance, keep the same Proportion in their Velocities, with which those *Mediums* give Way ; nor yet again, that in the same *Medium* Moveables of different Gravity, keep in their Velocities, the Proportion of those Gravities (speaking of different specific Gravities) ; I bethought myself to join those two Accidents together, and to consider what would happen if the Moveables of different Gravity were put into *Mediums* of different Resistance ; and I found that
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the Inequalities of the Velocities were always greater in the more resisting *Mediums* than in the less ; and that with this Diversity, that of the two Moveables, which falling in the Air, differ very little in Velocity of Motion : One will, in the Water, move ten times faster than the other ; and that that which in the Air descends swiftly, in the Water not only will not descend, but will be wholly depriv'd of Motion ; or, what is more, will move upwards : For sometimes such a kind of Wood may be found, or a Knot or Root of it, which in the Water will lie still, but in the Air will descend swiftly.

SAGR. I have often, with the utmost Patience, strove to bring a Ball of Wax, which of itself doth not sink, to the same Gravity as Water, by adding to it Grains of Sand, so that it might stand still in the Midst of the Water ; but I could never, with the utmost Care and Labour, succeed in the Attempt : Whence I doubt whether there be any Matter naturally so alike in Gravity to Water, as being put into it, to lie still where-ever you put it.

SALV. In this, as well as in a thousand other Operations, many Animals are more knowing than Men are : And in this Case of yours, the Fishes would have been able to have given you some Light, being so skilful in this Exercise, that at their Pleasure they preserve an Equilibrium, not only with one Sort of Water, but also with different, as such as either are naturally or accidentally muddy, or else are salt, which makes a great Alteration : They, I say, so accurately preserve an Equilibrium, that,

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without the least Motion they can lie still in every Place; and this they do, in my Opinion, by making use of an Instrument Nature has given them to that End; I mean a little Bladder which they have within them, and whose narrow Neck answers to their Mouth, thro' which, when they would lie still, they send forth Part of the Air contain'd in the said Bladder; and when they would rise to the Surface, or Top of the Water, they draw more Air in: by this Artifice making themselves at one time more, at another less heavy, and when they please they preserve an Equilibrium.

SAGR. I deceiv'd some of my Friends with another Device, to whom I had boasted, that I could reduce a Ball of Wax to an exact Equilibrium with the Water: Thus, viz by putting some Salt-Water in the Bottom of the Vessel, and then filling up the Vessel with the like Quantity of fresh Water; which done, I shew'd them a Ball at rest in the Midst of the Water, which being thrust to the Bottom, or raised to the Top of the Water, would continue in neither Place, but would always return to the Midst.

SALV. This Experiment of yours has its Uses: For when Physicians, in particular, treat of the different Qualities of Waters, and amongst these of their Difference of Levity and Gravity, they make use of such a Ball, so adjusted, or poised (that it may rest ambiguous, if I may so say) in the Water, between rising and sinking: If that Ball, upon the least Difference of Weight, between two Waters, sinks in one, in the other, which is more heavy, it will rise.

rise. And this Experiment is so exact, that if into six Pounds of Water, you put only two Grains of Salt, the same Ball, which before was sunk to the Bottom, will now rise to the Surface or Top of the Water. But farther, to confirm the Exactness of this Experiment, and clearly to prove the Non-resistance of Water to Division or Penetration ; I affirm to you, that not only by mixing with the Water some heavier Matter, the Water becomes heavier, with so notable a Difference ; but by warming or cooling it a little, the same Effect is produc'd, and with so subtle an Operation, that the infusing into the six Pounds but four Drops of other Water, a little warmer or a little colder, shall cause the Ball to sink or rise ; to sink upon Infusion of the Warm, and to rise upon the Infusion of the Cold. Now judge how much those Philosophers are deceiv'd, who would introduce in Water Viscosity, or other Conjunction of Parts, to make it resist Division or Penetration.

SAGR. I have seen many conclusive Discourses concerning this Argument, in a Treatise of our Academick ; yet there's one great Doubt rests with me, which I can't get rid of ; 'tis this : If there be nothing of Tenacity or Coherence amongst the Parts of Water, how can such large Heaps sustain themselves, as in a particular Manner are seen upon Cole-wort or Cabbage Leaves, without dispersing or levelling ?

SALV. Altho' it be true, that he that is Master of a true Conclusion, may resolve all Objections that can be brought against it ; yet will I not arrogate to myself the Power

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of doing so ; nor ought my Insufficiency to darken the Light of Truth. First then, I freely confess that I know not how it comes to pass, that those Globes of Water sustain themselves at such an Height and Bigness, altho' I certainly know that it does not proceed from any internal Tenacity that is between their Parts : whence I necessarily infer, that the Cause of that Effect is without the Water. That it is not internal, setting aside the Experiments already produc'd, I can prove by another most convincing one ; 'tis this : If the Parts of that Water which sustains itself in Globules whilst 'tis encompassed by the Air, had an internal Cause for so doing, they would much better sustain themselves encompassed by a *Medium*, wherein they would have less Propensity to descend than they have in Air : But now such a *Medium* would every Fluid heavier than Air, be ; as, for Instance, *Wine* ; and therefore if about that Globe of Water *Wine* be pour'd, this might raise itself on high about that, and yet the Parts of the Water, conglutinated by their internal Viscosity, would never be dissolved : But now this does not so happen in these Globules, which no sooner hardly doth the circumfused Liquor touch, but they dissolve and become flat, resting under the Wine, if it be Red : The Cause therefore of this Effect is from without, and perhaps from the ambient Air ; and a great Dissention observable between Air and Water, as I have found by another Experiment, and that is this ; If I fill a round Crystal Bottle with Water, whose Mouth is no bigger than that of a Straw, and after this turn its Mouth downwards, yet will not the Water, altho' very heavy and prone to descend in Air, nor the Air, as much disposed on the other Hand, as being

very light, to ascend thro' the Water ; yet will they not, I say, agree, that that should descend, issuing out of the Mouth, and this ascend, entering in at the same ; but both keep their Places, and yield not to each other. But on the contrary, if I apply to the Mouth of this Bottle a little Vessel of Red Wine, which is insensibly less heavy than Water, we shall see it in an Instant gently to ascend by red Streams thro' the Water ; and on the contrary, the Water, with the same Slowness, to descend thro' the Wine, without ever mixing with each other, till at length the Bottle will be full of Wine, and all the Water will sink to the Bottom of the Vessel that's underneath. What must we say now ? or what Argument must we use, unless this, that there is a Dissention, how I know not, between Water and Air ? But perhaps——

SIMP. I can't help laughing to see the great Antipathy *Salviatus* has to Antipathy ; that he does not think it worth naming, tho' it be so accommodate to resolve these Difficulties.

SALV. Well, for your Sake, *Simplicius*, let this solve our Doubt ; and leaving this Digression, let us return to our Purpose.

We have found that the Difference of Velocities in Moveables of different Gravities to be more and more as the *Mediums* are more and more resisting : Thus in a *Medium* of Quicksilver, Gold does not only sink to the Bottom more swiftly than Lead, but it is the only thing that will sink in it, all other Metals and Stones moving upwards therein, and floating on its Surface ; whereas between Balls

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of Gold, Lead, Brass, Porphyry, or any other heavy Matter, the Inequality of their Motion in the Air shall be almost wholly insensible, so that indeed a Ball of Gold, falling from an Height of an hundred Yards, in the End of its Fall does not out-strip one of Wax four Inches : And this being so, I have thought that if the Resistance of the *Mediums* be wholly taken away, all Matter would descend with equal Velocity.

SIMP. How, *Salviatus* ! You'll ne'er get me to believe, I think, that a Lock of Wool, and a Piece of Lead, would, even in a *Vacuum*, fall with the same Celerity.

SALV. Fair and softly, *Simplicius* : Your Scruple is not so abstruse, nor I so hasty, as you think, in my Assertion, as not to be able to justify it, and to answer your Doubt. That you may understand my Explanation, pray give me the Hearing.

The Business we are now upon is to examine what would befall Moveables exceedingly different in Gravity, in a *Medium* of no Resistance, so that all the Difference of Velocities, which is found between the said Moveables, ought to be referred to the sole Inequality of Gravity : And because only a Space altogether void of Air, and of every other the finest and most yielding Body, would be fit sensibly to shew us what we seek, and since we want such a void Space, let us observe what happens in the more subtile and less-resisting *Mediums*, and compare them with what we find happens in others less subtile and more resisting : For if we should really find Moveables different in Gravity, to differ less and less in their Velocities,

cities, according as the *Mediums* are less and less resisting ; and if at length that tho' they are extremely different in Gravity, the Difference of Velocities is found next to nothing, and in a manner inobservable, when they move in a *Medium* more subtile than any other, tho' not in a *Vacuum*, we may, methinks, reasonably conjecture that in a *Vacuum* their Velocities would be exactly equal : Let us consider, therefore, what happens in the Air ; and that we may have a Figure of an uniform Superficies, and of very light Matter, let us take a blown Bladder, in which the included Air, in a *Medium* of the Air itself, will weigh little or nothing, since therein it cannot much compress itself ; so that all its Gravity consists only in that of the little Skin, which is not a thousandth Part of the Weight of a Lump of Lead of the same Bigness with the blown Bladder. Now if these two, *Simplicius*, be let fall from the Height of four or six Yards, how far do you think would the Lead out-strip the Bladder in the Fall ? You may assure yourself that that would not fall three times, no nor twice, so fast as this, altho' you would have that it falls a thousand times as swift.

SIMP. It may possibly happen, that at the Beginning of their Fall, *i. e.* in so small a Length as four or six Yards, it may be as you say ; but in a long Continuation, I dare say, the Lead would leave the Bladder behind six, eight, if not ten Parts in twelve.

SALV. I believe so too, not doubting but that from very great Heights the Lead will have fallen a hundred Miles, before the Bladder has fell one : But this Effect

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which you propose, is so far from contradicting my Proposition, that 'tis the greatest Confirmation of it imaginable. My Intent is (to repeat it once more) to explain to you, that in Bodies of different Gravities the Cause of their different Velocities does in nowise consist in their different Gravities, but depends on exterior Accidents, and indeed, in particular, on the Resistance of the *Medium* they fall thro', so that this Resistance being taken away, all Moveables fall with the same Degrees of Velocity. And this I deduce chiefly from what you have just now admitted, and which is very true, namely, that of Moveables very different in Gravity, the Velocities differ more and more, according as the Lines of their Fall are greater and greater; which Effect would by no Means follow, if they depended on the different Gravities: For the Gravities being always the same, the Proportions betwixt the Lines of Descent would then also be always the same: But now we see this Proportion in the Continuation of the Fall constantly to increase, inasmuch as the heaviest Moveable in a Fall of one Yard will not leave the lightest behind the tenth Part of a Yard; but in a Fall of twelve Yards, it will leave it four Yards; and in one of a hundred Yards, it will leave it behind ninety Yards.

SIMP. Very well: Then (to make use of your own Way of arguing) if the Difference of Gravity in Moveables of different Gravity, does not cause the Difference of Proportion in their Velocities, their Gravities continuing the same; then neither can the *Medium*, which, in our Case, is supposed always to continue the same, cause any Alteration in the Proportion of the Velocities.

SALV. Seem-

SALV. Seemingly you have contradicted me ; but I'll endeavour to set this right, before I proceed to any thing else. I say then, that every heavy Body has by Nature an intrinsick Principle of tending towards the common Centre of heavy Things ; namely, to that of our Terrestrial Globe, and that with a Motion continually accelerated, and indeed always equally accelerated, so as in equal Times are equally added new Moments and Degrees of Velocity : And this must be understood to hold true when all accidental and external Impediments are remov'd, amongst which there is one we can't remove, I mean the Impediment of a full *Medium*, which must be cut and mov'd aside by the falling Body ; to which transverse Motion the *Medium*, tho' yielding and calm, opposes itself with a Resistance at one time less, at others greater and greater, according as it is more slowly or hastily to separate or open to give Passage to the Moveable, which because, as I have said, it of its own Nature moves with a continual Acceleration, consequently it continually encounters greater Resistance in the *Medium*, and therefore meets with Hinderances and Letts, in acquiring new Degrees of Velocity ; so that at length the Velocity arrives to that Pitch, and the Resistance of the *Medium* to that Strength, that ballancing each other they take away all further Acceleration, and bring the Moveable to an equable and uniform Motion, with which it afterwards continually goes on : There is therefore in the *Medium* an Augmentation of Resistance, not that it changes its Essence, but because the Velocity is chang'd, wherewith it ought to
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open and be mov'd laterally, to give Passage to the falling Body, which is continually accelerated.

Since now the Resistance of the Air to the *Momentum* or *Impetus* of the Bladder is very great, and to the great Weight of the Lead but very little, I am induc'd to conclude for certain, that if this Resistance should be wholly removed, (a great Advantage to the Bladder, and but little to the Lead) their Velocities would become equal ; Taking this Principle therefore for granted, that in that *Medium* wherein either because of a *Vacuum* or some other Reason, there is no Resistance to abate the Velocity of the Motion, so that the Velocities of all Moveables become equal ; we might aptly enough assign the Proportion of the Velocities of Moveables in the same, and in different, full, and therefore resisting, *Mediums*. And this we may obtain by considering how much the Gravity of the *Medium* takes from the Gravity of the Moveable, which Gravity is the Instrument with which the Moveable makes its Way, by forcing the Parts of the *Medium* aside : an Operation that can't take Place in a *Vacuum*, or empty *Medium* ; wherefore no Difference is to be expected from their different Gravities : And because 'tis manifest that the *Medium* takes away from the Gravity of the Body therein contain'd, as much as is the Weight of such another Mass of the *Medium's* own Matter ; if now in such Proportion we diminish the equal Velocities of Moveables, which (as it has been suppos'd) they would have in a non-resisting *Medium*, we shall obtain our Desire. Thus, for Example, suppose Lead to be ten thousand times heavier than Air, and Ivory only a thousand times ; Now as to the Velocities of these two Matters, which, absolutely
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taken, that is, all Resistance taken away, would be equal, the Air would take from the ten thousand Degrees of the Lead one Degree ; and from the thousand Degrees of the Ivory likewise one Degree ; or, if you will, from its ten thousand, ten. If therefore the Lead and the Ivory should fall from any Height in the Air, which they would do in the same Time, all Resistance of the Air being taken away, the Air will abate one Degree of the ten thousand of Velocity of the Lead : But of the ten thousand Degrees of Velocity of the Ivory, it will abate ten, that is, that Altitude from which the Moveables fall, being divided into ten thousand Parts, the Lead will come to the Ground, leaving the Ivory behind ten, or at least nine Parts of those ten thousand Parts. And what else is this, I pray, than that a Ball of Lead falling from a Tower two hundred * Cubits high, shall out-strip an Ivory one less than four Digits : The Ivory weighs, suppose, a thousand times more than Air, but the Bladder so blown but four times more ; wherefore the Air takes away one Degree of a thousand from the intrinsic and natural Velocity of the Ivory ; but from that of the Bladder (which taken absolutely would have been the same with that of the Ivory) the Air takes one Part of four : When therefore the Ball of Ivory, falling from the Tower, reaches the Ground, the Bladder has fall'n but three Quarters of the Way down. Lead is twelve times heavier than Water, but Ivory only twice as heavy ; the Water therefore, from their absolute Velocities which would be equal, takes from that of the

* One Cubit = 1 Foot 6 Inches. 1 Digit = 0 Foot 0 Inches $\frac{3}{4}$.

Lead a twelfth Part, but from that of the Ivory the half; When therefore the Lead shall have sunk in the Water eleven Parts, the Ivory shall have sunk but six : And following this Rule, we shall find, I believe, that such a Computation will be more agreeable to Experience than that *Aristotle* has given us.

And by the like Method we may find the Proportion between the Velocities of the same Moveable in *different* fluid *Mediums*, not by comparing the different Resistances of the *Mediums*, but by considering the Excess of the Gravity of the Moveable above the Gravities of the *Mediums* : Thus, for Example, Tin is a thousand times heavier than Air, and ten times heavier than Water ; wherefore the absolute Velocity of Tin divided into a thousand Degrees it shall move in the Air which takes away a thousandth Part from it, with Nine hundred ninety-nine Degrees, but in the Water with Nine hundred only, the Water taking off the tenth Part of its Gravity, and the Air the thousandth.

Again, suppose a Solid something heavier than Water, for Instance, Oak ; and let a Ball of it weigh, *e. g.* a thousand Drachms, and let an equal Mass of Water weigh Nine hundred and fifty, and an equal Mass of Air two : 'Tis manifest now, that if we suppose its absolute Velocity to be a thousand Degrees, in Air there would remain only Nine hundred ninety-eight, and in Water only fifty ; *viz.* because the Water takes off from the thousand Degrees of Gravity Nine hundred and fifty, leaving only fifty : Wherefore such a Body will move twenty times as swift in Air as in Water, as the Excess of its Gravity above that

of the Water is = the twentieth Part of its own *. And here I would have it consider'd, that since in Water no Matter can sink, unless they are specifically heavier than Water, and consequently many hundreds of times heavier than Air; when we are enquiring into the Proportion of those Velocities in Air and in Water, we may, without subjecting our Calculation to any considerable Error, consider the Air as taking nothing at all from the absolute Gravity, and consequently nought from the absolute Velocities of such Matter; wherefore having found with Ease the Excess of their Gravities above the Gravity of Water, we may say that their Velocities in the Air, to their Velocities in Water, have the same Proportion as their whole Gravity has to the Excess of this above the Gravity of Water: For Example, If a Ball of Ivory weigh twenty Ounces, and an equal Mass of Water weigh seventeen Ounces; then the Velocity of the Ivory in the Air, to its Velocity in Water, will be very nearly as 20 to 3.

SAGR. I have long busied myself in a Matter of itself curious enough, wherein, tho' without much Benefit, I have often thought till I've been weary; tho' I must own there's nothing wanting to reduce these Speculations of yours to Practice, could we but find what Gravity the Air has, in Comparison to that of Water, and consequently to those of other Matters.

* *i. e.* As 1000 — 950 = 50 is = 1000 ÷ 20 = 50.

SIMP. But

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SIMP. But if we should find that the Air, instead of Gravity, has Levity ; what must we say of the foregoing Discourses, otherwise very ingenious ?

SALV. Why, if so, then what I have been saying may be truly accounted airy, light, and vain. But can you doubt that the Air has Weight, when *Aristotle* expressly affirms it ? Who says, that all the Elements, even the Air itself, has Gravity ; and subjoins this as a Proof, *viz.* a * Bladder after it's blown, weighs more than before.

SIMP. I don't believe that the Gravity of a blown Bladder proceeds from the Gravity that is in the Air, but from the very many thick Vapours intermix'd with it, in these our lower Regions : What if I should say, that hence may be deduc'd the additional Gravity of the Bladder ?

SALV. I would not have you say so, much less introduce *Aristotle* as saying so ; because if he, (speaking of the Elements) in order to persuade me that the Element of Air has Weight, and in order to prove this by Experiment, should bid me take a Bladder and fill it with gross Vapours, and I shall find the Weight of the Bladder will be encreased ; I might answer him thus, That the Bladder would weigh more if he filled it with Bran ; and might tell him farther, That these Experiments prove indeed that Bran and gross Vapours have Weight ; but as to the Element of Air, I should still be in the same Doubt. The

* Un Otro Gonfiato.

Experiment therefore of *Aristotle* is good, and his Proposition true. But I can't say so much of another Reason I have met with in another Philosopher, whose Name I have forgot ; but I am sure I have read it : who argues, that the Air is rather heavy than light, because it more easily presses heavy Bodies downwards, than it lifts the light ones upwards.

SAGR. Very good, truly ! So, according to this Way of Reasoning, the Air must be much heavier than Water, since all heavy Bodies are carried more easily downwards thro' the Air than in Water ; and all light ones more easily upwards in This than in That ; nay, infinite Matters ascend in Water which in Air descend. But to go on : The Weight of the Bladder, *Simplicius*, whether it be ascrib'd to gross Vapours, or to pure Air, is nothing to our Purpose, who are seeking what happens to Moveables, that are moved in this our vaporous Region. To return then to that which more concerns me, that I may be fully and absolutely instructed in this Matter, I would be thoroughly assured, not only that the Air has Gravity, which I believe it has, but I would, if possible, know what Gravity it has. Therefore, *Salviatus*, if you can satisfy me in this Point, I beg you will.

SALV. That the Air has positive *Gravity*, and not, on the contrary, as some have thought, *Levity*, which perhaps is found in no Matter whatsoever, the Experiment of the * blown Bladder produc'd by *Aristotle* demonstrates

* Del Pallone Gonfiato.

to us ; for if the Quality of absolute and positive Levity was in the Air, then the Air being multiplied, if I may so say, and compressed, its Levity would, by so doing, increase, and consequently its Propensity to ascend : but Experience shews the contrary.

As to the other Part of the Question, *viz.* To find what the Gravity of the Air is ; I'll tell you the Method I have taken to do it. I took a pretty big Glass Bottle, with a narrow Neck, and tied very close to its narrow Neck a Leathern Cover ; to this Cover, and within Side, I put a Valve ; thro' this with a Syringe I forced a great Quantity of Air, of which, because it admits of great Condensation, twice or thrice as much may be forced in as the Bottle naturally holds : Then I very carefully weigh'd in a most exact Ballance the Bottle with the compressed Air within it, adjusting their Weight by very fine Sand ; then opening the Valve, I let out the Air which was violently contain'd in the Vessel ; I put the Bottle again into the Scales, and finding it much lighter than before, I took out of the other Scales so much Sand (keeping it by itself) until the Sand and the Bottle were in *Equilibrio* : Now there can be no Room to doubt but that the Weight of the Sand taken out is = the Weight of the Air which was violently forced into the Bottle, and which afterwards was let out.

But this Experiment assures me of no more than this, *viz.* that the Weight of the Air violently compressed in the Bottle, is equal to the Weight of that reserved Sand : But I have not yet determin'd how much the Air absolutely weighs in respect of Water, or any other heavy Matter ; nor can I know this, unless I measure the Quantity of the compressed

compressed Air ; which may be done by either of the two Ways following.

The former is this : Take such another Bottle, with a Neck exactly of the same Size with that of the former ; round which Neck tie very fast another Leather, the other End of which tie also closely over the former Bottle's Neck. Now the Bottom of this second Bottle must be drill'd or bor'd thro', so that thro' the Hole a Wire may be put, wherewith at Pleasure the Valve of the former Bottle may be open'd, to let out the superfluous Air after it hath been weigh'd : But now this second Bottle must be fill'd with Water. All things thus prepar'd, and the Valve open'd by Help of the Wire, the Air issuing out with Impetuosity, and entering the Bottle of Water, shall drive the Water out by the Hole at the Bottom. Now 'tis manifest that the Quantity of Water forc'd out in this manner, is = the Bulk and Quantity of Air that issued out of the other Bottle ; Wherefore keeping that Water, let the Bottle, now lighten'd of the compressed Air, be again weigh'd, (for I suppose it to have been weigh'd before, together with the compressed Air) and then 'tis manifest that the superfluous Sand, reserv'd as before order'd, is exactly equal to the Weight of such a Mass of Air, as is the Mass of Water forc'd out and preserv'd, which, if weigh'd, we shall see how many times its Weight shall contain the Weight of the reserv'd Sand ; and we may safely affirm, that the Water is so much heavier than the Air, which will not be only ten times, as seems to be *Aristotle's* Opinion, but nearly 400 times, as this Experiment shews us.

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The other Method is more expeditious, and by making use of one Vessel only ; and that is the former fitted as before directed, excepting that I would have no more Air in it than what is naturally in it ; but I would have Water injected without letting out the least Air, which being forced to give Way to the supervenient Water, must of Necessity be compress'd : Having gotten in as much Water as possible, (which without much Force may be about three Fourths of what the Bottle will hold) put it into the Scales, and weigh it very exactly ; which done, holding the Vessel with the Neck upwards, and opening the Valve, let the Air out, of which there will issue such a Quantity as there is Water in the Bottle. The Air being gone out, put the Vessel again into the Scales, which, because of the Air let out, will be found lighter ; and taking out of the other Scale the Over-weight, it will give us the Weight of such a Mass of Air as is that of the Water in the Bottle.

SIMP. I must confess your Contrivances are very subtle and ingenious : But whilst, on one Hand, they seem thoroughly to satisfy my Understanding, yet, on the other, they really confound me. For it being undoubtedly true, that the Elements in their proper Regions are neither light nor heavy ; I can't conceive how and whence that Portion of Air, which weigh'd, for Instance, four Drachms of Sand, should afterwards have that same Gravity in the Air, in which the Sand, which is weigh'd against it, easily retains it ; Wherefore methinks this Experiment ought not to be made in the Element of Air, but in some *Medium* where

where the Air itself might exercise its Quality of Gravitation, if it really have such Quality.

SALV. A smart Objection, I must own, *Simplicius*, which should be thoroughly answer'd, or own'd unanswerable. That that Air which, compress'd, I demonstrated to weigh as much as that Sand, when set at Liberty in its Element, does no longer gravitate, as, on the contrary, the Sand does, is a thing most manifest : Wherefore to make such an Experiment, we must chuse a Place and *Medium* where the Air, as well as the Sand, may do so ; because, as has been often said, the *Medium* takes off from the Weight of every Matter immers'd therein, so much as is the Weight of a Mass of the same *Medium*, which is = the immersed Mass ; so that I can't deny but that the Air depriveth the Air of all its Gravity. The Experiment therefore ought to be tried in a *Vacuum*, where all Bodies would exercise their Faculties without any Diminution : Wherefore, *Simplicius*, if we weigh any Portion of Air in a *Vacuum*, would you then be convinc'd in this Matter ?

SIMP. Yes, truly ; but this is to desire or expect an Impossibility.

SALV. Surely then you'll own yourself highly oblig'd to me, if for your Sake I do that which is impossible : But I won't offer to sell now what I have already given you : Know then, that in the foregoing Experiment we actually weigh'd Air in a *Vacuum*, and not in Air, or any other full *Medium*. But that from the Gravity, *Simplicius*, of a Mass immers'd in a fluid *Medium*,

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is taken something by that *Medium*, comes to pass from hence, that it resists its being open'd and driven aside, and at length its being rais'd and held up, as we plainly see by its Proneness to return and fill the Space up again, which the immersed Mass occupied, as soon as ever the Mass is taken out, when otherwise, if that *Medium* suffer'd nothing from such Immersion, it would never operate against, or contrary to it.

But now when you hold in the Air, the aforesaid Bottle fill'd with the Air which is naturally contain'd therein, tell me, pray, what Division, Impulse, or, in a Word, what Mutation the external ambient Air receives from that second Air, which was newly forc'd into the Vessel? Doth it enlarge the Bottle whereby the ambient Air must recede to give it Place? No, certainly. Therefore we may certainly say, that that second Air is not immersed in the ambient Air, it not occupying any Space therein, but is as if in a *Vacuum*: nay, it is really so, interposing itself in Vacuities, not quite fill'd up with the former uncondensed Air. And I know not how to discern any Difference between the two Constitutions of the Inclosed and Ambient, because in this the Ambient does no-ways press the Inclosed, and in that the Inclosed doth not in the least oppose the Ambient: And such are the Dispositions of any Matter *in Vacuo*, and of the second Air compressed in the Bottle: Wherefore the Weight which is found in such condens'd Air is the same, which that same Air would have, were it freely distended in a *Vacuum*. 'Tis true, indeed, that the Weight of the Sand which is weigh'd against it, inasmuch as 'tis in the open Air, would *in Vacuo* weigh something more; so that we may say that

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the Air weigh'd is in reality something lighter than the Sand which counterpoises it, *i. e.* so much as an equal Mass of Air would weigh *in Vacuo*.

SIMP. I really thought that there had been something wanting in the foregoing Experiment ; but now I am thoroughly satisfied.

SALV. The Things I have hitherto alledg'd, and particularly this, That the Difference of Gravity, tho' never so great, has not the least to do in altering the Velocities of Moveables, but that all things would move with equal Celerity, let their Difference of Gravity be what it will, is so new, and, at the first thinking, so remote from all Probability, that could I not make it apparent, and as clear as the Sun itself, I had better never have mention'd it : Wherefore since I have mention'd it, I think I should omit neither Experiment nor Reason to establish it.

SAGR. Not only this, but many other also of your Propositions, are so remote from the Opinions and Doctrines commonly receiv'd, that if they were publish'd they would gain you many Adversaries ; the innate Disposition of Men being such, that they look with an envious Eye upon such as discover Truth or Fallacies which they themselves could not find out, calling them Innovators of Doctrines, a Character but few like ; endeavouring thus to cut the Knots they can't untie, and with subterraneous Mines to blow up those Structures which have been erected with ordinary Tools by industrious Architects : But with us here, who are far from any such Thoughts,
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your Experiments and Arguments are fully satisfactory : But nevertheless if you can produce any Experiments more palpable, or Arguments more convincing, we would gladly receive them.

SALV. That Experiment made with two Moveables as different as may be in Weight, by letting them fall from a Place on high, in order to observe whether their Velocities are equal, is subject to some Difficulty ; because if the Height be great, the *Medium* which must be open'd, and laterally repell'd by the *Impetus* of the falling Body, will more resist the small *Momentum* of a very light Moveable, than the Violence of a very heavy one ; whence in a long Fall the heavy one will leave the light one behind ; and in a short Fall there is either in Reality no Difference, or if any, 'tis next to inobservable. Wherefore I have often been thinking to repeat that Descent from a small Height so often, and to collect together those very small Differences of Times which are betwixt the heavy Body and the light one's coming to the Ground, until these so conjoined would make a time not only observable, but very easily so.

Moreover, that I might subject to Observation, Motions as slow as possible, in which the Resistance of the *Medium* operates less as to the Change of the Effect, that depends on simple Gravity, I resolv'd to let go the Moveables upon a declining Plane, not much rais'd above the Plane of the Horizon ; for upon this, no less than perpendicularly, we may find that which is done by heavy Bodies different in Weight : And proceeding farther, that I might free myself from any Impediment which might arise from the Contact of those Moveables, and the declining Plane, I took two

Balls, one of Lead, and the other of Cork, That above a hundred times heavier than This, and fasten'd them on high with two equally fine Threads, each equally four or five Yards long ; then moving each Ball from their perpendicular State, I let them both go at the same Moment ; and they descending by the Circumferences of Circles describ'd by the equal Threads as Semidiameters, presently return the same Way back, till at length it doth, in nearly a hundred Vibrations to and fro, sensibly shew that the heavier Ball moves so exactly in the time the lighter one does, that it does not in a hundred, no nor in a thousand Vibrations out-run it by one Moment of Time, but they keep moving by equal Paces : and here may be noted the Operation of the *Medium*, which in some measure hindring the Motion, doth much more diminish the Vibrations of the Cork than of the Lead, not that it makes more or less of them in Number ; but altho' the Arches run thro' by the Cork are not above five or six Degrees, but those describ'd by the Lead are fifty or sixty Degrees, yet they both describe them in the same time.

SIMP. If this be so, how can it be that the Velocity of the Lead is not greater than that of the Cork, when that runs a Course of sixty Degrees, in the same time that this runs barely six ?

SALV. But what would you say, *Simplicius*, if they should vibrate in the same time, when the Cork removed thirty Degrees from its Perpendicular, should pass over an Arch of sixty Degrees ; and the Lead removed from the same Point only two Degrees, should pass over an
Arch

Arch of four Degrees ; would not then the Cork be so much swifter than the Lead ? And yet Experience shews this to be the Case : Pray observe, therefore, that the *Pendulum* of Lead being mov'd, *v. g.* fifty Degrees from the Perpendicular, and then let go, it swings nearly fifty Degrees beyond the Perpendicular, describing an Arch in the whole of nearly an hundred Degrees ; and then returning of itself back again, it describes an Arch a little less ; and so by continually repeating those Vibrations, after a great Number have been made, it at length is reduced to rest. Now all these Vibrations are made in the same time, as well those of ninety Degrees, as those of fifty, twenty, ten, or four ; and consequently the Velocities of the Mover constantly lessen, because in equal times it doth successively run thro' Arches lesser and lesser.

The like way the self-same Effect is produc'd by the Cork hanging by a String of the like Length, excepting that after a less Number of Vibrations it returns to Rest, as being, on account of its Levity, less fit to overcome the Resistance of the Air ; and yet all the Vibrations great and small, are made in equal times, which are also equal to the times of the Vibration of the Lead : Wherefore it is true, that if whilst the Lead runs thro' an Arch of fifty Degrees, the Cork moves thro' one of but ten Degrees, the Cork is then slower than the Lead : But, on the other hand, it will also happen, that the Cork moveth thro' an Arch of fifty Degrees, whilst the Lead runs thro' an Arch of ten or six ; and so the Lead shall be swifter one while than the Cork, and at another the Cork swifter than the Lead : But if the same Moveables shall also in the same
times

times move thro' equal Arches, we may, without any Scruple, conclude their Velocities to be equal.

SIMP. This Discourse of yours seems to me at the same time to demonstrate the thing, and not to demonstrate it : And I find such a Confusion in my Mind, arising from the same Moveables being mov'd one while swiftly, another slowly, and another while extremely slowly ; that I own I'm in Doubt whether it be true or no that their Velocities are always equal.

SAGR. Pray, *Salviatus*, let me put in a Word or two, and tell me, *Simplicius*, whether you grant that it may be said with absolute Verity, that the Velocities of the Cork and Lead are equal, in case that both of them departing at the same Moment from Rest, from the same Distances from the Perpendicular, do run over equal Spaces in equal Times?

SIMP. This admits of no Doubt ; nor can it be contradicted.

SAGR. It happens now in the *Pendulums* that each of them moves thro', now sixty Degrees, now fifty, now thirty, now ten, eight, four and two Degrees : And when each of them moves over an Arch of sixty Degrees, they do it in the same time, and each Moveable spends the same time in passing thro' an Arch of fifty Degrees ; and so on in Arches of thirty, ten, and the rest Degrees : Whence we may conclude, that the Velocity of the Lead in the Arch of sixty Degrees, is equal to the Velocity of the

the Cork in the same Arch of sixty Degrees: And that the Velocities of each in the Arches of fifty Degrees, are also equal to each other, and so on in the rest. But it cannot be said that the Velocity which is exercised in an Arch of sixty Degrees, is equal to that observ'd in the Arch of fifty Degrees, nor this equal to that which is an Arch of thirty: But in lesser Arches the Velocities will always be less, as is collected from our seeing the same Moveable, as to Sense, spend exactly the same time in running thro' the greater Arch of sixty Degrees, as it requires to run thro' the lesser Arch of fifty, or a less of ten Degrees; and, in a Word, in their being all run thro' always in equal times. It is therefore true, that both the Lead and Cork continually retard their Motions according to the Diminution of the Arches, but that their Harmony is not alter'd in keeping an Equality of Velocity thro' all the same Arches run thro' by them.

Thus much I thought fit to say, more that I might hear whether I have rightly understood *Salviatus's* Meaning, than that I imagined *Simplicius* to stand in need of any clearer Explication than what *Salviatus* gave; which is (as indeed all that he advances are) as clear as possible; and when he often resolves Questions, not only apparently obscure, but even seemingly repugnant to Nature and Truth, he does it by Reasons, Observations, or Experiments so common and familiar to every one, that it has given Occasion to one of the most celebrated Professors of our Age, to esteem less his Novelties as depending on too mean and common Fundamentals; as if the most admirable and most valuable Property of the demonstrative Sciences, was not their arising and flowing from most known Principles,
such

such as every Person understands and grants. But let us go on and feast upon this Diet so light of Digestion. And supposing *Simplicius* to be convinced, that the internal Gravity of different Moveables has no Part in altering their Velocities, but that all Moveables, for any thing that has to do with it, would move with the same Velocities; tell us, *Salviatus*, in what you place the sensible and apparent Inequalities of Motion, and pray answer to that Instance which *Simplicius* produces, and which I agree with him in; I mean when we see a Cannon Ball move more swiftly than a Bird Shot, that that Difference of Velocities is but small, in respect of that which I object against you, in Moveables of the same Matter, of which some of the greater will descend in one Second that Length which others will not run thro' in the same *Medium* in one, in four, or in twenty Hours; such are Stones and small Grains of Sand, especially that very fine Sand which makes Water muddy: In which *Medium* it will not descend two Yards in many Hours, which Stones, and those not very large ones, will pass thro' in one Second of Time.

SALV. What Effect the *Medium* produces in retarding Moveables, more according as they are amongst themselves of less specifick Gravity, has been already declared, by shewing it to be done by Subtraction or Diminution of Weight: But to explain how the same *Medium* can with so much Difference diminish the Velocity in Moveables, differing only in Magnitude, altho' they are of the same Matter and Figure, requires a more subtile Discourse than that which suffices to make us understand how the more

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dilated Figure of a Moveable, or the contrary Motion of the *Medium* to the Moveable, retards the Velocity of the said Moveable:

And for my Part I take the Reason of the present Problem to be the Scabrosity and Porosity, which is commonly and generally found in the Surfaces of solid Bodies, and which, whilst the Body keeps moving, continually push and thrust against the Air, or other ambient *Medium*: An evident Sign of which we have when we hear Bodies, tho' made as round as possible, to hum as they pass swiftly thro' the Air; nay, which indeed we hear not only hum, but also to hiss or whistle, if any more than ordinary Cavity or Prominence be found in them. Thus all round Bodies, in their turning round, we see excite a little Wind. But what need of many Words? Don't we hear a *Top* to make a notable humming, and in a very acute Tone, whilst it turns round on the Ground the swiftest? The Acuteness or Shrillness whereof becomes continually flatter or lower as the Velocity of its Whirling about doth gradually slacken, which is an Argument that the Scabrosities, tho' very small on their Surfaces, do necessarily encounter and strike against the Air. Thus 'tis not to be doubted but these, in the Descent of Moveables, rubbing against the fluid *Medium*, must retard their Velocities, and so much the more, by how much the Surface is greater; as are the Surfaces of lesser Solids when compared with those of greater.

SIMP. Hold a little, I pray, for here I find myself at a Loss; for tho' I understand and grant that that mutual Confrication of *Medium* and Surface must retard the

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Motion

Motion of the Moveable, and more so where, *Cæteris Paribus*, the Surface is greater ; yet I don't comprehend upon what Grounds it is that you assert, that the lesser Solids have the greater Surfaces. And furthermore, if, as you affirm, the greater Surface infers the greater Retardation, the greater Solids, I should think, should be the slower ; the contrary of which is true : But this may be easily solv'd by saying that altho' a greater Solid has a greater Superficies, yet it hath also a greater Gravity, and therefore the Impediment which the greater Surface opposes to the greater Gravity, does not exceed the Impediment which the lesser Surface objects to the lesser Gravity ; so that the Velocity of the greater Solid becomes not less. Therefore I see no Reason why the Equality of the Velocities should be alter'd, since so much as the moving Weight is diminish'd, so much the Faculty of the retarding Surface is also diminish'd.

SALV. I'll answer all your Objections at once. You'll admit, *Simplicius*, without Controversy, that if of two equal Moveables of the same Matter, and alike in Figure, (which Bodies, without doubt, would move with equal Velocity) as well the Gravity as Surface of one of them be diminished, (retaining their Likeness in Figure) that yet the Velocity does not decrease in that which is diminish'd.

SIMP. I think it ought to follow as you say, still having regard to our Doctrine, which teaches that neither the greater nor the lesser Gravity operates at all in the Acceleration or Retardation of Motion.

SALV. Right ;

SALV. Right ; whence it seems to me to follow, that if the Gravity diminishes more than the Surface, in a Moveable so diminished there may be some Retardation of Motion introduc'd, and that greater and greater, by how much in Proportion the Diminution of the Weight was greater than the Diminution of the Surface.

SIMP. I don't at all question it.

SALV. Now you must know, *Simplicius*, that the Surfaces of Solids can't be diminished so much as the Weights, provided they keep the same Likeness of Figure : For it being manifest, that when a heavy Body is diminished, it loses as much of its Weight as of its Bulk, whenever the Bulk is diminished more than the Surface, (the Similitude of Figure being retain'd) the Weight is diminished also more than the Surface. But *Geometry* teacheth us, that the Proportion in like Solids, between Bulk and Bulk, is greater than that between Surface and Surface : which, that you may the better see thro', I will explain by the following Example.

Conceive a Cube, the Length of each of whose Sides is two Inches, one of its Faces then will be four square Inches ; and thence all the six Faces, that is the whole Surface of the Cube will be twenty-four square Inches. If then you imagine the same Cube to be cut by three Sections into eight lesser Cubes, the Length of the Side of any one of them will be one Inch, and each of its Faces one square Inch ; and therefore the whole Superficies of each of these will be six square Inches ; of which the

whole Cube contained in its Superficies twenty-four. Hence you see now, since the superficial Content of the lesser Cube is a fourth Part of the Surface of the greater, (six being the fourth Part of twenty-four) but the solid Content of the said Cube is only the eight Part; that therefore the Bulk, and consequently the Weight, is much more diminished than the Superficies.

But now if you subdivide that Cube into eight others, for the whole Surface of one of these we shall have one square Inch and a half, which is but the sixteenth Part of the Surface of the first Cube; but its Bulk is only the sixty-fourth Part of the Bulk of that. You see, therefore, how that in only these two Divisions the Bulks are diminished four times more than their Surfaces are: And if we should prosecute this Subdivision until the first Solid be reduced into fine Dust, we should find the Gravity of the most minute Particles to be diminished above a hundred times more than their Surfaces.

And this which I have exemplified in Cubes, happens in all like Solids, the Bulks of which are in a *sesquialteral* Proportion of their Surfaces. You see therefore in how much a greater Proportion the Impediment which arises from the mutual Contact of the Moveable's Surface and the *Medium*, increases in lesser Moveables than in greater: And if we add that the Scabrosities in the very small Surfaces of those Corpuscles are not, may be, less than those which are in the Surfaces of greater Solids, which are carefully polish'd; you see how fluid that *Medium* must be, to give Passage to so feeble a Power. Observe therefore, *Simplicius*, that I did not equivocate when just now

I said that the Surfaces of lesser Solids, are greater in Comparison to those of greater Solids.

SIMP. I am thoroughly satisfied, and, believe me, were I to begin my Studies again, I would follow *Plato's* Advice, and begin with Mathematicks, which I see proceed with the utmost Caution, and refuse to admit any thing as certain but what they demonstrate.

SAGR. This Discourse has given me great Pleasure ; but before we go any farther, I should be glad to understand a *Term* you us'd, *Salviatus*, just now, when you said that similar Solids are to one another in a *sesquialteral Ratio* of their Surfaces : I have, indeed, seen and understood that Proposition and its Demonstration, wherein 'tis prov'd that the Surfaces of similar Solids are in a *duplicate Ratio* of their Sides ; and that other also which proves the same Solids to be in a *triplicate Ratio* of the same Sides : But I don't remember that I ever till now heard so much as mention'd the Proportion between Solids and their Surfaces.

SALV. You have answered yourself, and cleared your own Doubt : for that which is the *Triple* of any thing whereof any other is *Double*, doth not that *Triple* become the * *Sesquialter* of that *Double* ? Certainly it does. Now if Surfaces are in a double Proportion of those Lines of

* One and a half : thus in the Numbers 4, 2, and 6 ; 6 is the Triple of 2, whereof 4 is the Double, then 6 is Sesquialter, or $1\frac{1}{2}$ of 4.

which

which the Solids are in triple Proportion, may not we say that Solids are in a sesquialteral Proportion of their Surfaces ?

SAGR. I understand you now very well : And tho' I have some other Enquiries to make, relating to some particular Matters already treated of, yet by running from Digression to Digression, we should make it too long before we come to the Questions principally intended, which concern the Diversity of Accidents of the Resistances of Solids against Fraction. If you please, therefore, let us resume the first Thread, or what was proposed at the Beginning of this Discourse.

SALV. Well mention'd. But so many and such different Things as we have examin'd, have stolen away so much of our Time, that there's but little of the Day left for our principal Business ; wherein many Geometrical Demonstrations are necessary, and which must be consider'd with Attention : wherefore I should think it better to adjourn our Meeting till to Morrow, as well on the aforesaid Account, as because then I might bring with me some Papers, wherein I have set down in order those Theorems and Problems, wherein the different Passions of that Subject are proposed and demonstrated, which may be would not in a proper Method come now into my Mind.

SAGR. I very gladly comply with you, and so much the more willingly, in that, for a Conclusion to this Day's Conference, I shall have Time to hear you explain some Doubts which remain with me, concerning the Matter

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last handled. One of which is this, Whether we are to esteem the Impediment of the *Medium* sufficient to assign Bounds to the Acceleration of the greatest and heaviest Bodies, which are of a spherical Form : I instance in such as are of a spherical Form, as that which is comprehended under the least Surface, and thence the less subject to Retardation. Another shall be touching the Vibrations of *Pendulums* ; and this hath many Heads: One shall be, Whether all great, mean, and very small, are made really and exactly in equal Times : And another, What the Proportion is of the Times in Moveables, when fasten'd to unequal Strings ; I mean the Times of their Vibrations.

SALV. These are Questions of great Moment ; and, as it happens in all Truths, I doubt not but which ever of them we handle, it will draw after it so many other Truths and curious Consequences, that the Remainder of the Day will hardly suffice for the discussing them all.

SAGR. If they are but as pleasant as the foregoing ones, I could with all my Heart spend as many Days as we have Hours to Night : And I dare say *Simplicius* will be no more tired with such Reasonings than I shall be.

SIMP. No indeed ; especially when the Questions treated on are Physical, and which no other Philosophers have touch'd, or given us their Opinions about.

SALV. To the first then : I positively affirm that there is no Sphere so big, or of Matter so heavy, but that the
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Renitence of the *Medium*, tho' never so thin, checks its Acceleration, and, in the Continuation of the Motion, reduces it to Equability ; an Argument of which we may learn from Experience. For if any falling Moveable, in the Continuation of its Motion, be able to attain to any Degree of Velocity, no Velocity conferred upon it by the external Mover could be so great, but it would get rid of and free itself from it by means of the Impediment of the *Medium*. Thus a Cannon Ball, that hath descended in the Air, *e. g.* four Yards, and hath acquired, for Instance, ten Degrees of Velocity, with which it enters the Water ; if the Impediment of the Water were not able to restrain its *Impetus*, it would acquire a greater Velocity, or with the same would move to the Bottom, which yet we don't see to follow : But the Water, altho' but a few Fathoms deep, does restrain and weaken it, so that it hits the Bottom of the River or Lake very gently : 'Tis therefore manifest, that that Velocity which the Water had Power to deprive it of, in so little a Depth, it would never suffer it to acquire again, tho' in the Depth of a thousand Fathoms : For why should it permit it to acquire that in a thousand Fathoms, which afterwards it will take from it in four ? But what need of more ? Don't we see the immense *Impetus* of the Ball shot from the Cannon itself to be so deaden'd by passing thro' a few Fathoms of Water, that, hardly reaching the Ship, it hits it without any manner of Damage ? Nay, even the very Air, altho' very yielding, doth nevertheless repress the Velocity of the falling Moveable, tho' it be very heavy, as we may collect from like Experiments : For if from the Top of a very high Tower, we should discharge a Musket downwards,

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this will make a less Impression on the Ground, than if it should be discharged at the Height of but four or six Yards above the Plane ; an evident Sign that the *Impetus* wherewith the Bullet rushes from the Gun, discharged on the Top of the Tower, in its Descent thro' the Air, is continually diminished : Therefore the Descent from any great Height whatsoever, will not suffice to make it acquire that *Impetus*, of which the Resistance of the Air depriv'd it, and which had been by any Means whatsoever conferr'd upon it. In like manner I don't believe the Blow of a Bullet, shot perpendicularly from an immense Height, will be with that Force as it would was the Blow of the same Bullet to strike at the Distance of twenty Yards. I believe therefore there is a Bound or Term in every natural Moveable that departs from Rest, and that the Impediment of the *Medium* at length reduces it to Equability, which afterwards it always maintains.

SAGR. These Experiments are truly, in my Opinion, much to the Purpose : And nothing remains, unless the Adversary should stoutly deny that they will hold true in very great and very heavy Masses ; and that a Cannon Ball coming from the Concave of the Moon, or from the upper Region of the Air, would make a greater Percussion than if shot from a Cannon.

SALV. No doubt but many Things may be objected, which Experience cannot solve : Yet in this Objection, methinks, there is something may fall under Consideration ; to wit, that it is very likely that a heavy Body falling from so great a Height, by that Time it reaches the Earth,

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has got such an *Impetus* as is required to throw it to that Height it fell from, as is plainly seen in a *Pendulum* reasonably heavy, which, being mov'd fifty or sixty Degrees from the Perpendicular, acquires such a Velocity and Power as exactly sufficeth to carry it again to the same Height, abating only that little which is taken from it by the Impediment of the Air. To place then a Cannon Ball at such a Height, in falling from which it may acquire such an *Impetus* as it receives when fired out of the Cannon, it would suffice to shoot it upwards perpendicularly from the same Cannon, and then observing whether in its Fall it makes an Impression equal to that Percussion made near at Hand on its issuing forth from the Piece. But indeed I believe it would be much weaker ; Wherefore I hold that the Velocity with which the Ball is carried at its being just sent out of the Piece, is such as the Impediment of the Air will never suffer it to acquire whilst it descends with a natural Motion, leaving the State of Rest from any very great Height.

I come now to the other Questions relating to *Pendulums*, Matters which to some would seem very frivolous, and more especially to those Philosophers who are continually employ'd in the more profound Questions of natural Philosophy : Yet I am far from contemning them, encouraged by *Aristotle's* Example, in whom I can't help admiring this, *viz.* that he hath left nothing unhandled, as one may say, that's worth one's Consideration. And as to your Questions, I think I can give you a Conceit of my own, relating to some Problems concerning *Musick*, a noble Subject, of which so many great Men, and even *Aristotle* himself, have written ; and touching which he considers

considers many curious Problems : And if I, from easy and sensible Experiments, can deduce Reasons of the wonderful Accidents of Sounds, I may hope my Discourse may be grateful to you.

SAGR. Not only grateful, but by me in particular most desirable ; for I take great Delight in all Musical Instruments : And altho' I have thought much concerning Consonances, yet I never could comprehend whence it arises, that one should more please me, and give me greater Delight than another ; that some others do not only not delight me, but, on the contrary, highly offend me. And again, that common Problem concerning two Strings set to an Unison, one of which actually soundeth upon striking the other, I, as yet, cannot solve ; nor do I clearly understand the Forms, and some other Particulars relating to Consonances.

SALV. I will see whether from these our *Pendulums* I can deduce any thing to answer all these Difficulties.

And first as to your Question, *viz.* Whether the same *Pendulum* doth really and exactly perform all its Vibrations, greatest, mean, and least, in Times precisely equal, I refer myself to that which I have long since learnt of our *Academic*, who plainly demonstrates, that a Moveable descending along the Chord subtending any Arch, passes them all in equal Times, *i. e.* as well the Subtense of 180° , (*i. e.* the whole Diameter) as the Subtense of 100° , or 60 , 10 , 2 , $\frac{1}{2}$ a Degree, or of four Minutes, supposing them all to terminate in the lowest Point, which touches the horizontal Plane : Again, concerning Descendents by

the Arches of the same Chords, elevated above the Horizon, Experience likewise shews that they all are passed over in equal Times, but in shorter than the Times in which the Chords are run thro'; an Effect so wonderful, that at first View one would think the contrary should happen; for the Terms of the Beginning and Ends of the Motion being common, and a right Line being the shortest of those which are comprehended between the same Terms, it seems reasonable that the Motion made thro' it should be perform'd in the shorter Time; which yet it quite otherwise, and the shortest Time, and consequently the swiftest Motion, is that which is made thro' the Arch of which that right Line is the Chord.

In the next Place: As to the Times wherein the Vibrations of Moveables, suspended by Strings of different Lengths are performed, they are in subduple Proportion of the Lengths of the Strings; or, if you will, the Lengths are in a duplicate Ratio of the Times, *i. e.* as the Squares of the Times; so that supposing, for Example, the Time of a Vibration of one *Pendulum*, to be double the Time of a Vibration of another *Pendulum*, it necessarily follows, that the Length of the String of that *Pendulum* be quadruple of the Length of the String of this: And then in the Time of one Vibration of that, another will make three Vibrations, when the String of that is nine times as long as that of the other: From whence it follows, that the Lengths of the Strings have the same Proportion to one another, as have the Squares of the Numbers of the Vibrations made in the same Time.

SAGR. So

SAGR. So that, if I understand you aright, I may easily know the Length of a String hanging from any never so great a Height, tho' I can't see its upper End, by which it is fasten'd on high, but only its lower End: For if I fasten Weight enough to the said String here below, and set it a vibrating to and fro, letting some Friend number some of its Vibrations. Then if I at the same time count the Vibrations of another Moveable hung by a String exactly a Yard long; then from the Number of Vibrations which those *Pendulums* make in the same time, I shall find the Length of that String: Thus, for Example's Sake, suppose that in the time that my Friend hath counted 20 Vibrations of the longer String, I have counted 240 of my String that is a Yard long: The Numbers 20 and 240 being squar'd, which are 400 and 57600, I can pronounce the longer String to contain 57600 of those Measures whereof my String contains 400; and because this is but one Yard in Length, I will divide 57600 by 400, and the Quotient will be 144: And so many Yards I affirm that String is in Length.

SALV. And you'll not be mistaken one Inch, especially if you take a sufficient Number of Vibrations.

SAGR. You give me frequent Occasion to admire the Riches, and withal the extraordinary Bounty of Nature, whilst from Things so common and mean, you deduce, one way or other, many curious and new Notions, and such oftentimes as are remote from all Imagination. I have very often carefully consider'd the Vibrations in particular

ticular of the Lamps, which in some Churches hang by very long Ropes, and which have been by Chance stirr'd or mov'd by any one : But the most that I inferr'd from that Observation was this, that their Opinion was very improbable, who will have that such Motions are maintain'd and continu'd by the *Medium*, *i. e.* by the Air : For I should think the Air endued with great Judgment, and with the greatest Ease, to cause an hanging Weight to vibrate for Hours together with so much Regularity : But I should never have learnt from hence, that if the same Moveable, suspended by a String a hundred Yards long, now elevated above the lowest Point ninety Degrees, now only one, or half a one, that, I say, it should spend as much time in moving thro' the least Arch, as it does in moving thro' the greatest ; I don't think this would ever have entered into my Head, and, to be plain, I still think it next to impossible. But now I stand prepar'd to hear how from these the most simple and plain Things can be assign'd me such Reasons concerning those Musical Problems, as in some measure to give me Satisfaction about them.

SALV. Above all things you must know that every *Pendulum* hath the Time of its Vibrations so limited and prefixed, that it is impossible to make it vibrate in any other Period than that which is natural to it : For let any one take the String the Weight is fasten'd to in his Hand, and let him try to increase or lessen the Number of its Vibrations, and he shall find his Labour to be in vain : But we may, on the contrary, to a *Pendulum*, tho' it be heavy, and at Rest, give a Motion by only blowing upon

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it, and, repeating the Blast, a Motion considerable ; but this second Blast must be made at an Instant suitable to the Motion of the Vibrations ; so that if at the first Blast we move it half an Inch from the Perpendicular, and then again, if when it has returned to us, and is beginning its second Vibration, we add a second Blast, we shall confer a new Motion to it ; and thus by successively repeating the Blasts at proper Times, but not when the *Pendulum* is moving towards us, (for so we should check and not help the Motion) we may confer upon it such an *Impetus*, that a much greater Force than our Breath will be required to stop it again.

SAGR. I have observ'd, when I was a Boy, that one Man alone by such Impulses, given at right Times, has been able to raise a very great Bell ; and when four or six have taken hold of the Rope to cease the Bell from Motion, they have been all raised from the Ground ; they all being not able to withstand the *Impetus*, which one Man alone, with regular Pulls, had conferred upon the Bell.

SALV. An Example that declares my Meaning as fully as my foregoing one is suitable to render the Reason of that admirable Problem of the Strings of the Viol or Lute, which moves and makes not that only to sound, which is tun'd to an Unison with it, but that also which is set to an Eighth, and to a Fifth : The String being touch'd or struck, its Vibrations begin and continue so long as its Sound lasts ; these Vibrations give a Vibration and Tremor to the adjacent Air, whose Tremors and Circulations,
extending

extending themselves a great Way, strike upon all the Strings of the same Instrument, as also upon the Strings of any others near it : That String that is set to an Unison with that touched, being disposed to make its Vibrations in the same Time, at the first Impulse begins to move a little ; and upon the Addition of a second, a third, and twentieth, and many more, all made in fit and periodic Times, it receives at last the same Tremulation with that first touched, and, as plainly may be seen, it dilates its Vibrations exactly according to the Dilatation of those of its Mover.

This Undulation that extends itself thro' the Air, moves and makes to vibrate, not only the Strings, but likewise any other Body disposed to Tremulation, and to vibrate in the same time with the trembling String ; wherefore if upon the Sides of the Instrument, several small Particles of Hair, or Bristles, or any other flexible Matter, be laid, sounding the Viol, we shall see now this, now that Corpuscle tremble, as this or that String is struck, whose Vibrations are made in the same Time ; but the others will not move at the Striking or Sound of this String ; nor will that Corpuscle move at the Sounding of any other String.

If the Base String of a Viol be smartly struck with a Bow, and a thin and smooth Drinking-Glass be set by it ; if the Tone of the String be an Unison with the Tone of the Glass, the Glass will tremble and sensibly resound : Again, we plainly see the Circulation of the *Medium* about the resounding Body, to diffuse to a large Space, by making a Glass to sound, that has some Water in it, by rubbing the Rim or Edge of it with the Tip of one's
Finger :

Finger : For we shall thus see the Water in the Glas to undulate in a most regular Order ; which Effect will yet more clearly be seen, if we put the Foot of the Glas in the Bottom of a Vessel of reasonable Bigness, and fill it with Water nearly to the Glas's Rim, and then make it sound by rubbing it round, as before, with the Tip of one's Finger, for then we shall see the Circulations in the Water to be most regular, and with great Velocities to spread to a great Distance round about the Glas : Nay, I have often happen'd to see, in making a pretty big Glas, almost full of Water, to sound as before, the Waves form'd with an exact Equality ; but the Tone of the Glas happening sometimes to rise an Eighth higher, I have seen at that very Instant every one of the said Waves to divide themselves into two : which Accident most plainly proves the Form of the Octave to be the Double.

SAGR. The same Thing has happen'd to me more than once, to my great Delight and Advantage too : For I stood a long time in Doubt concerning the Forms of Consonance, not thinking the Reasons commonly brought by the learned Authors, who have hitherto wrote of Musick, sufficiently demonstrative. They tell us that the *Diapason*, that is the *Octave*, is contain'd by the *Double* ; and that the *Diapente*, which we call *the Fifth*, is contain'd by the *Sesquialter* : For if a String, stretch'd upon the marine Trumpet, be sounded open, and afterwards placing a Bridge under the Midst of it, its half only be sounded, you'll hear an Eighth : And if the Bridge be placed under one third of the String, and you then strike the two thirds open, it soundeth a Fifth to that of the whole String struck

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when open : whereupon they infer that the Eighth is contain'd between two and one, and the Fifth between three and two. But I don't think we can conclude from hence, that the *Double* and *Sesquialteral* can naturally assign the Forms of the *Diapason* and *Diapente* : And my Reason for't is this : There are three Ways by which we may sharpen the Tone of a String, *viz.* by shortning it, by stretching it, or by making it thinner : If now, retaining the same Tension and Thicknes, we would hear an Eighth, we must make it shorter by half, *i. e.* we must first sound the whole String, and then its half. But if keeping the same Length and Thicknes, we would have it rise to an Eighth from its present Tone, by stretching it, or screwing it higher, 'tis not sufficient to stretch it with a Double, but with four times the Force : Thus, if at first it was stretch'd by a Weight, suppose of one Pound, we must hang a four Pound Weight to it, in order to raise its Tone an Eighth. And lastly, if keeping the same Length and Tension, we would have a String to sound an Eighth, this String must be but one fourth of the Thicknes of the other it must sound an Eighth to. And this that I say of the Eighth, *i. e.* that its Form taken from the Tension or Thicknes of the String in a duplicate *Ratio* to that which it receiveth from the Length, I would have understood of all other Musical Intervals ; because if that which the Length gives us in a *Sesquialter* Proportion, *i. e.* when first by striking it open, and then its two thirds, we would do the same from the Tension, or Tenuity, or thinning the String, we must double the *Sesquialter* Proportion, by taking the double *Sesquiquartan* : And if the grave String be stretch'd by a four Pound Weight to the sharp one, we must hang not
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fix but nine Pounds : And as to the Thickness ; we must make the grave String thicker than the acute, in the Proportion of nine to four, to gain the Fifth.

These Things being really so in Fact, I saw no Reason why these sage Philosophers should rather constitute the Form of the *Eighth Double* than *Quadruple* ; and the Form of the *Fifth* rather *Sesquitertian*, than the *double Sesquiquartan*. But because 'tis impossible to number the Vibrations of the String, which, while it sounds, are made very swiftly ; I should always have been in Doubt whether or no it were true, that the String sounding the higher *Eighth* did perform double the Number of Vibrations, in the same Time to those of the graver String, unless the Undulations, which may be continued as long as you please, by keeping the Glass sounding and vibrating, had sensibly shewn me, that in the very Instant wherein sometimes we hear a Tone to jump to an *Eighth* ; there are seen to arise other lesser Undulations, which, with infinite Accuracy divide each of the former into two.

SALV. An excellent Observation this ! whereby we can distinguish one by one the Undulations produced by the Tremor of the sounding Body ; which are those which, diffusing themselves thro' the Air, cause a Titillation on the Drum of our Ear, whence in our Soul is produc'd a Sound. But since we can't see and observe those Undulations in the Water, any longer than that Rubbing with the Tip of the Finger lasteth, and they being not at a Stay, but in constant Motion all that Time, wou'd'nt it be a great Thing to contrive such as we might exquisitely

observe, as long as we please, for Months if we will, nay, for Years, whence we should be able exactly to measure, and easily to count them?

SAGR. I assure you I should highly esteem such an Invention.

SALV. The Discovery was merely accidental; the Observation and applying it to Use was my own: I own I like the Thing, and have often consider'd it with great Satisfaction, tho' the Experiment at first View may seem to be of little or no Account or Worth.

Scraping a Copper Plate with an Iron Chizel to take out some Spots in it, upon moving the Chizel quick to and fro, amongst the many Attritions or Rubbings, I, more than once, heard it send forth a whistling Noise or Sound; and then looking upon the Plate, I espied a long Row of small Streaks, parallel to one another, and exactly equidistant. Returning to my Scraping again, I found, by several Trials, that the Chizel left those Streaks upon the Plate, at those Scrapings, and those only that yielded the aforesaid Noise: But when the Attrition was made without such Whistling, there appear'd not the least Sign of any such Streaks: Repeating the Experiment, for my farther Satisfaction, several times afterwards, scraping one while with greater, another with less Velocity, the Whistling I found to have one while a sharper Tone, and another a graver; and I observ'd the Marks made at the sharper Sounds, to be more, and closer together; and fewer, and farther asunder, at the grave Sounds: And furthermore, as the Swiftneſs towards the End of the self-same Scrape
grew

grew greater than that at the Beginning, the Sound was heard continually to grow sharper, and the Streaks were observ'd to stand thicker, but always made with the greatest Regularity, and exactly equidistant from one another : And furthermore, when in scraping such Sound was heard, I felt the Chizel to tremble in my Hand, and a certain Shivering to run along my Arm ; and, in short, I observ'd the very same Thing effected by this Tool, which may be observ'd in ourselves when we whisper, and afterwards speak aloud ; for when we send forth our Breath without forming a Sound, we do not perceive any Motion in the Throat or Mouth, in Comparison of that Tremor which we feel in the Wind-Pipe and Jaws, upon sending forth the Voice, especially if the Tones emitted be grave and strong. I have, moreover, at such a time observ'd, amongst the Strings of the Viol, two to sound Unisons; to two of the Sounds made by scraping after the Manner aforesaid, and exact Fifths to one another : then measuring the Intervals of the Streaks of both the Attritions or Scrapes, I found forty-five Spaces of the one to be equal to thirty of the other, which, indeed, is the Form attributed to the *Diapente*.

But now, before I proceed any farther, I must tell you, that of the three Ways of making a Tone more acute, that which you refer to the Slenderness or Fineness of the String, may with more Truth be attributed to its Weight : The Alteration taken from the Thickness, answers indeed when the Strings are of the same Matter, and so a String of Gut to sound an Eighth, must be four times as thick as the other Gut-string, below which it is to sound an Eighth ; and one of Wire also four times thicker than
another

another of Wire : but if betwixt two Strings, one of Wire, and the other of Gut, I would get an Eighth, I must not make it four times thicker, but four times heavier ; so that, as to the Thickness, the Wire one shall not be four times thicker, but four times heavier than the other ; for then the Wire one will be finer than that of Gut which sounds an Eighth above it. Whence it comes to pass, that stringing one Instrument with Gold Strings, and another with Brass ones, provided the Strings of the one Instrument be of the same Length, Thickness, and Tension with those of the other, the Tone of the Gold Strings shall be about a Fifth lower than that of the Brass ones, Gold being nearly double the Weight of Brass. And here I would have you observe, that the Gravity of the Moveable more resists the Velocity of the Motion, than the Thickness does, contrary to what one at first would think ; for, indeed, in Appearance, 'tis more reasonable that the Velocity should be retarded, by the Resistance of the *Medium*, against dividing or opening, more in a thick and light Moveable, than in a heavy and small one : and yet in this Case it happens quite contrary. But to pursue our first Purpose ; I say, that 'tis neither the Length, nor the Tension, nor the Thickness, that constitutes the nearest immediate Reason of the Forms in Musical Intervals, but the Proportion of the Number of Vibrations and Percussions in the Undulations of the Air, striking the Drum of our Ear, which itself also doth tremulate under the same Measures of Time.

Having establish'd this Point, we may perhaps be able to assign a just Reason whence it comes to pass, that of Sounds differing in Tone, some Pairs are heard with great
Delight,

Delight, others with less ; and that others are very offensive to the Ear. This we may do, I say, by seeking the Reason of the more or less perfect Consonances, and also of Dissonances. The Offence these give, proceeds, I believe, from the discordant and jarring Pulsations of two different Tones, which, without any Proportion, strike the Drum of the Ear : And the Dissonances will be extreme harsh, in case the Times of the Vibrations are incommensurable. And of such this is one ; When one of two Strings, Unisons, is sounded with such a Part of another, as is the Side of a Square of its Diagonal ; which Dissonance is like to the *Tritone* or *Semi-diapente*.

Those Pairs of Sounds shall be Consonances, and will be heard with Pleasure, which strike the *Timpanum* in some Order ; which Order requires, in the first Place, that the Percussions made in the same Time be commensurable in Number, that the Cartilage of the *Timpanum* or Drum may not be subject to a perpetual Torment of bending itself two different Ways, in Submission to the ever disagreeing Percussion.

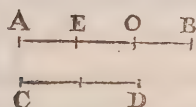
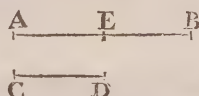
The first and most grateful Consonance will be the *Eighth*, since for every Stroke wherewith the grave String strikes the Ear, the acute one gives two ; so that *both*, in every second Vibration of the sharp String, strike at once, or together ; wherefore half the whole Number of Strokes agree in striking together ; but the Strokes of the Strings that are *Unisons*, are always struck together, and therefore are but as one only String, and make no Consonance.

The *Fifth* also is delightful, where, for every two Strokes of the grave String, the sharp one gives three ; whence it follows, that if we count the Vibrations of the acute String,

String, we shall find the third Part of the whole Number to agree, and strike together ; that is, between each Pair of Consonances, there are two single Sounds : and in the *Diatefferon* there are three single ones interpos'd. In the second, that is, in the *Sesquioctave* Tone in nine Pulsations there's only one that strikes in Concord with the other of the graver String : and all the rest are Discords, and fall upon the *Timpanum* irregularly and troublesomely, and thence by the Ear are esteem'd as Dissonances.

SIMP. I wish you would explain this Discourse a little more.

SALV. Let the Line AB be the Space and Dilatation of one Vibration of the grave String, and the Line CD of the acute one, which are Eighths to each other : Bisect AB in E ; then 'tis manifest that if the Strings begin to



move in the Terms A and C, that when the acute Vibration shall be come to the Term D, the other will be extended only to its half, or to its middle Point E, which not being the Term or Bound of the Motion, it doth not yet strike, but yet a Stroke is made in D. Then whilst

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the Vibration returns from D to C, the other passes from E to B, upon which the two Percussions of B and C now strike both together upon the *Timpanum*; and because they continue to repeat in like manner the subsequent Vibrations, we may conclude that the Union of the Percussions of the Line CD, with the Percussions of the Line AB happen every other Time: But the Pulsations of the Terms AB are always accompanied, and are always the same with one of CD; which is manifest from hence, that supposing A and C to strike together, whilst A passes to B, C goes on to D, and goes back to C; so that the Strokes B and C are made together.

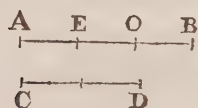
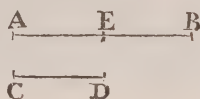
But now let AB and CD be two Vibrations which produce the *Diapente*, whose Times are in a *Sesquialteral* Proportion, and divide the Line AB of the grave String into three equal Parts, by the Points E and O; and suppose the Vibrations to begin at the same Time from the Terms A and C; then it is manifest, that when the Stroke is made in D, the Vibration of AB is got only to O; wherefore the *Timpanum* receives the Pulsation D only. Then whilst it returns from D to C, the other Vibration is pass'd from O to B, and return'd to O, first making a Pulsation in B, which is also made alone, and in Counter-time (a Thing to be consider'd) for since we suppos'd the first Pulsations to be made at the same Moment in the Terms A and C, the second which was made alone from the Term D was made so long after as is the Time of Transition thro' CD, that is AO: but the following one which is in B, is distant from the other, as much as is the Transition OB only, which is the half: but it goes on from O to A, in the Time the other passes from C to D,

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and

and thus are two Pulsations made together in A and D : There afterwards follow other Periods like these, that is, with the Interposition of two single Pulsations of the acute String by itself; and one single Pulsation of the grave String, alone or by itself, between those two single Strokes of the acute one.

Wherefore supposing the Time divided into Moments, *i. e.* into its least equal Particles, and supposing that in the two first Moments, the concordant Pulsations made in A and C, to pass to O and D, and that in D a Stroke is made; and that in the third and fourth Moment, one passes from D to C, and strikes in C; and the other goes on from O to B, and returns to O, striking in B: and in the fifth and sixth Moment the one passes from C to D, and the



other from O to A; then striking together we shall receive the Pulsations on the *Timpanum* distributed in this Order, that supposing the Pulsations of the two Strings to be made in the same Moment, two Moments after it shall receive one solitary Percussion, in the third Moment another, also solitary, in the fourth, another; and two Moments after, that is, in the sixth, two together; and here ends the Period, and *Anomaly*, if I may so say; which Period is often, after the same Manner, repeated.

SAGR. I can hold no longer! but must loudly express the Pleasure I have to hear such apposite Reasons assigned of Effects, concerning which I have been so long in the Dark. Now I understand how it comes to pass, that an *Unison* differs not from a single Tone: Now I see why the *Octave* is the principal Consonance, and so like to the *Unison*, that it is taken and conjoin'd with others as an *Unison*. It resembles an *Unison*, because, as the Pulsations of Strings set to an *Unison* all strike together, the Pulsations of the grave String, which sounds an Eighth, are all accompany'd with Beats of the acute String, and of these one solitary one is interpos'd, and in equal Distances, and in a certain Order, without any Variety; whence such Consonance, without the least Allay, becomes most melodious.

But the *Fifth*, with those its Counter-times, and which the Interpositions between the Pairs of conjoin'd Pulsations of two solitary ones of the acute String, and one solitary one of the grave one, and these three in such Interval of Time, as is its half, produces such a Titillation upon the Cartilage of the *Timpanum*, that, allaying the Sweetness by a Mixture of Tartness, it seems at one and the same Time to kiss and bite.

SALV. Since I see you are so pleas'd with these Noveltyes, I'll shew you a Method whereby not the Ear only, but the Eye also, may be recreated, in beholding the same Sports the Ear feels or hears.

Hang Balls of Lead, or other heavy Matter, by three Strings of different Lengths, in such Manner, that in the Time the longest makes two Vibrations, the shortest makes

four, and the middle one three; which will happen when the longest contains sixteen Feet, or other Measures, such as the middle one contains nine, and the shortest four. And removing them all at once from Perpendicularity, and then letting them go, you'll see a various Intermixture of those *Pendulums* with various Accidents, but so that at every fourth Vibration of the longest, all the three will concur together in one and the same Term; which they afterwards leave and then again repeat the same Period; And this Mixture of Vibrations is the same with that which being made in Strings of Instruments, presents to the Ear an *Eighth*, with an intermediate *Fifth*. And if we order the Length of other Strings in the like Disposition, so that their Vibrations answer to those of other musical consonant Intervals, we shall still see new Connections, and those always such as all the Strings (be they three or four) will, in determinate Times, and after determinate Numbers of Vibrations, concur in the same Moment, at the Term of their Vibrations, and thence they again begin another like Period; But if the Vibrations of two or more Strings, are either incommensurable, so that by vibrating to and fro, they never harmoniously perform a determin'd Number of Vibrations; or if they are not incommensurable, and yet return not till after a long Time, and after a great Number of Vibrations, then the Sight is confounded by the irregular and confus'd Order of irregular Intermixtures, as the Ear with Regret receives the intemperate Impulses of the Air's Tremulations, which, without Order or Rule, successively strike its *Timpanum* or *Drum*.

But whither, Gentlemen, have we been transported for so many Hours, by various Problems, and unlook'd for Discourses?

Discourses? We have made it Night, and yet have handled few or none of the Points first propos'd; but, on the contrary, have so lost our Way, that I hardly remember where we begun, and what that small Introduction was, which, by Way of *Hypothesis*, we laid down as a Principle to build upon in the Demonstrations which were to follow.

SAGR. True, it will be convenient, therefore, that we put an End to this Day's Conference, and refresh our Minds by the Night's Repose, and To-morrow (if you please so far to favour us) we will resume the Discourses desir'd, and at first chiefly intended.

SALV. I'll not fail to be here at the usual Hour, to enjoy your Company, and do you what Service I can.

The END of the FIRST DAY'S CONFERENCE.

GALILÆUS



G A L I L Æ U S
H I S
D I A L O G U E S.

D I A L O G U E II.

*Concerning the CAUSE of the COHERENCE
in SOLIDS.*

SALVIATUS, SAGREDO, *and* SIMPLICIUS.

SAGR.  HILST *Simplicius* and I waited your coming, we have been recollecting that last Consideration, which, as the Principle and Supposition whereon you ground the Conclusions which you proposed to demonstrate to us, was that Power by which all Bodies resist *Fractiō*, depending on that Cement which connects and holds together their Parts, so as not to suffer them to give Way and separate without a strong Attraction : Afterwards we began
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an Enquiry into the Cause of that Coherence, which in some Solids is very strong : and I remember that was proposed as the Chief, which is deduced from a *Vacuum* ; which afterwards occasion'd so many Digressions, as employ'd us the whole Day, and carried us so far from the Matter first proposed, which was the Contemplation of the Resistances of Solids to Fræction.

SALV. I remember all this very well : To return then to the Place where we began : Whatever we suppose this Resistance of Solids to Fræction, by a violent Attraction, to be ; it is sufficient to us that there is undoubtedly in Solids such a Thing, and which, tho' it be very great, against the Strength of one that draws them directly, or strait out, yet 'tis observ'd to be less, if the Force be applied transversely or sideways.

Thus we see, for Example, a Rod of Steel or Glass, to sustain lengthways a Weight of a thousand Pounds, which, fasten'd at right Angles to a Wall, will break with a Weight of fifty Pounds only. Now this second Resistance shall be the Subject of our present Discourse, where we shall enquire in what Proportion it is found in Prisms and Cylinders of like or unlike Figure, Length and Thickness, provided they be of the same Matter : In which Enquiry, I take for a Maxim that, which in *Mechanics* is demonstrated amongst the Properties of the *Vectis*, which we call the *Leaver* ; namely, that in the Use of the Leaver, the Force is to the Resistance in a reciprocal Proportion of their Distances, *i. e.* the Distances of the Force and Resistance from the *Fulcrum*.

SIMP. *Aristotle*

SIMP. *Aristotle* first of all demonstrated this in his *Mechanics*.

SALV. I allow the Precedency to *Aristotle*, as to Time, but as to the Firmness of Demonstration, I think *Archimedes* ought to be preferr'd before him : and that on Account of a single Proposition by him demonstrated in his Book *De Equiponderantibus*, whereon depends the Reasons, not only of the Leaver, but also of the greater Part of the other Mechanical Instruments.

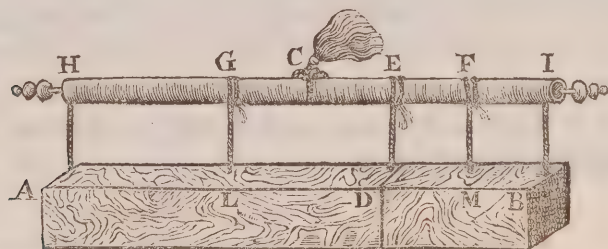
SAGR. But since this Principle is the Foundation of all that you intend to demonstrate to us, it would be requisite, and very kind of you, if it ben't too much Trouble, to give us its Demonstration, and thereby a full and perfect Satisfaction concerning it.

SALV. However necessary this is, yet it will be better that I introduce you into the Field of all our future Speculations, by an Entrance different from that *Archimedes* used : and here I shall take nothing for granted, but that equal Weights put into a Ballance of equal Arms, make an *Equilibrium* (a Principle likewise supposed by *Archimedes* himself); afterwards, I may demonstrate to you, that it is no less true, that unequal Weights make an *Equilibrium* in a Ballance of Arms unequal, according to the Proportion of those Weights reciprocally suspended : and that it is the same thing to place equal Weights at equal Distances, as to place unequal Weights at such Distances, which are in a reciprocal Proportion to their Weights.

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Now to demonstrate these Things clearly, Describe a Cylinder or Prism as AB, hung by its Ends to the Line HI, and sustain'd by the Cords HA and IB. Now 'tis manifest, that if I suspend the Whole by the Cord C, plac'd in the Midst of the Ballance HI, the Prism AB will remain in *Equilibrio*, one half of its Weight being on one Side, and the other half on the other of the Point of Suspension C, by the Principle by us presupposed. Now suppose the Prism to be divided into unequal Parts, by a Plane passing thro' D, and let DA be the greater Part, and DB the lesser : And that such Division



being made, the Parts of the Prism may remain in the same Situation and Constitution in respect of the Line HI, apply the Cord ED, which tied in the Point E, may sustain the Parts DA and DB of the Prism : Then there can be no Doubt, since there is no Local Mutation made of the Prism, in respect of the Ballance HI, but it will remain in the same State of Equilibration : But it will rest in the same Situation, if the Part AD of the Prism, which is now suspended at the two Extremes by the two Cords AH and DE, be hang'd at one single Cord placed in the Middle ; And in like manner the other Part DB will

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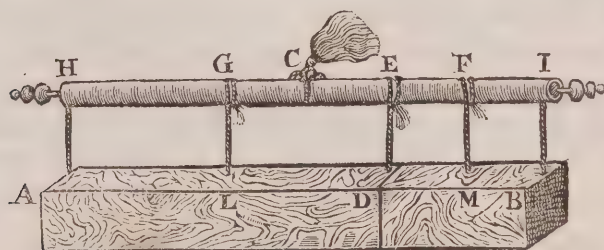
not change its State, if suspended by the Middle, it be sustained by the Cord FM : So that the Cords HA, ED, and IB being untied, and only the two Cords GL and FM being left, the *Equilibrium* will still remain ; as if the Suspension was still made at the Point C.

Here now let us consider that we have two heavy Bodies AD and DB, hanging from the Terms G and F of the Beam GF, in which the *Equilibrium* is kept at the Point C, in such manner that the Distance of the Point of Suspension G, from which the Weight AD hangs from the Point C, may be the Line CG ; and the other Part CF be the Distance at which the other Weight DB hangs. It remains then only to be demonstrated, that those Distances have the same Proportion to one another, as the Weights themselves have, but reciprocally taken ; that is, that the Distance GC is to the Distance CF, as the Prism DB to the Prism DA : Which I thus demonstrate ;

Since the Line GE is half the Line EH, and EF the half of EI, therefore GF will be the half of the whole Line HI, and thence will be = CI ; and taking away the common Part CF, the Remainder GC will be = the Remainder FI, that is to FE : Then adding the common Part CE, the two Lines GE and CF will be equal to each other : And therefore it will be $GE : EF :: FC : CG$; but as GE to EF, so the Double to the Double ; that is, so is HE to EI, that is the Prism AD to the Prism DB. And therefore by Equality and Conversion of Proportions, as the Distance GC to the Distance CF, so is the Weight BD to the Weight DA. *Q. E. D.*

If you understand what has hitherto been said, I believe you will easily admit, that the two Prisms AD and DB

make an *Equilibrium* in the Point C, because one half of the whole Solid AB, is on the right Hand of the Point of Suspension C, and the other half to the left Hand ; and that in this manner there are exhibited two equal Weights, disposed and extended at two equal Distances. Again, that those two Prisms AD and DB, afterwards reduced into two Cubes, or two Balls, or into two any other Figures,



(provided that they keep the same Points of Suspension G and F) do retain an *Equilibrium* in the Point C, I believe you don't at all doubt, since 'tis manifest that Bodies change not their Weight, if the same Quantity of Matter is retain'd : From which we may gather this general Conclusion, That any two Weights whatever make an *Equilibrium*, at Distances answering reciprocally to their Gravities.

This Principle then being establish'd, before I proceed, I am to examine how these Forces, Resistances, *Momenta*, and Figures, may be consider'd abstractedly, or separate from Matter ; as also in Concrete, or conjoin'd with Matter : and by these Means, those Accidents that agree with Figures consider'd as immaterial, shall receive certain Modifications, when we join Matter to them, and consequently Gravity.

Gravity. As for Example ; suppose a Leaver as BA, apply'd to the *Fulcrum* E, in order to raise the heavy Stone D.

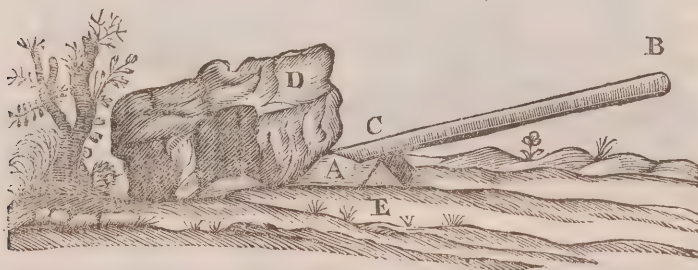
Now 'tis manifest, from the Principle before demonstrated, that the Power plac'd at the Extremity B, is sufficient to equal the Resistance of the Weight D, if so be its *Momentum* have the same Proportion to the *Momentum* of the said Weight D, as the Distance AC has to the Distance CB : and this is as true, if we consider no other *Momenta* than those of the simple Power in B, and of the Resistance in D ; as if the said Leaver was immaterial.



and destitute of all Gravity. But if at the same time we consider the Gravity of the Instrument or Leaver itself, which may be made either of Wood or Iron, 'tis manifest that the Weight of the Leaver, join'd with the Power in B, alters the Proportion ; which it will be requisite to declare in other Terms : Wherefore, before we go any farther, 'tis necessary that we distinguish between these two Ways of Consideration ; calling that a taking it absolutely, when we understand the Instrument in Abstract, that is, separate from the Gravity of its own Matter. But if with the simple and absolute Figures we conjoin Matter and Gravity, these

these Figures so join'd with Matter, we'll call the *Momentum* or compounded Power.

SAGR. I must of necessity break the Resolution I had taken, *viz.* to give you no Occasion of Digression ; for I shall not be able to apply myself with Attention to what follows, unless you remove a Scruple that just offers itself: 'Tis this ; You seem to me to make Comparison between the Force plac'd in B. with the total Gravity of the Stone D, one Part of which Weight, methinks, if not the greater Part, rests upon the Plane of the Horizon, so that ———

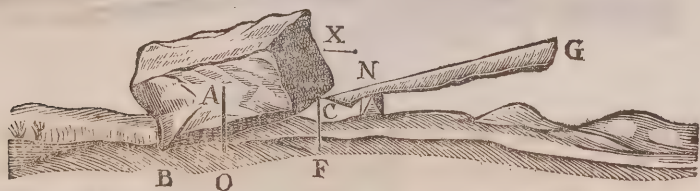


SALV. I understand your Meaning, so that you need say no more ; only take Notice that I nam'd not the total Gravity of the Stone, but spake of the *Momentum* that it has and exercises at the Point A, the extreme Term of the Leaver BA, which is always less than the whole Weight of the Stone, and is variable according to the Figure of the Stone, and according as it is more or less rais'd.

SAGR. I

SAGR. I am satisfied in that Particular ; but I have another Request, and that is this, that for my entire Satisfaction, you'll please to demonstrate to me, if it can be done, how I may find what Part of the whole Weight that is which is sustained by the subjacent Plane, and what that which gravitates upon the Leaver at the Extremity A.

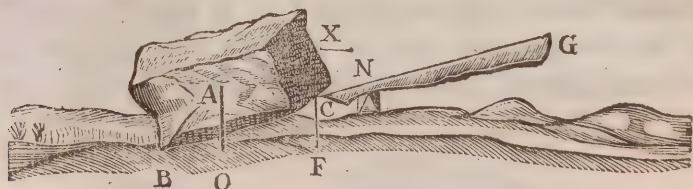
SALV. Since I can satisfy you in a few Words, I'll do it : In the annexed Figure suppose the Weight, whose Centre of Gravity is A, to rest upon the Ground with one



End B, and at the other End lifted up by Help of the Leaver CG, upon the *Fulcrum* N, by a Power plac'd in G ; then from the Centre A and Term C let fall to the Horizon two Perpendiculars AO and CF : Then, I say, that the *Momentum* of the whole Weight has to the *Momentum* of the Power in G a Proportion compounded of the *Ratio* of the Distance GN to the Distance NC, and of the *Ratio* of FB to BO.

Make as the Line FB to the Line BO, so NC to X ; And since the whole Weight A is sustain'd by the two Powers plac'd in B and C, the Power B will be to the Power C, as the Distance FO to the Distance OB ; and by Composition, the two Powers B and C together ; that is,

is, the whole *Momentum* of the whole Weight A, is to the Power in C, as the Line FB to the Line BO, that is, as NC to X : but the *Momentum* of the Power in C, is to the *Momentum* of the Power in G, as the Distance GN to the Distance NC : Therefore, by Perturbation of Proportion, the whole Weight A will be to the *Momentum* of



the Power in G, as GN to X. But the Proportion of GN to X, is compounded of the *Ratio* of GN to NC, and of the *Ratio* of NC to X, that is of FB to BO : Therefore the Weight A, to the Power which sustains it in C, has a Proportion compounded of the Proportion of GN to NC, and of the Proportion of FB to BO.
Q. E. D.

And now to return to our first Purpose : If the Things hitherto declared are well understood, it will be easily conceiv'd, That,

PROP. I.

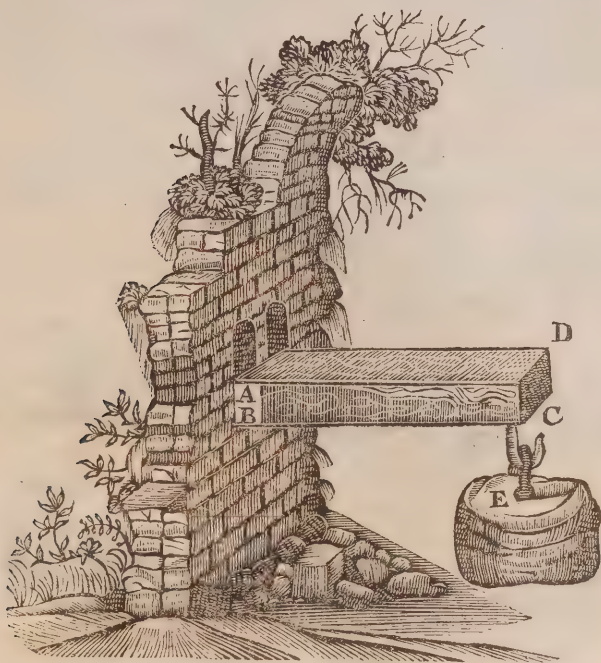
A Prism or Cylinder of Glass, Steel, Wood, or any other frangible Matter, which, suspended lengthways, will sustain a very great Weight hang'd thereat, will, side-

ways

†

ways or transversely (as we just now said) be broken in Pieces by a far less Weight, according as its Length shall exceed its Thickness.

Imagine the Prism ABCD to be fix'd into a Wall at the End AB, and at the other End imagine the Weight E to be hang'd, (the Wall here I understand to be perpendicular to the Horizon, and the Prism or Cylinder to be perpendicular to the Wall) then 'tis manifest, if it must break,

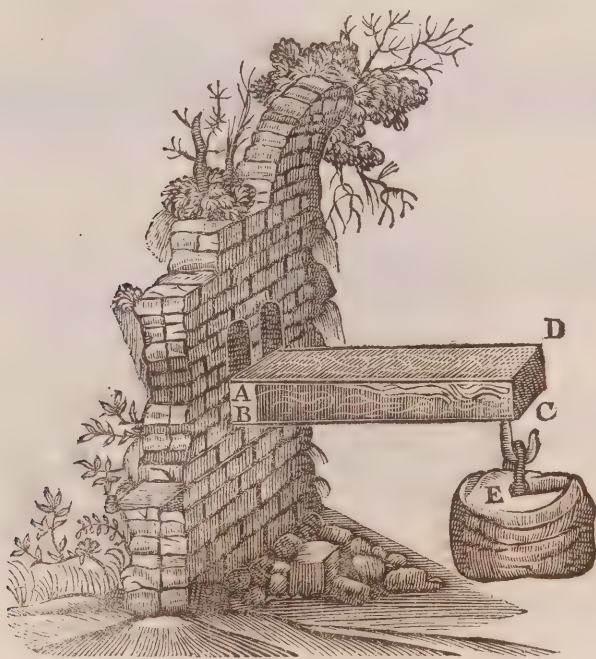


it will be in B, where the Mortise in the Wall is in the Room of a *Fulcrum*, and the Line BC that Part of the Leaver where the Force is plac'd, and BA, the Thickness of the Solid, is the other Part of the Leaver, wherein is

Z

the

the Resistance to that Separation which must be made of BD, the Part of the Solid hanging out of the Wall, from that which is let into the Wall : And by what has been already shewn, the *Momentum* of the Power plac'd in C, to the *Momentum* of the Resistance, which is in the Thickness of the Prism, *i. e.* in the Connection of the Base BA with the contiguous Parts, hath the same Proportion as the Length CB has to half of BA ; and therefore the absolute



Resistance against Fraction in the Prism BD, (the absolute Resistance is that which is made if the Prism be drawn directly ; for then the Motion of the Mover is the same with that of the Body mov'd) to the Resistance against
 Fraction

Fraction by Help of the Leaver BC, has the same Proportion as the Length BC has to half AB in the Prism, which Half in the Cylinder is the Semidiameter of its Base. And this is the first Proposition. But note that what I have here said must be understood without the Consideration of the Weight proper to the Solid BD, which Solid I took as depriv'd of all Weight. But in case we would consider its Weight, together with the Weight E, we must add to the Weight E half the Weight of the Solid BD, so as the Weight of BD being, *e. g.* two Pounds, and the Weight E ten Pounds, we must take the Weight E as if it was eleven Pounds.

SIMP. And why not as if it was twelve, I pray?

SALV. The Weight E, *Simplicius*, hanging at the End C, gravitates in respect of the Leaver BC, with all its *Momentum* of ten Pounds, when if only BD hang there, it would gravitate with its whole *Momentum* of two Pounds; but, as you see, that Solid is uniformly distributed thro' its whole Length BC; wherefore its Parts near the End B, gravitate less than those farther off: So that in the End, those being compensated with these, the Weight of the whole Prism is brought to operate under the very Centre of Gravity which answers to the Middle of the Leaver BD: But a Weight hanging at the End C, has a *Momentum* double of that it would have hanging from the Middle: And therefore the half of the Weight of the Prism must be added to the Weight E, when we use the *Momenta* of both, as if they were placed at the End C.

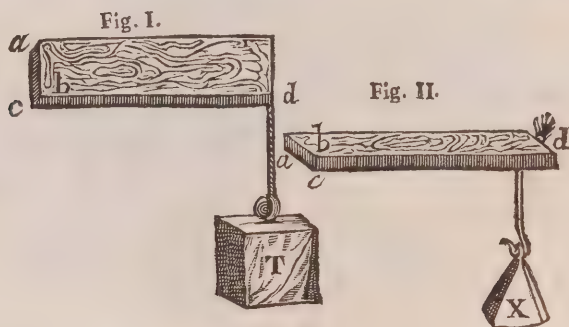
SIMP. I apprehend you very well ; and farther, unless I'm deceiv'd, the Power of both Weights BD and E so placed, would have the same *Momentum* as if the whole Weight of BD, with twice the Weight E, were hung in the Middle of the Leaver BC.

SALV. You're very right ; and pray remember this. Now we may easily understand,

P R O P. II.

How, and with what Proportion, a Ruler or a Prism broader than thick, resists Fraction more if forc'd according to its Breadth, than according to its Thickness.

The better to understand this, imagine a Prism or Ruler *ad*, whose Breadth is *ac*, and Thickness much less *bc* ;



Now the Thing here enquired after is this, *viz.* how it comes to pass that it will resist the great Weight *T*, if we attempt to break it edgeways, as in *Fig. I.* but plac'd
flatways,

flatways, as in *Fig. II.* it will not resist the Weight *X*, which is less than *T* : And the Thing is plain, since we suppose the *Fulcrum* in one case to be under the Line *ac*, and in the other, under *bc*, and the Distances of the Forces to be equal in both Cases, *viz. cd* ; But in the former Case the Distance of the Resistance from the *Fulcrum*, which is half the Line *ca*, is greater than the Distance in the other Case, which is half the Line *cb* : Wherefore the Power of the Weight *T* must be necessarily greater than the Weight *X*, as much as the half of the Breadth *ca* is greater than half the Thickness *bc*, that serving for a Counter-Leaver to *ca*, and this to *cb*, to overcome the same Resistance, *i. e.* the Quantity of Fibres of the whole Base *ab*.

Wherefore we conclude that the same Ruler or Prism, which is broader than it is thick, resists breaking more if plac'd edgeways than flatways ; and that in Proportion of the Breadth to the Thickness.

'Tis requisite now that we begin,

P R O P. III.

To find in what Proportion the *Momentum* of the proper Gravity increases in a Prism or Cylinder, in relation to the proper Resistance against Friction, whilst keeping its Parallelism to the Horizon it is made longer and longer : And this *Momentum* I find increases continually in a *duplicate* Proportion of that of the Prolongation.

For

For Demonstration whereof, imagine a Prism or Cylinder AB firmly fasten'd in the Wall at the End A, so as to be parallel to the Horizon, which afterwards imagine to be prolong'd to E, by adding thereto the Part BE. It is manifest that the Prolongation of the Leaver AB to C



by itself alone, *i. e.* taken absolutely, increases the *Momentum* of the Force pressing against the Resistance to Fraction or Separation to be made in A, according to the Proportion of CA to BA; but moreover the *Weight* of the Solid BE adjoin'd to the *Weight* of the Solid AB, increaseth the *Momentum* of the pressing Gravity, according to the Proportion of the Prism AE to the Prism AB, which Proportion is the same with that of the Length AC

to
†

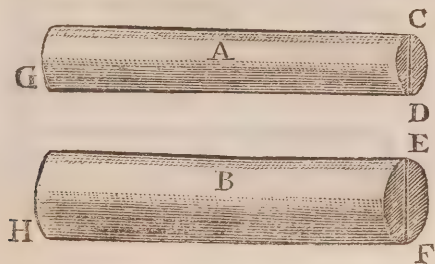
to the Length AB. Wherefore 'tis manifest that if the two Augmentations of both Lengths and Gravities be put together, the *Momentum* compounded of both is in a *duplicate Ratio* of either of them. We conclude, therefore, that the *Momenta* of the Forces of Prisms and Cylinders of equal Thickness, but unequal Lengths, are to one another in a *duplicate Ratio* of that of their Lengths; that is, are as the Squares of their Lengths.

We will shew in the second Place, in what Proportion the Resistance to Friction in Prisms and Cylinders increases, supposing them of the same Length, but to increase in Thickness: And here I assert, That,

P R O P. IV.

In Prisms and Cylinders of equal Length, but unequal Thickness, the Resistance against Friction increases in a *triplicate* Proportion of the Diameters of their Thicknesses, that is, of their Bases.

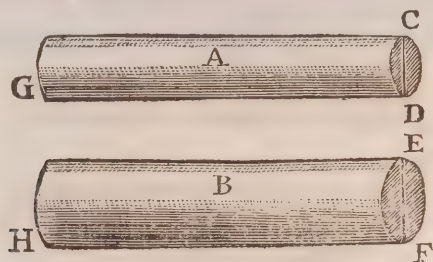
Let the two Cylinders be A and B, whose equal Lengths are DG and FH, and whose unequal Bases are the Circles



whose Diameters are CD and EF. I say that the Resistance of the Cylinder B against Friction, to that of the Cylinder

Cylinder A, is in a *triplicate* Proportion of that of the Diameter FE, to the Diameter DC : For if we consider the absolute and simple Resistance that resides in the Bases, *i. e.* in the Circles EF and DC to Fraction, by being forcibly pull'd directly or endways ; without all Doubt the Resistance of the Cylinder B is so much greater than that of the Cylinder A, as is the Circle EF greater than the Circle DC, for the Fibres, Filaments, or tenacious Parts, which hold together the Parts of the Solids, are so many the more.

But if we consider that we, when we apply the Force transversely, do make use of two Leavers, of which the Parts, or Distances at which the Forces are applied, are the Lines DG, FH, the *Fulcra* are in the Points D and F, but the other Parts or Distances, at which the Resistances are plac'd, are the Semidiameters of the Circle DC, EF ; because since the Filaments are dispersed thro' the whole Superficies of the Circles, it is the same thing as if they were all reduced into the Centres : If, I say, we consider



such Leavers as these, we shall find the Resistance in the Centre of the Base EF, opposed to the Force of H, to be
so

so much greater than the Resistance of the Base CD, against or opposed to the Force plac'd in G, (since the Forces in G and H are of equal Leavers DG and FH) as much as the Semidiameter FE is greater than the Semidiameter DC: the Resistance against Friction therefore in the Cylinder B, is more than the Resistance of the Cylinder A, according to both the Proportions, *i. e.* of the Circles EF and DC, and of their Semidiameters, or, if you will, of their Diameters: but the Circles are in a *duplicate* Proportion of their Diameters; therefore the Proportion of their Resistances, which is compos'd of them, is *triplicate* of that of their Diameters. *Q. E. D.*

But now because Cubes are in a *triplicate* Proportion of their Sides, we may likewise conclude, *That the Resistances of Cylinders of equal Length, are to one another as the Cubes of their Diameters.*

From what has been demonstrated, we may likewise conclude, That,

C O R O L L A R Y.

The Resistances of Prisms and Cylinders of equal Length are in *Sesquialter* Proportion of that of the Cylinders themselves.

This is manifest, because the Prisms and Cylinders of equal Height, have the same Proportion to each other as their Bases have, that is, *duplicate* of the Sides or Diameters of their Bases. But the Resistance (as hath been demonstrated) have a *triplicate Ratio* of the same Sides or Diameters: Therefore the Proportion of the Resistances is *Sesquialter* to that of the said Solids, and consequently of the Weights of the said Solids.

SIMP. Before we go any farther, I desire you'll answer me one Doubt, and that is this : I have not heard mentioned or consider'd another Sort of Resistance, that, in my Opinion, is diminished in Solids as they are longer and longer, and not only in taking them transversely, but also directly or longways ; just as we see a very long one less fit to sustain a great Weight than a shorter is : Wherefore I believe a Ruler of Wood or Iron would sustain a much greater Weight, if it be short, than it would do if it were very long, provided it be always used lengthways, and not transversely, its own Weight comprehended, which in the longer is greater.

SALV. I fear, *Simplicius*, that you, with many others, are out in this Point, if I rightly apprehend your Meaning ; You mean, I suppose, thus, that a Rope, *v. g.* forty Yards long, cannot sustain so great a Weight as one or two Yards of the same Rope will.

SIMP. I do mean so, and to me the Proposition seems very probable.

SALV. But I think it not only improbable, but false, as I shall easily make it appear : Suppose then this Rope AB to be fasten'd on high, by the End A ; and at the other End let hang the Weight C, by the Force of which the Rope shall break ; pray, *Simplicius*, assign me the particular Place where it will break.

SIMP. Let it be in D.

SALV. And

SALV. And why, pray, must it break in D?

SIMP. Because the Rope is not strong enough in that Part to sustain, *e. g.* an hundred Pounds, which is the Weight of the Part of the Rope BD, together with that of the Stone C.



SALV. Whenever then we shall apply to this Rope the Weight of an hundred Pounds, it will break in D?

SIMP. I think so.

SALV. But tell me now, if one should hang the same Weight not at the End of the Rope B, but to any Point as E, near the Point D; or else should fasten the Rope not in A, but just above the same Point D, as at F: Tell me, I say, whether the Point D would feel the same Weight of an hundred Pounds?



SIMP. It would feel it, provided to the Stone C be added the Part EB of the Rope.

SALV. If then the Rope be drawn by the said hundred Pounds of Weight, it will break in D, by your own Concession; and yet FE is but a small Part of the long Rope

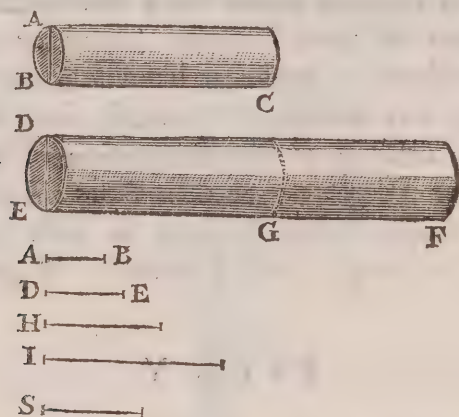
Rope AB: Why therefore do you continue in the Opinion that the long Rope is weaker than the short one? I hope you're now satisfied, and that I have freed you from an Error, wherein you have had many Companions, and these otherwise very learned Men. Now let us proceed; and having now demonstrated that the *Momentum* of Prisms and Cylinders increases above their Resistances, according to the Squares of their Lengths (provided they are of the same Thickness); And that likewise if they are of the same Length, but different Thickness, they increase their Resistances according to the Proportion of the Cubes of their Sides, or Diameters of their Bases: Let us proceed, I say, and enquire what happens in such Solids, which differ both in Length and Thickness: Where I observe, That,

P R O P. V.

The Resistances against Friction of Prisms and Cylinders of different Length and Thickness, are in a Proportion compounded of the Proportion of the Cubes of the Diameters of their Bases, and of the Proportion of their Lengths reciprocally taken.

Let

Let two such Cylinders be ABC and DEF : I say, the Resistance of the Cylinder AC, shall be to the Resistance of the Cylinder DE, in a Proportion compounded of the Proportion of the Cube of the Diameter DE, and of the Proportion of the Length EF to the Length BC.



Suppose $EG = BC$, and let H be a third Proportional, and (I) a fourth, between the Lines AB and DE : And as EF is to BC , so let (I) be to S . Since now the Resistance of the Cylinder AC is to the Resistance of the Cylinder DG , as the Cube of AB to the Cube of DE ; that is, as the Line AB to the Line (I) : and the Resistance of the Cylinder DG to the Resistance of the Cylinder DF , as the Length FE to the Length EG ; that is, as the Line (I) to the Line S : It will be, by Equality of Proportion, as the Resistance of the Cylinder AC to the Resistance of the Cylinder DF , as the Line AB to the Line S : But the Line AB has to the Line S a Proportion compounded

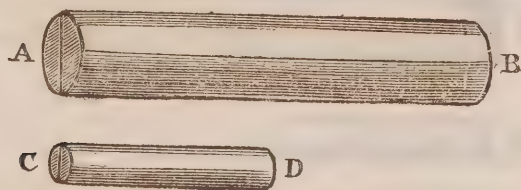
pounded of the Proportion of AB to (I), and of the Proportion of (I) to S: Therefore the Resistance of the Cylinder AC, is to the Resistance of the Cylinder DF, in a Proportion compounded of the Proportion of AB to (I), *i. e.* of the Cube of AB to the Cube of DE, and of the Proportion of the Line (I) to the Line S, *i. e.* of the Length EF to the Length BC. *Q. E. D.*

This Proposition being demonstrated, let us consider what happens in similar Cylinders and Prisms; concerning which we will demonstrate, That,

PROP. VI.

Of like Cylinders and Prisms the *Momenta* compounded, *i. e.* those resulting from their Gravities, and from their Lengths, which are of the Nature of Leavers, have to one another a Proportion *Sesquialter* to that the Resistances of their Bases have.

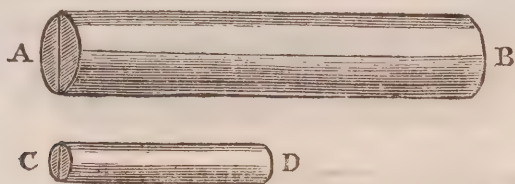
In order to demonstrate which, describe two similar Cylinders AB and CD: Then I say that the *Momentum*



of the Cylinder AB, by which 'tis able to overcome the Resistance of its Base B, hath a Proportion to the Resistance of the Base D, *Sesquialter* to that which the same Resistance of the Base B has to the Resistance of the Base D:

And

And because the *Momenta* of the Solids AB and CD, to overcome the Resistances of their Bases, are compounded of their Gravities, and the Forces of the Leavers ; and the Force of the Leaver AB is = the Force of the Leaver CD, because the Length AB has the same Proportion to the Semidiameters of the Base B, (the Cylinders being similar) as the Length CD has to the Semidiameter of the Base D : It remains that the whole *Momentum* of the Cylinder AB, be to the whole *Momentum* of CD, as the sole Gravity of the Cylinder AB to the sole Gravity of the Cylinder CD, *i. e.* as the Cylinder AB to the Cylinder CD : But these Cylinders are in a *triplicate* Proportion of the Diameter of their Bases B and D : and since the Resistances of the same Bases are to each other as the Bases themselves are, they are consequently in a *duplicate* Proportion of the Diameters of their Bases : Therefore the *Momenta* of Cylinders are in a *Sesquialter* Proportion of that which the Resistances of their Bases have.



SIMP. This Proposition, indeed, is not only new to me, but unexpected, and at first Sight very remote from the Judgment I should have made concerning it ; for these Figures being in all other Respects alike, I should have
 certainly

certainly concluded, That their *Momenta* also should have observ'd the same Proportion to their proper Resistances.

SAGR. This is the Demonstration of that Proposition, which, at the Beginning of our Discourse, I thought I had a Sort of a Glimpse of.

SALV. Some time ago I myself thought as you did; I believ'd that the Resistances of like Solids were alike, until a certain Observation, tho' not altogether to be relied on, seem'd to represent to me that similar Solids do not preserve the same Tenor in their Toughness, but that the bigger are less apt to sustain violent Accidents, or Shocks; as in falling, lusty Men are more liable to Hurt than little Children. And (as we said at our Beginning) as we see a great Beam or Column, falling from on high, to break, whilst a small Stick does not, tho' falling from the same Height. This is the Observation which put me upon enquiring after that wonderful Property which I am now going to demonstrate to you, *viz.* That of the infinite Number of similar Solids, there are not two whose *Momenta* keep the same Proportion to their proper Resistances.

SIMP. Now you bring into my Mind something which *Aristotle* has among his Mechanical Questions, where he endeavours to give a Reason why Beams become weaker, and bend more, the longer they are, altho' the longer are thicker than the shorter: And, if I remember right, he reduces it to that of the simple Leaver.

SALV. 'Tis very true ; And because that Solution seem'd to leave some Room for Doubt, Monsignore *di Guevara*, who in his truly learned Commentaries hath greatly illustrated this Work, not to mention his other subtile Speculations, sets himself to remove all these Difficulties : Yet he himself remain'd still doubtful in this Point, *viz.* Whether the Lengths and Thicknesses of such Solids, increasing in the same Proportion, ought to retain the same Proportion of Force and Resistance against Fraction and Flexion.

After a long Consideration of this Matter, I hit upon what I am going to tell you : And first I will demonstrate, That,

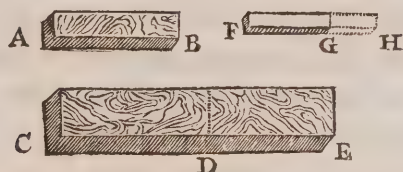
P R O P. VII.

In similar and heavy Prisms and Cylinders, there is one and but one, that (charg'd with its own Weight) is reduced to the ultimate State betwixt breaking and not breaking ; so that every greater, as being unable to resist its own Weight, will break ; and every lesser will be able to resist some additional Force apply'd to break it.

Let AB be a heavy Prism, of the utmost Length it can possibly be to hold without breaking, so that if it be lengthen'd never so little, it will instantly break : I say, that this is the only one, amongst all those that are similar to it, (which are infinite in Number) which is capable of being reduced into such a doubtful and ticklish State, that every greater will break by its own Weight, and every lesser will not ; but, on the contrary, will be able to resist or bear some additional Weight.

First ;

First ; Let the Prism CE be similar and greater than AB : I say, this cannot consist, but must break, being overcome by its own Gravity : Suppose the Part CD to be of the same Length with AB : Now because the Resistance of CD, is to that of AB, as the Cube of the Thickness of CD to the Cube of the Thickness of AB, *i. e.* as the Prism CE to the Prism AB, (since they are similar) and therefore the Weight of CE is the utmost that can be sustain'd at the Length of the Prism CD ; but the Length CE is greater ; therefore the Prism CE will break.



Secondly ; Let FG be a similar Prism, and less than AB ; we will likewise demonstrate, (supposing FH = BA) that the Resistance of FG is to the Resistance of AB, as the Prism FG to the Prism AB, provided the Distance AB, *i. e.* FH be = FG ; but it is greater : therefore the *Momentum* of the Prism FG, plac'd in G, is not sufficient to break the Prism FG.

SAGR. The Demonstration is short and clear ; and now the Truth and Necessity of the Proposition plainly appears, which at first Sight seem'd very improbable : It would therefore be requisite to alter sufficiently the Proportion betwixt the Length and Thickness of the greater Prism, by making it thicker or shorter, in order to reduce

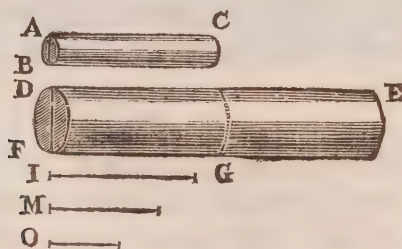
it to that nice and dubious State between breaking and not breaking : And now the Investigation of this State would, methinks, be no less subtle and ingenious.

SALV. Nay, rather more, as it is more laborious : I'm sure I spent no little time in finding it out ; and I will now impart it to you.

PROP. VIII. PROB. I.

A Cylinder or Prism of the greatest Length, not to be broken by its own Weight, and also a greater Length being given, to find the Thickness of another Cylinder or Prism, which is the only one and biggest of that given Length that can resist its own Weight.

Let the Cylinder BC be the greatest that can resist its own Weight, and let DE be a Length greater than AC ; it is required to find the Thickness of the Cylinder that is the greatest of the Length DE that can resist its own Weight. Let (I) be a third Proportional to the Lengths



DE and AC ; and as DE is to (I), so let the Diameter FD be to the Diameter BA ; and make the Cylinder FE :

Then

Then I say, that this is the greatest and the only one amongst all that are similar to it, that can sustain its own Weight. To the Lines DE and (I), let M be a third Proportional, and O a fourth ; and suppose FG to be = AC : Because the Diameter FD is to the Diameter AB, as the Line DE to (I), and O is a fourth Proportional to DE and (I), the Cube of FD will be to the Cube of BA as the Line DE to O : But as the Cube of FD to the Cube of BA, so is the Resistance of the Cylinder DG to the Resistance of the Cylinder BC ; therefore the Resistance of the Cylinder DG is to the Resistance of the Cylinder BC, as the Line DE to the Line O.

And because the *Momentum* of the Cylinder BC is = its Resistance, if we shew that the *Momentum* of the Cylinder FE is to the *Momentum* of the Cylinder BC, as the Resistance DF is to the Resistance BA, *i. e.* as the Cube of FD to the Cube of BA, *i. e.* as the Line DE to the Line O, we have our Desire.

The *Momentum* of the Cylinder FE is to the *Momentum* of the Cylinder DG, as the Square of the Line DE to the Square of AC, *i. e.* as the Line DE to (I) : But the *Momentum* of the Cylinder DG is to the *Momentum* of the Cylinder BC, as the Square of DF to the Square of BA, *i. e.* as the Square of DE to the Square of (I), *i. e.* as the Square of (I) to the Square of M, *i. e.* as the Line (I) to O ; therefore by Equality of Reason : As the *Momentum* of the Cylinder FE to the *Momentum* of the Cylinder BC, as the Line DE to O, *i. e.* as the Cube of DF to the Cube of BA, *i. e.* the Resistance of the Base DF to the Resistance of the Base BA ; which is what was sought.

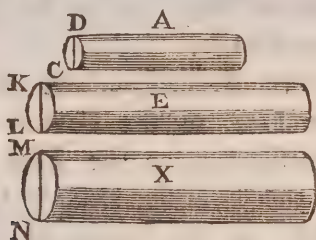
SAGR. This

SAGR. This is a long Demonstration, and hard to be remember'd at once hearing ; therefore pray, *Salviatus*, be pleas'd to repeat it.

SALV. With all my Heart ——— But may be I had better give you a more concise and shorter one ; but then we must have a Figure somewhat different.

SAGR. This will be still more obliging : But pray let me have this last Demonstration in Writing, that I may consider it at Leisure.

SALV. You shall have it : ——— Now suppose a Cylinder A, the Diameter of whose Base let be the Line DC,



and let this A be the greatest that can sustain itself without breaking, than which we would find a greater similar to the Cylinder A, which likewise shall be the greatest, and the only one that can sustain itself at a Length equal to a given Line ; and let this Cylinder be E, the Diameter of whose Base let be KL ; and let MN be a third Proportional to the Lines DC and KL, which MN let be the Diameter

Diameter of the Base of the Cylinder X, equal in Length to the Cylinder E : Then I say, that X is the Cylinder sought,

Because the Resistance DC is to the Resistance KL as the Square of DC to the Square of KL, *i. e.* as the Square of KL to the Square of MN, *i. e.* as the Cylinder E to the Cylinder X, *i. e.* as the *Momentum* E to the *Momentum* X : But the Resistance KL to the Resistance MN, is as the Cube of KL to the Cube of MN, *i. e.* as the Cube of DC to the Cube of KL, *i. e.* as the Cylinder A to the Cylinder E, *i. e.* as the *Momentum* A to the *Momentum* E : and thence by Perturbation of Proportion, as the Resistance DC to the Resistance MN, so the *Momentum* A to the *Momentum* X : and thence the Cylinder X is in the same Constitution of *Momentum* and Resistance with the Cylinder A : But let's make this Problem more general ; and let the Proposition be this,

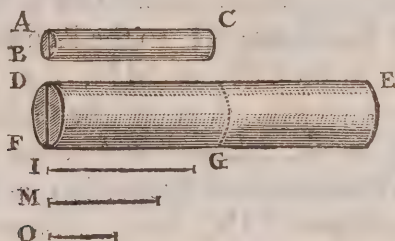
P R O P.

The Cylinder AC being given with any Proportion at Pleasure of its *Momentum* to its Resistance ; and any Length DE being also given ; to find the Thickness of the Cylinder whose Length is DE, and whose *Momentum* has the same Proportion to its Resistance, as the *Momentum* of the Cylinder AC has to its Resistance.

Resuming the foregoing Figure, Page 188. and nearly the same Progress, we say,

Because

Because the *Momentum* of the Cylinder FE is to the *Momentum* of the Part DG, as the Square of ED to the Square of FG, *i. e.* as the Line DE to (I) : and the *Momentum* of the Cylinder FG to the *Momentum* of the Cylinder AC, is as the Square of FD to the Square of AB,



i. e. as the Square of DE to the Square of (I), *i. e.* as the Square of (I) to the Square of M, *i. e.* as the Line (I) to O : Therefore, *ex Equali*, the *Momentum* of the Cylinder FE is to the *Momentum* of the Cylinder AC, as the Line DE to O, that is, as the Cube of DE to the Cube of (I), that is, as the Cube of FD to the Cube of AB, that is, as the Resistance of the Base FD to the Resistance of the Base AB. Q. E. F.

Now from what has been demonstrated, you may plainly see how impossible it is, not only for Art, but also for Nature herself, to increase her Machines to an immense Bigness : That immensely great Ships, Palaces, Temples, can't be built by Art, whose Oars, Yards, Beams, * Iron Bolts, and, in a Word, whose other Parts will consist or hold together : Nor can Nature produce Trees

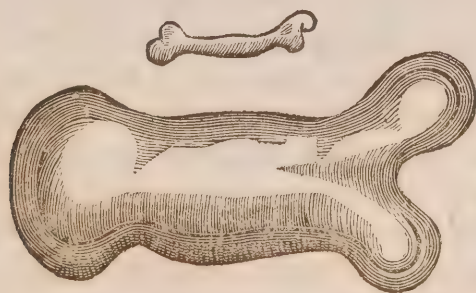
* Catene di Ferro.

of a most immense Bigness, since their Arms or Boughs would at length break by their own Weight : As it is also impossible for her to give Bones for Men, Horses, or other Animals, so made as to subsist, and proportionally to perform their Offices, when such Animals should be enlarged to immense Heights, unless she uses Matter much harder and more resisting than she commonly does ; or else should make such Bones of a Thickness out of all Proportion ; whence the Figure and Look of the Animal must become monstrous : Which, may be, my most witty Poet hinted at, where, describing a huge Giant, he says,

Non si può compartir quanto sià lungo
Si smisuratamente è tutto grosso.

*Whatever Height we to the Giant give,
He cannot without equal Thickness live.*

To explain this a little by an Example, I have here drawn the Figure of a Bone, three times longer than a common one, and increased in Thickness in such Proportion, as that it may, in its great Animal, perform its Office proportionally to what the lesser Bone does in the lesser Animals : The Figures are these ;



C c

Where

Where you see what a Figure, out of all Proportion, that of the augmented Bone makes.

Whence 'tis manifest, that he that would, in an huge Giant, keep the same Proportion that the Members have in an ordinary siz'd Man, must either find Matter much harder, and more resisting, to make Bones of, or else must admit, that its Consistency or Strength is in Proportion much weaker than what is found in Men of a middle Stature. Otherwise, nlairing the Giant to an immense Height, he would be oppress'd by, and fall under his own Weight: Whereas, on the contrary, when Bodies are diminished, we don't see the Strength to diminish in the same Proportion: nay, in the lesser we observe the Robustness or Strength to increase in a greater Proportion: So that I believe that a little Dog could carry two or three Dogs of equal Size with himself, on his Back; but I don't think an Horse could carry so much as one single Horse of his own Size.

SIMP. I must own I don't thoroughly agree with you in this; and the immense Bulks that we see in Fishes have occasion'd it: Thus, for Instance, a Whale, as I have heard, is as big as ten Elephants, yet sustains itself.

SALV. The Objection you have started, *Simplicius*, hints to me another Condition I did not before think of, by Help whereof, even Giants, and other vast Animals may consist, and move themselves as easily as smaller do: And this will happen, not only when greater Strength is added to the Bones and other Parts, whose Office it is to sustain their own Weight, and the Weight that's upon

them;

them ; but the Structures of the Bones keeping the same Proportions, the same Fabricks would just in the same manner, nay, indeed, would more easily consist, if the Gravity of the Matter of those Bones, as also that of the Flesh, or of whatever else is to rest itself upon the Bones, be diminish'd in that Proportion : And indeed Nature has made use of this Artifice in the framing of Fishes, making their Bones and Muscles, not only very light, but without any Gravity.

SIMP. I see, *Salviatus*, whither your Discourse tends : You'll say, That since the Element of Water is the Habitation of Fishes, which by its Corpulency, or, as others will, its Gravity, diminishes the Weight of Bodies immersed in it ; for this Reason the Matter of Fishes, not weighing any thing, may, without any Detriment to the Bones, be sustained. But this is not enough, because, altho' the rest of the Substance of the Fish weigh not, yet without doubt the Matter of the Bones must gravitate, or must have its Weight : Who will say that the Rib of a Whale, that's as big as a Beam, does not weigh a great deal, and when in Water does not sink to the Bottom ? It ought therefore to follow from what you have said, that in so vast a Bulk, these should not be able to subsist.

SALV. A shrewd Objection : But to answer your Doubt ; pray tell me whether you have ever observ'd Fishes, at Pleasure, to stand or lie immoveable under Water, so as neither to descend to the Bottom, nor rise to the Top, without stirring their Fins.

SIMP. I have often observ'd it.

SALV. Fishes then being able to stay themselves without any Motion, in the Midst of the Water, 'tis an infallible Argument that the Composition of their corporeal Mass, equalleth the specifick Gravity of the Water; so that if there be found in them some Parts heavier than Water, it is necessarily required that there be others so much lighter, as to procure an *Equilibrium*.

Wherefore if the Bones be heavy, 'tis necessary that the Muscles and other Matter be light, that these with their Lightness may compensate the Weight of the Bones: Thus in Aquatick Animals, the contrary happens to what befalls the Terrestrial Ones; namely, in the latter 'tis the Office of the Bones to sustain their own Weight with that of the Flesh; in the former the Flesh sustains its own Weight, with that of the Bones: Let's no longer wonder then that there may be vast Animals in the Water, but not on the Land, that is, in the Air.

SIMP. I am satisfied; and can't help observing that those Animals which we call Terrestrial, ought, with greater Reason, to be call'd Aerial, since they in reality live in the Air, and are surrounded by the Air, and breathe therein.

SAGR. *Simplicius's* Doubt and its Solution pleases me mightily: And I easily comprehend farther, that one of these huge Fishes being hauled on Shore, could not long sustain itself; but that the Joints or Connexions of the

Bones

Bones being relaxed, its Mass would be crush'd under its own Weight.

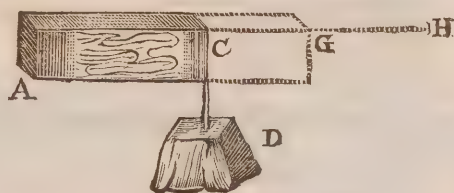
SALV. At present I'm of the same Opinion : and I almost think that the same thing would happen to that huge Ship, which, whilst floating on the Sea, is not dissolv'd by its own Weight, and the great Burden of its Lading and Artillery ; but was it on dry Ground, and surrounded with Air, perhaps it would fall to Pieces.

But let us proceed and demonstrate how,

P R O P. IX. P R O B. II.

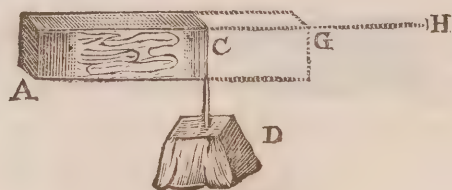
A Prism or Cylinder with its Weight, and the greatest Weight it can sustain being given, to find the greatest Length, beyond which if it be prolong'd, it would break by only its own Weight.

Let there be given the Prism AC, with its Weight, and likewise let the greatest Weight D be given that can be



sustain'd at its Extremity C ; 'tis required to find the greatest Length to which the said Prism may be prolong'd without breaking : Make the Line AH in the same Proportion to the Length AC, as the Compound of the Weight
of

of AC, and the double Weight of D, is to the Weight of the Prism AC; and let AG be a mean Proportional between AC and AH: Then I say that AG is the Length sought. For the gravitating *Momentum* of the Weight D in C, is equal to the *Momentum* of the double Weight of D, if it be plac'd in the Middle of AC, where is also the Centre of the *Momentum* of the Prism AC: The *Momentum* therefore of the Resistance of the Prism AC which is in A, is equivalent to the Gravitation of the Double of the Weight D, together with the Weight of AC, hang'd also in the Middle of AC: And since it has been made as the *Momentum* of the said Weights so plac'd, *i. e.* of the Double of D with AC, to the *Momentum* of AC, so the Line HA to the Line AC, between which the Line AG is a mean Proportional; therefore the *Momentum* of the Double of D, with the *Momentum* of AC, will be to the *Momentum* of AC, as the Square of GA to the Square of AC: But the pressing *Momentum* of the Prism GA, is to the *Momentum* of AC, as the Square of GA to the Square



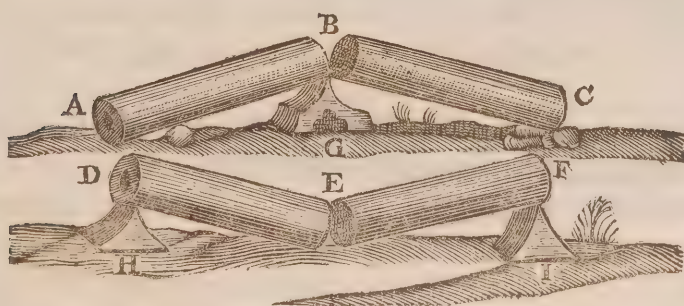
of AC; Therefore that Length AG is the greatest sought, *i. e.* that to which the Prism AC being prolong'd, sustains itself, but beyond which Length were it produc'd, it would break. Hitherto we have consider'd the *Momenta* and Resistances

Resistances of Prisms and Cylinders, one End whereof has been supposed fix'd, and to the other only the Force of a pressing Weight apply'd, considering either that Weight by itself, or join'd to that of the Solid : or else the sole Gravity of the said Solid. Now let us say something of those same Prisms and Cylinders, supposing them fix'd or sustain'd at both Ends, or else to rest upon one sole Point taken between the Ends : And first, I say that,

P R O P. X.

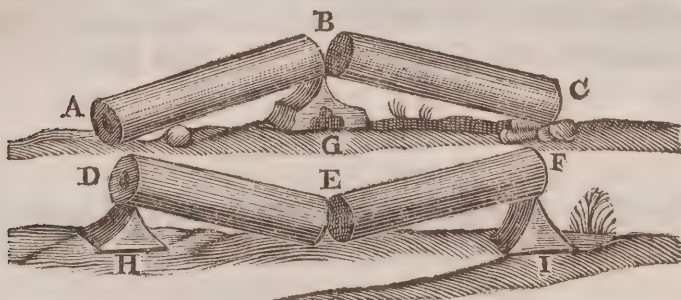
The Cylinder that charg'd with its own Weight is reduced to its greatest Length, beyond which it could not sustain itself, whether it be sustain'd in the Midst by one *Fulcrum*, or by two at its Extremities, may be in Length double of that fasten'd in a Wall as before, *i. e.* which is sustain'd only at one End.

This is very manifest ; for if we suppose the half AB of the Cylinder ABC to be the utmost Length it can



sustain itself at, whilst fix'd at the End B, it will sustain itself in the same manner, if, being laid on the *Fulcrum* G,
it

it be counterpoised by its other Half BC : And in like manner, if the Length of the Cylinder DEF be such that only one half of it can be sustain'd, whilst 'tis fix'd at the End D, and consequently the other EF fix'd in the Point F, 'tis manifest that if the *Fulcra* H and (I) be plac'd under



the Extremities D and F, any *Momentum* of Force or Weight added in E, will there make a Fracture.

That which requires a more subtile Speculation, is this ; when abstracting the proper Gravity of those Solids, 'tis propos'd to us,

PROP. XI. PROB. III.

To find whether that Force or Weight, which apply'd to the Middle of the Cylinder, supported at its Ends, would suffice to break it, could do the same applied to any other Place not in the Middle of that Cylinder.

As for Example : If going to break a Staff, we hold it at each End in our Hand, and clap our Knee against the Middle of it ; whether the same Force which suffices to break it after this manner, would also suffice in case the
Knee

Knee were set not against the Middle of it, but nearer to one End.

SAGR. I think *Aristotle* has mention'd this Problem in his *Mechanical Questions*.

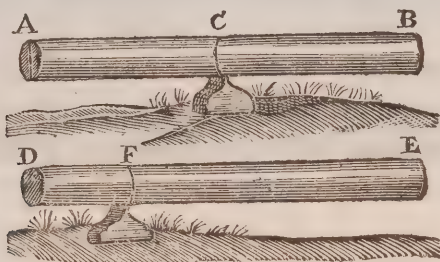
SALV. His is not quite the same ; for he only seeks a Reason why less Strength is required to break the Staff with one's Knee, holding it in our Hands at the very Ends, that is, as far distant as possible from the Knee, than if we hold it nearer its Middle ; and he gives a general Reason for the Thing, reducing it to the *Leaver* ; which Leavers are longer when the Arms are extended, and the very Ends grasp'd : But we seek more than this, viz. Whether setting the Knee to the Midst of the Staff, or to any other Part of it, always keeping the Hands at its Ends, the same Force be sufficient in all Situations to break it.

SAGR. At first View, methinks, the same Force should serve, because those two Leavers do as it were retain the same *Momentum*, seeing that so much as the one is shorten'd, the other is lengthen'd.

SALV. You see now how ready at Hand Equivocations are, and how cautiously and circumspectly we ought to proceed, lest we run into them : What you say, and what indeed at first sight seems so probable, is, in Strictness, so false, that whether the Knee, which is the *Fulcrum* of the two Leavers, be plac'd or not plac'd against the Middle, makes such an Alteration, that 'tis not enough to apply

four, nor ten, nor an hundred, no nor a thousand times that Force which is sufficient to break it in the Middle, to break it in some other Place : First, we will consider this in general, then come to the particular Determination of the Proposition, *viz.* To find how the Forces are alter'd according as the Staff is to be broke in this or that Point.

Let AB be a Staff to be broke in the Midst, upon the *Fulcrum* C; and let DE be just such another, to be broke upon the *Fulcrum* F, distant from its Middle : First, 'tis manifest; since the Distances AC and BC are equal, that the Force at the Ends B and A, is equally divided.



Secondly, because the Distance DF is less than the Distance AC, the *Momentum* of the Force plac'd in D is less than the *Momentum* in A ; that is plac'd at the Distance CA, and that according to the Proportion of the Line DF to the Line AC; and consequently it must be increased to equal, or exceed the Resistance of F : But now the Distance DF, in respect of the Distance AC, may be diminish'd *ad infinitum* : Therefore 'tis requisite that it be possible to increase the Force which shall be applied in D also *ad infinitum*, to equal the Resistance in F.

But,
‡

But, on the contrary, as the Distance FE increases above CB, we must lessen the Force in E, to equal or compensate the Resistance in F; but the Distance FE, in respect of CB, cannot increase *ad infinitum*, by removing the *Fulcrum* F towards the End D, nor indeed to its Double: Wherefore the Force in E, to equal or compensate the Resistance in F, will be always more than half the Force in B. Thus we easily see the Necessity of augmenting infinitely the *Momenta* of the collected Forces in E and D, to equal or exceed the Resistance plac'd in F, according as the *Fulcrum* F continually approaches nearer and nearer to the End D.

SAGR. What shall we say, *Simplicius*? Must we not own *Geometry* to be a more powerful Instrument than all others to sharpen the Wit, and to dispose it to reason rightly? And that *Plato*, with great Reason, desired his Scholars in the first Place to be grounded in the *Mathematicks*? For my Part, tho' I before well understood the Nature of the Leaver, how, its Length increasing and decreasing, the *Momentum* of the Force and of the Resistance is increased or diminished; yet in the Determination of the present Proposition I was infinitely deceiv'd.

SIMP. The Truth is, I begin to see that *Logic*, how instrumental soever it be to direct and regulate our Discourse, does by no means reach the Acuteness of *Geometry*, in stirring up our Minds to Invention.

SAGR. *Logic* seems to me to teach us how to discover whether Discourses and Demonstrations already made and

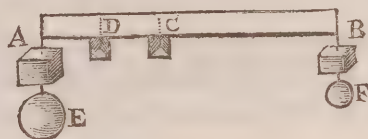
invented, be solidly conclusive ; but that it teaches how to discourse and reason conclusively, indeed I don't believe : But I wish *Salviatus* would demonstrate to us in what Proportion the *Momenta* of Forces increase, to overcome the Resistance to Friction of the same Staff, in Places farther and farther from the Middle.

SALV. This Proportion proceeds in such Manner, That,

P R O P. XII.

If in the Length of a Cylinder two Places be mark'd, in which we would have that Cylinder break, the Resistances at those two Places have the same Proportion to each other, as have the Rectangles made by the Distances of those Places reciprocally taken.

Let the Forces A and B be the least that can break the Cylinder in C, and let the Forces E and F be the



least that can break it in D : then, I say, that the Forces A and B have the same Proportion to the Forces E and F, as the Rectangle ADB has to the Rectangle ACB.

For the Forces A and B have to the Forces E and F a Proportion compounded of that of the Forces A and B, to the Force B ; of that of the Force B to the Force F ; and of that of the Force F to the Forces F and E : But as the Forces

Forces A and B are to the Force B, so is the Line BA to AC; and as the Force B is to the Force F, so is the Line DB to the Line BC; and as the Force F is to the Forces F and E, so is the Line DA to the Line AB; therefore the Forces A and B have to the Forces E and F a Proportion compounded of these three, *i. e.* of the right Line BA to AC; of the right Line DB to BC; and of the right Line DA to AB: But of the two Proportions DA to AB, and BA to AC, is composed the Proportion of DA to AC; therefore the Forces A and B have to the Forces E and F a Proportion compounded of the Proportions DA to AC, and of DB to BC: But the Rectangle ADB hath to the Rectangles ACB a Proportion compounded of the same Proportions of DA to AC, and of DB to BC; therefore the Forces A and B are to the Forces E and F, as the Rectangle ADB to the Rectangle ACB; which is as much as to say that the Resistance against Friction in C, has the same Proportion to the Resistance against Friction in D, as the Rectangle ADB to the Rectangle ACD. *Q. E. D.*

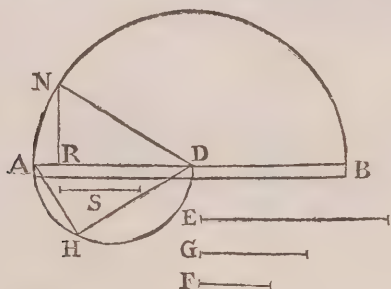
As a Confectary of this Theorem we may easily solve the following curious Problem.

P R O P. XIII. P R O B. IV.

The greatest Weight being given that can be sustain'd at the Middle of a Cylinder or Prism, where its Resistance is least; and a Weight being likewise given greater than that; to find in the said Cylinder or Prism, a Point at which the given greater Weight may be the greatest can be there supported:

Let

Let the given Weight, that is greater than the greatest Weight that can be supported at the Middle of the Cylinder AB, be to the said greatest Weight in Proportion as the Line E is to the Line F. Now 'tis required to find the Point in the Cylinder, at which the greater given Weight is the greatest that can be sustain'd.



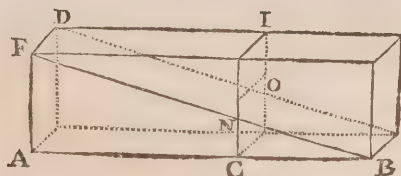
Between the two Lines E and F let G be a mean Proportional ; and as E is to G, so let AD be to S ; then S will be less than AD : Let AD be the Diameter of the Semicircle AHD, in which draw the Line AH = S, and join H and D, make DR = HD ; then I say, that R is the Point sought, at which the given Weight, which is greater than the given greatest that can be sustain'd at the Middle of the Cylinder D, would be the greatest that could be there supported.

On the Line BA describe the Semicircle ANB, and raise the Perpendicular RN, and join N and D : Now because the two Squares of NR and RD are = to the Square of ND, *i. e.* to the Square of AD, *i. e.* to the two Squares AH and HD ; and the Square of HD is equal to the Square of DR ; therefore the Square of NR, *i. e.* the
Rectangle

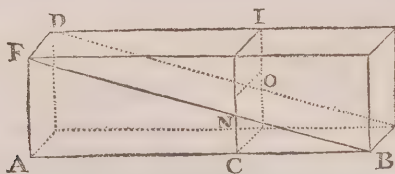
Rectangle ARB is = to the Square of AH , *i. e.* to the Square of S : But the Square of S is to the Square of AD , as the Line F to the Line E , *i. e.* as the greatest supportable Weight at D to the given greater Weight : Therefore this greater shall be supported in R , and be the greatest Weight that can there be supported ; which is what was sought.

SAGR. I understand you very well ; and am further thinking, that since the Prism AB becomes so much the stronger, and resists more against Pression in those Parts that are more and more distant from its Middle, that, I say, in very great and heavy Beams we may cut or saw off a good deal from towards their Ends, which would much lessen their Weight ; a thing in Beams for very large Rooms very commodious, and of no small Advantage. Now how curious it would be to find what Figure that Solid must be of, to have an equal Resistance in all its Parts ; so that it should not be by any Weight more easily broke in the Middle than in any other Place ?

SALV. I was just going to mention a notable Thing to you, and a very pleasant one, to this Purpose : And the better to explain my Meaning, see the following Figure.



The Figure DB is a Prism, whose Resistance against Fraction in the Extremity AD, by a Force pressing at the End B, is so much less than the Resistance that would be found in the Place CI, as the Length CB is less than the Length CA, as has been just now demonstrated.



Now suppose the said Prism to be cut diagonally along the Line FB, so as the opposite Surfaces may be two Triangles, one of which towards us is FAB : Now this Solid is of a Nature contrary to that of the Prism, *i. e.* it resists Fraction by the Force plac'd in B, so much less at the Term C than at the Term A, as the Length CB is less than the Length BA ; as we shall easily prove thus ;

The Section CNO being supposed parallel to the other AFD, the Line FA in the Triangle FAB has the same Proportion to the Line CN, as the Line AB to the Line CB ; and therefore if we suppose the *Fulcrum* of the two Leavers to be plac'd in the Points A and C, whose Distances BA, AF, BC, and CN will be similar ; and therefore such *Momentum* as the Force plac'd in B, at the Distance BA, has above the Resistance plac'd at the Distance AF, such will the same Force in B, at the Distance BC, have above the same Resistance, were it plac'd at the Distance CN : but the Resistance to be overcome in the *Fulcrum* C, plac'd at the Distance CN, by the Force B, is so much less than
the

Again, that the Prism being cut according to the *parabolic* Line FNB, a third Part of it will be taken off, appears thus;

Because the *Semiparabola* FNBA and the Rectangle FB are Bases of two Solids comprehended between two parallel Planes, *i. e.* between the Rectangles FB and DG; therefore they are in the same Proportion to each other as their Bases are: but the Rectangle FB is *Sesquialter* of the *Semiparabola* FNBA; therefore by cutting the Prism according to the *parabolic* Line, the third Part of it is taken off.

Whence 'tis manifest, that we may lessen the Weight of Beams Thirty-three Pounds in the Hundred, without weakening them at all, which in building large Ships may be of great Benefit: for in such Kind of Fabricks, Lightness is of infinite Advantage.

SAGR. So many and so great are the Advantages, that it would be tedious, nay, 'tis impossible to mention them all. Let's pass them therefore by, for I should rather desire to be convinc'd that that Diminution of Weight is made, according to the assign'd Proportion: That the diagonal Section takes away half the Weight, I well know, but that the other *parabolic* one takes away a third Part of the Prism, (tho' I can believe it on the Word of *Salviatus*, who always speaks Truth) yet I would rather chuse to see it demonstrated.

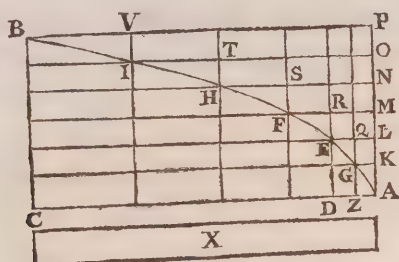
SALV. You'd have me demonstrate to you, that the Excess of the Prism above the *parabolic* Solid, is a third Part of the whole Prism. This I am sure I have formerly demon-

demonstrated : I'll try whether I can put the Demonstration together again ; to which Purpose I remember I made use of a certain *Lemma* of *Archimedes*, which he has in his Book of *Spirals* ; and 'tis this ;

L E M M A II.

If any Number of Lines exceed one another equally, and the Excess be equal to the least of those Lines ; and if there be as many more, each of them = the greatest : the Squares of these shall be less than *triple* the Squares of those Lines which exceed one another : but the same shall be more than *triple* those other that remain after the Square of the longest is taken away.

This being granted : In this Rectangle ACBP, inscribe the *parabolic* Line AB : Then we are to prove the mix'd Triangle BAP, whose Sides are BP and PA, and whose Base is the *parabolic* Line BA, to be the third Part of the whole Rectangle CP.



Now, if it be not the third Part, it must be more or less ; If it be, suppose, less, then let the Space X be supposed = its Defect, or to what it wants of this third Part ; then dividing

†

viding the Rectangle CP continually into equal Parts, with Lines parallel to the Sides BP and CA, we shall at length arrive at such Parts, one of which shall be less than the Space X. Now let one of those Parts be the Rectangle OB, and thro' the Points where the other Parallels cut the *parabolic* Line, draw Lines parallel to AP: here now I'll suppose, described about our Mixt-lin'd Triangle, a Figure composed of Rectangles; which are BO, IN, HM, FL, EK, GA, which Figure will certainly be less than the third Part of the Rectangle CP, since the Excess of this Figure above the Mixt-lin'd Triangle, is much less than the Rectangle BO, which BO is less than the Space X.

SAGR. Hold a little, pray: for I'm not satisfied that the Excess of this circumscribed Figure above the Mixt-lin'd Triangle, is considerably less than the Rectangle BO.

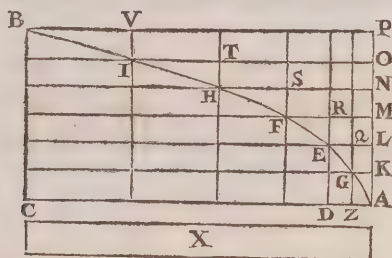
SALV. Is not the Rectangle BO = all those Rectangles thro' which our *parabolic* Line passes, I mean these: BI, IH, HF, FE, EG, GA; of which one Part only lies without the Mixt Triangle? And was not this Rectangle BO given or supposed less than the Space X?

Therefore if the Triangle, together with the Space X, as our Adversary supposes, be = the third Part of the Rectangle CP, the circumscribed Figure which adds to the Triangle, so much less than the Space X, will be yet less than the third Part of the said Rectangle CP: but this is impossible, because it is more than a third Part; therefore it is not true that our Mixt-lin'd Triangle is less than the third Part of the Rectangle.

SAGR. You

SAGR. You have solved the Doubt I started ; but you have still to demonstrate to us, that the circumscribed Figure is more than a third Part of the Rectangle CP ; which I believe will be pretty difficult.

SALV. Not at all : for in the *Parabola* the Square of the Line DE has the same Proportion to the Square of ZG, as the Line AD has to AZ, which is the same as the $\square$ KE has to the $\square$ AG, (their Heights AK and KL being equal) therefore the same Proportion as the Square of ED



has to the Square of ZG, *i. e.* the Square of LA has to the Square of AK, the same the $\square$ KE has to the $\square$ KZ : And in the self-same Manner may we prove that the other Rectangles LF, MH, NI, OB, are to each other as the Squares of the Lines MA, NA, OA, PA.

Let us now consider the circumscribed Figure to be composed of certain Spaces, that are to one another as the Squares of the Lines that exceed one another by an Excess = the least Line, and the Rectangle CP to be composed of so many other Spaces, each of which are equal to the longest Line, which are all the Rectangles equal to OB.

Then,

the Resistance in A, as the Rectangle CO is less than the Rectangle AD, *i. e.* as the Line CN is less than the Line AF, *i. e.* as the Line CB is less than the Line BA; therefore the Resistance of the Part OCB against Friction in C, is so much less than the Resistance of the Whole DAO against Friction in A, as the Length CB is less than the Length AB.

Wherefore we have taken from the Beam or Prism one half, by cutting it diagonally, and we have the Wedge or triangular Prism FBA left; which two Solids, that is, the whole Prism, and that left, are of contrary Qualities, *i. e.* that, *viz.* the Prism DB, the shorter it is, the more it resists; and this, *viz.* the Wedge FBA, the shorter you make it, the weaker it grows: Now this being so, it seems very reasonable, nay necessary, that it may be so cut, that, taking off what is superfluous, a Solid may remain of such Figure as in all its Parts shall have the same Resistance.

SIMP. It must unavoidably be so; for where there's a Transition from a bigger to a less, we must needs meet with the Equal.

SAGR. But now the Business is to find the Curve along which the Prism must be cut to have this Effect.

SIMP. This seems to me to be easy; for if by cutting the Prism diagonally, taking off half of it, the Body that remains be of a Nature (as to its Resistance against Friction) contrary to that of the whole Prism; so that in all Places wherein this acquires Strength, that in the same Proportion

E e loses

loses it ; methinks if we take a *Medium*, and take off only one half of that half, the remaining Figure will neither gain nor lose any Strength, in all those Places in which the two other Bodies lost and gain'd an equal Portion of theirs.

SALV. You are out, *Simplicius*, for I will prove to you, that what may be cut off from the Prism without weakening it, is really not only a fourth, as you suppose, but a third Part of it.

Now it remains (as *Sagredus* hinted)

PROP. XIV. PROB. V.

To find according to what Line the Section is to be made ; and this I will demonstrate to be *Parabolical*.

But first I must demonstrate the following *Lemma*.

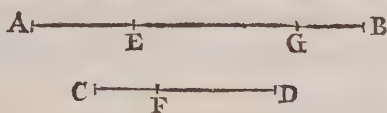
LEMMA I.

If there be two Ballances or Leavers divided by their *Fulcra*, in such Sort that the two Distances, at which the Forces are to be plac'd, may have to each other a *duplicate* Proportion of the Distances, at which the Resistances are ; which Resistances are to each other as their Distances : I say, the sustaining Powers will be equal.

Let AB and CD be two Leavers, divided upon their *Fulcra* E and F, in such Sort that the Distance EB has to the Distances FD a *duplicate* Proportion of that which the Distance EA has to FC : I say, the Powers which in B
and

and D shall sustain the Resistances A and C, shall be equal to each other.

Let EG be supposed a mean Proportional between EB and FD : then it will be $BE : EG :: EG : FD$, and so is AE to CF; And so is supposed the Resistance of A to the Resistance of C : And because it is $EG : FD :: AE : CF$, it will be by Permutation $GE : EA :: DF : FC$, and

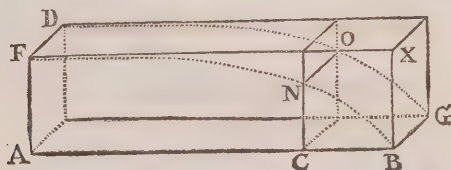


therefore (because the two Leavers DC and GA are divided proportionally in the Points F and E) in case the Power which plac'd in D, countervails or equals the Resistance of C, was plac'd in G, it would equal the same Resistance of C plac'd in A : But by what has been granted, the Resistance of A has to the Resistance of C, the same Proportion as AE has to CF, *i. e.* as BE has to EG ; therefore the Power G, or, if you will, D, plac'd at B, will sustain the Resistance C plac'd in A. *Q. E. D.*

This being understood ; on the Surface FB of the Prism DB, let the *parabolic* Line be describ'd, whose Vertex is B ; according to which let the Prism be cut or saw'd, so that there may remain the Solid comprehended between the Base AD, the Rectangle AG, and the Surface DGBF, bent according to the Curvity of the *parabolic* Line FNB. Then I say, that this Solid is throughout or every where of equal Resistance.

DEMONSTRATION.

Let this remaining Solid be cut by a Plane CO, parallel to AD; and imagine two Leavers divided and placed upon the *Fulcra* A and C: And let the Distances of one be BA and AF, and of the other BC and CN: Then



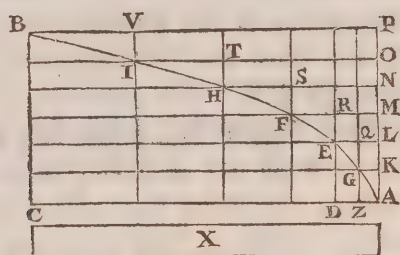
because in the *Parabola* FBA it is $AB : BC :: FAq : CNq$; 'tis manifest that the Distance BA of one Leaver has to the Distance BC of the other a *duplicate* Proportion of that which the other Distance AF has to the other CN: And because the Resistance to be equal'd by the Leaver BA, has to the Resistance to be equal'd by the Leaver BC, the same Proportion as the Rectangle DA has to the Rectangle OC; which is the same as the Line AF has to the Line NC, which are the two other Distances of the Leavers: It is manifest by the foregoing *Lemma*, that the same Force which apply'd to the Line BG equals the Resistance DA, will also equal the Resistance CO: And the same may be demonstrated, if that Solid be cut in any other Place; therefore such a Solid is every where of equal Resistance.

Again,

Then, by the foregoing *Lemma*, the circumscribed Figure is more than the third Part of the Rectangle CP ; but it was supposed less, which you see is impossible : therefore the Mixt-lin'd Triangle is not less than the third Part of the $\square$ CP. I say likewise, that it is not more, for if it be more than one third Part of the $\square$ CP, suppose the Space X to be = the Excess of the Triangle above the third Part of the $\square$ CP ; then dividing and subdividing the Rectangle into Rectangles lesser and lesser, but always equal to each other, we shall meet at last with such, one whereof shall be less than the Square X : Let this Division be made, and let the Rectangle BO be less than the Space X ; and having describ'd the Figure as before, we shall have in the Mixt-lin'd Triangle an inscrib'd Figure compos'd of the Rectangles VO, TN, SM, RL, QK, which also shall not be less than the third Part of the great Rectangle CP. For the Mixt-lin'd Triangle exceeds the inscribed Figure by a far less Excess than it exceeds the third Part of the $\square$ CP ; because the Excess of the Triangle, above the third Part of the Rectangle CP, is = to the Space X, which is less than the $\square$ BO : and this still considerably less than the Excess of the Triangle above the inscribed Figure : For to the Rectangle BO, all the small Rectangles AG, GE, EF, FH, HI, IB, are equal ; much less yet than the Half of which is the Excess of the Triangle above the Figure inscribed ; Wherefore the Triangle exceeding the third Part of the Rectangle CP, by much more (it exceeding it by the Space X) than it exceedeth its inscribed Figure, that same Figure shall also be more than the third Part of the Rectangle CP ; but it is less by the foregoing *Lemma* : the Rectangle CP taken as the Aggregate of all the biggest Rectangles, having the

F f
same

same Proportion to the Rectangles composing the inscribed Figure, as the Aggregate of all the Squares of the Lines = the longest, to the Squares of the Lines which exceed one another equally, the Square of the longest being first taken away, and therefore (as it happens in Squares) the total Aggregate of the biggest (*i. e.* of the Rectangle CP) is more than the triple of the Aggregate of those that exceed one another by an Excess = the least of them, the Square of the greatest being first taken away, which compose the inscribed Figure: Therefore the Mixt-lin'd Triangle is neither greater nor less than the third Part of the Rectangle CP; it is therefore equal to it.



SAGR. A very curious Demonstration; and so much the more so, as it affords us the Quadrature of the *Parabola*, proving it to be *Sesquitertial* of the Triangle inscrib'd in it; the same indeed that *Archimedes* demonstrated by two very different, but both very admirable Processes of many Propositions: And the same that has lately been demonstrated by *Lucas Valerius* the *Archimedes* of our Age; which Demonstration is given us in a Treatise of his, concerning the Centre of Gravity of Bodies.

SALV. A

SALV. A Treatise as excellent as any that has been written by the famous Geometricians of the present, or any the past Ages ; which, when our *Academic* read, he desisted to prosecute his Discoveries, which he was then writing about, concerning the same Subject ; he there finding the whole Business ready done, and most happily demonstrated to his Hands by the said *Lucas Valerius*.

SIMP. This our *Academic* told me himself, and I often intreated him to let me see the Demonstrations he had hit upon, before he met with *Valerius's* Book ; but I never saw them.

SALV. I have a Copy of them, and will shew them you, because it's pleasant to, see the different Methods Authors take to investigate the same Conclusions, and their Demonstrations ; where some of the Conclusions have different Explanations, altho' in Effect they are equally true.

SAGR. I should be very glad to see them ; pray be so kind as to bring them with you the next time you come. But now, *Salviatus*, since the Matter is thus as you have prov'd, in Relation to the Resistance of a Solid made from a Prism, by a *parabolic* Section, an Operation not only Elegant but Useful, in many Mechanical Operations ; it would be of great Service to Artificers, had they an easy and ready Way to describe this *parabolic* Line upon the Surface of the Prism.

SALV. There are several Ways of describing such Lines, but two above all the rest are very expeditious ones; and those I'll now shew you: The former of which is really admirable; for by it, in less time than any one can describe neatly with a Pair of Compasses on Paper, four or six Circles of different Magnitudes, I can describe thirty or forty *parabolic* Lines, as accurately and exactly as the Circumferences of his Circles are. 'Tis thus: I take a Brass Ball exquisitely round, no bigger than a Nut, or a Boy's Marble; this thrown upon a Steel Mirror, plac'd not perpendicular, but something inclined to the Horizon, so that the Ball may run along it, pressing lightly upon it, it will then leave us a *parabolic* Line finely and exactly describ'd; and this wider or narrower, according as the Projection is more or less elevated: Whereby we have prov'd, by a clear and sensible Experiment, that the Motion of *Projectts* is made in *parabolic* Lines; which Effect our *Academic* was the first that observ'd, a Demonstration whereof he has given us in his Book of *Motion*, which we will peruse at our next Meeting. Now this Ball by which we say a *parabolic* Line is described, in the Manner aforesaid, must be first a little warm'd and moisten'd, by holding it in the Hand; for thus it will leave its Track more apparent upon the Mirror.

The other Way of describing the Line sought upon the Prism is this;

Let two Nails be drove at a convenient Height into a Wall, both equidistant from the Horizon, and distant from each other by twice the Breadth of the Rectangle, upon which the *Semiparabola* is to be describ'd; then upon these

two

†

two Nails, hang a small Line of such Length, as that its Curvature might reach the whole Length of the Prism : then that Line will hang in the Curve of a *Parabola* ; and if this Curve be mark'd by Points upon the Wall, we shall have a whole *Parabola* there described, which, a Perpendicular hanging from a Point in the Midst between these two Nails, will divide into two equal Parts ; which Curve any Artificer that knows any thing, will be able, without any Difficulty, afterwards to transfer to the opposite Surfaces of the said Prism. The same Line might also be drawn upon the Side of the Prism, by Points, or by Help of the Geometrical Lines, delineated upon the Compass of our *Friend*, without any Trouble.

Hitherto we have demonstrated the various Conclusions belonging to the Contemplation of these Resistances in Solids against Fraction ; and have first open'd the Way to this *Science*, by supposing the direct Resistance known, which may be prosecuted farther, to the gaining, by their Help, still new Conclusions, with their Demonstrations, which in Nature are infinite. But to put an End to this Day's Conference, I will only add the Contemplation of the Resistances of hollow Solids, which are made Use of by *Art*, and more by Nature in a thousand Operations ; where, without increasing the Weight, she greatly augments the Strength ; As is seen in the Bones of Birds, and in most Tubes, or hollow Trunks, which, tho' light, yet greatly resist against bending and breaking.

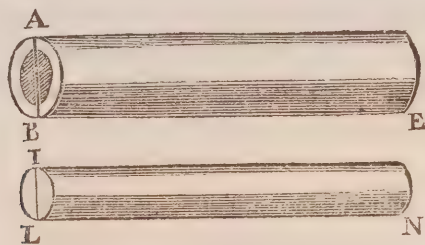
Thus, if a Wheat Straw, which supports an Ear that's heavier than the whole Stalk, was made of the same Quantity of Matter, but solid, it would much easier bend or break than it does : And with the same Reason *Art* has observ'd,

observ'd, and Experience confirm'd, that an hollow Cane, or Trunk of Wood or Metal, is much stronger and more firm, than if, being of the same Weight and Length, it was solid, which then would consequently be not so thick : And therefore *Art* has contriv'd a Way to make Canes hollow within, when they are required to be light and strong. We will next then demonstrate, That,

P R O P. XV.

The Resistances of two Cylinders of equal Weight and Length, one of which is hollow, and the other solid, have the same Proportion to each other, as their Diameters have.

Let the Tube or hollow Cylinder AE, and the solid Cylinder IN, be equal in Weight and Length : Then I say, that the Resistance of the Tube AE, against Friction, to the Resistance of the solid Cylinder IN against Friction,



has the same Proportion as the Diameter AB has to the Diameter IL. This is very manifest, for the Tube and Cylinder having the same Length, the Circle IL, that is, the

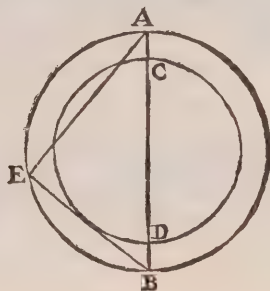
the Base of the Cylinder shall be equal to the Ring AB, of the Tube AE, (that Superficies which remains, when a lesser Circle is taken out of a greater concentric with it, I call a Ring) and therefore their absolute Resistances shall be equal ; but because in breaking crossways the Cylinder IN, we make use of the Length IN for a Leaver, and of the Point L for a *Fulcrum*, and of the Semidiameter or Diameter LI for a Counter-leaver ; and because in the Tube the Part of the Leaver, or the Line BE, is = the Line LN ; but the Counter-leaver beyond the *Fulcrum*, is the Diameter or Semidiameter AB : It is manifest that the Resistance of the Tube exceeds the Resistance of the solid Cylinder, as much as the Diameter AB exceeds the Diameter IL, which is what was sought : Therefore the Strength of the hollow Tube exceeds the Strength of the solid Cylinder, in Proportion of their Diameters ; I mean when they are of the same Matter, Weight, and Length : It would not be amiss, I think, in the next Place, to try to investigate what happens, indifferently in other Cases, between Tubes and solid Cylinders of equal Lengths, but unequal as to Weight, and more or less hollow'd out.

And first, we will demonstrate how,

PROP. XVI. PROB. VI.

A hollow Tube being given, to find a solid Cylinder equal to it.

This Operation is very easy: For let the Diameter of the Tube be AB , and the Diameter of its Concave be CD ; in the greater Circle draw the Line AE equal to the Diameter CD , and draw EB : Now because in the Semicircle AEB , the Angle E is right, the Circle whose Diameter is



AB , shall be = the two Circles, whose Diameters are AE and EB : But AE is the Diameter of the Concave of the Tube: therefore the Circle, whose Diameter is EB , will be = the Ring $ACDB$; and therefore the solid Cylinder, the Circle of whose Base has the Diameter EB , shall be = the Tube, they being of the same Length.

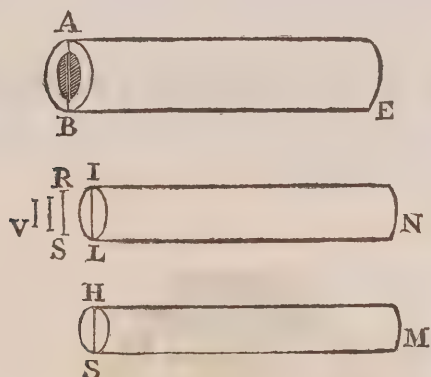
This being demonstrated, we shall easily be able,

PROP. XVII.

PROP. XVII. PROB. VII.

To find what Proportion the Resistances of any Tubes and Cylinders whatever, provided their Lengths be equal, have to one another :

Let the Tube ABE, and the Cylinder RMS, be of equal Length, it is required to find what Proportion their Resistances have to each other.



By the foregoing Proposition, find a Cylinder as ILN, of an equal Length with, and equal to the Tube ; and to the Lines IL and RS, (which are the Diameters of the Bases of the Cylinders IN and RM) let the Line V be a fourth Proportional : Then I say, that the Resistance of the Tube AE, to the Resistance of the Cylinder RM, is as the Line AB to the Line V : For the Tube AE being = and of the same Length with the Cylinder IN, the Resistance of the Tube shall be to the Resistance of the

G g Cylinder,

Cylinder, as the Line AB to the Line IL : But the Resistance of the Cylinder IN, is to the Resistance of the Cylinder RM, as the Cube of IL to the Cube of RS, *i. e.* as the Line IL to the Line V : Therefore, *ex Æquo*, the Resistance of the Tube AE, to the Resistance of the Cylinder RM, has the same Proportion as the Line AB has to the Line V : Which is what was sought.

The END of the SECOND DAY'S CONFERENCE.



GALILÆUS



G A L I L Æ U S
H I S
D I A L O G U E S.

D I A L O G U E III.
of L O C A L M O T I O N.

SALVIATUS, SAGREDO, *and* SIMPLICIUS.



HERE we advance a very new *Science* on a very old *Subject*. May be nothing in Nature was earlier than MOTION; and many and great Volumes we see have been written by Philosophers concerning it: But yet I find many Properties of it worth our knowing, and which hitherto have not been observed, much less demonstrated.

Some flight ones have been indeed noted, as for Example, That the natural Motion of falling Bodies is continually accelerated: But according to what Proportion that Acceleration is made, hath not hitherto been shewn; for no

one, that I know of, hath ever demonstrated, that the Spaces run thro' by a Body falling from Rest in equal Times, are in the same Proportion as the odd Numbers, which follow in order from the *Unit* or *One*.

Again, it has been observ'd, indeed, that *Projectts* (or Things thrown with Violence) do describe a Line, after some manner or other, curv'd; but that this is the Curve of a *Parabola* no body has told us.

Now that these are so, with many other Things no less worthy our Knowledge, I will here demonstrate: And, what is still more, I will open a Passage to a most ample and excellent *Science*, of which these our Labours are as Elements; into the Depths of which *Science* more piercing Wits than mine will be better able to penetrate.

This Tract we'll divide into three Parts.

In the first Part, we'll consider those things that respect *equable* or *uniform* Motion.

In the second, we'll treat of Motion *naturally accelerated*: And,

In the third, of *violent* Motion, or of *Projectts*.

Of EQUABLE MOTION.

Concerning equable or uniform Motion, we have need but of one Definition, which I give as follows;

DEFINITION.

An equable or uniform Motion, is that when a Moveable, in all equal Times whatsoever, passes thro' equal Spaces.

N. B. I

N. B. I thought it proper to add to the old Definition (which simply calls that an equable Motion, whereby equal Spaces are passed thro' in equal Times) these Words, *All whatsoever* ; thus, In *all* equal Times *whatsoever* : For it may happen that a Moveable may pass thro' equal Spaces in equal Times, and yet the Spaces may not be equal which it hath gone thro' in lesser, tho' equal Parts of the same Time.

On this Definition depend four *Axioms*, viz.

A X I O M I.

In the same equable Motion, that Space is greater which is passed over in a longer Time than that Space passed over in a shorter.

A X I O M II.

In the same equable Motion, that Time is greater wherein a greater Space is passed thro', than that wherein the Moveable passes thro' a lesser Space.

A X I O M III.

The Space passed thro' with a greater Velocity in any Time, is greater than that passed thro' with a lesser Velocity in the same Time.

A X I O M

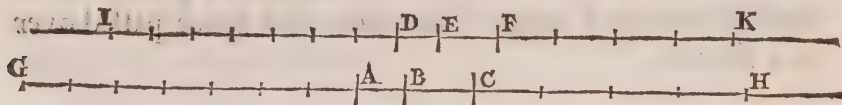
Axiom IV.

The Velocity wherewith a greater Space is passed over in any Time, is greater than the Velocity wherewith a lesser Space is passed in the same Time.

PROP. I. THEOR. I.

If a Moveable carried equably, and with the same Velocity, pass thro' two Spaces, the Times of the Motion shall be to each other as the said Spaces are.

Let the Moveable, by an equable Motion, pass thro' with the same Velocity, the two Spaces AB and BC, and



let the Time of the Motion thro' AB be DE ; and the Time of the Motion thro' BC let be EF : then I say, that as the Space AB to the Space BC, so is the Time DE to the Time EF.

Produce on both Sides the Spaces and Times towards G, H, and I, K ; and in AG take any Number of Spaces = AB, and in DI the like Number of Times, each = DE : And again, in CH take any Number of Spaces = CB, and in FK as many Times = the Time EF : then will the Space BG, and the Time EI, be *Equimultiples* of the Space BA, and the Time ED, be the Number of the forementioned Spaces and Times taken = AB and DE, what it will.

And

And in like manner the Space HB and the Time KE, will be *Equimultiples* of the Space CB, and of the Time FE, at whatever Rate they be equimultiply'd : And because DE is the Time of the Motion thro' AB, the whole Time EI shall be the Time of the Motion thro' BG, since the Motion is supposed Equable, and since in EI there are as many Times = DE, as in BG there are Spaces = BA ; In like manner we may conclude KE to be the Time of the Motion thro' HB. Now since the Motion is supposed Equable, if the Space GB was = BH, the Time IE would be also = the Time EK : and if GB be greater than BH, IE also will be greater than EK ; and if less, less : There are therefore four Magnitudes, AB the first, BC the second, DE the third, and EF the fourth : And of the first and fourth, *viz.* of the Space AB, and of the Time DE, are taken *Equimultiples*, according to any Multiplication, *viz.* the Time IE, and the Space GB ; and it hath been demonstrated, that these do at the same time, either equal, or want of, or exceed the Time EK, and the Space BH, which are *Equimultiples* of the second and fourth ; therefore the first hath to the second, *i. e.* the Space AB hath to the Space BC, the same Proportion as the third has to the fourth, *i. e.* as the Time DE has to the Time EF. *Q. E. D.*

P R O P. II. T H E O R. II.

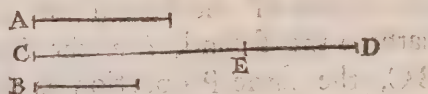
If a Moveable, in equal Times, pass thro' two Spaces, those Spaces will be to each other as the Velocities : And if the Spaces are as the Velocities, the Times will be equal. In the foregoing Figure, let the two Spaces AB and BC be

be pass'd over in equal Times ; the Space AB with the Velocity DE, and the Space BC with the Velocity EF : then I say, that the Space AB is to the Space BC, as the Velocity DE to the Velocity EF ; for taking, as before, *Equimultiples* both of the Spaces and the Velocities, according to any Multiplication whatever, *viz.* GB and IE of the Space AB, and of the Velocity DE, and likewise HB and KE of the Space BC, and of the Velocity EF, we may conclude, as above, that the *Multiples* GB and IE, both at the same time, fall short of, are equal to, or exceed the *Equimultiples* BH and EK ; therefore the Proposition is manifest.

PROP. III. THEOR. III.

The Times wherein the same Space is pass'd thro' by unequal Velocities, are in a reciprocal Proportion to the Velocities.

Let the unequal Velocities be A the greater and B the



lesser ; and according to both these, let a Motion be made thro' the same Space CD :

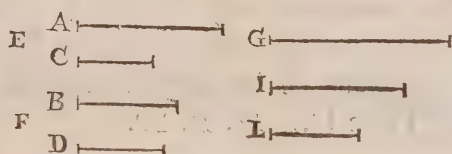
Then I say, that the Time in which the Velocity A passes thro' the Space CD, shall be to the Time in which the Velocity B passes thro' the said Space, as the Velocity B to the Velocity A. For make CD to DE, as A is to B, then, by the foregoing Proposition, the Time wherein the Velocity A passes CD, shall be the same with the Time where-

wherein the Velocity B passes CE. But the Time wherein the Velocity B passes CE, is to the Time in which it passes CD, as CE to CD ; therefore the Time wherein the Velocity A passes CD, is to the Time in which the Velocity B passes the same CD, as CE to CD ; that is, as the Velocity B to the Velocity A. *Q. E. D.*

PROP. IV. THEOR. IV.

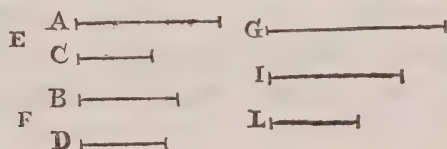
If two Moveables, move with an equable Motion, but with unequal Velocities, the Spaces which they pass in unequal Times, are to each other in a Proportion compounded of the Proportion of the Velocities, and of the Proportion of the Times.

Let the two Moveables, moving with an equable Motion, be E and F ; and let the Proportion of the Velocity



of the Moveable E, be to the Velocity of the Moveable F, as A is to B : And let the Time in which E is mov'd, be unto the Time wherein F is mov'd, as C is to D : Then I say, that the Space passed by E, with the Velocity A in the Time C, is to the Space passed thro' by F, with the Velocity B, in the Time D, in a Proportion compounded of the *Ratio* of the Velocity A to the Velocity B, and of the *Ratio* of the Time C to the Time D.

Let the Space passed by E, with the Velocity A, in the Time C, be G; and let G be to (I) as the Velocity A is to the Velocity B; and let (I) be to L as the Time C is to the Time D: then 'tis manifest, that (I) is the Space thro' which F is moved, in the same Time wherein E is moved thro' G; since the Spaces G and (I) are as the Velocities A and B, and since it is as the Time C to the Time D, so is (I) to L: And since (I) is the Space passed



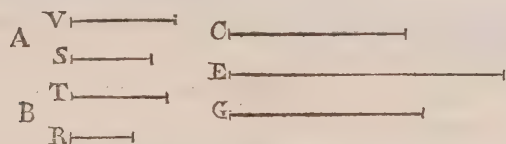
thro' by the Mover F, in the Time C, L will be the Space which is passed thro' by F in the Time D, with the Velocity B: But now the *Ratio* of G to L is compounded of the *Ratio* of G to (I), and of (I) to L, *i. e.* of the *Ratios* of the Velocity A to the Velocity B, and of the Time C to the Time D. ... Q. E. D.

PROP. V. THEOR. V.

If two Moveables move with an equable Motion, but with unequal Velocities, and if the Spaces passed be also unequal, the Proportion of the Times will be compounded of the Proportion of the Spaces, and of the Proportion of the Velocities taken reciprocally.

Let

Let A and B be two Moveables, and let the Velocity of A be to the Velocity of B, as V to T, and let the Spaces passed be as S to R : Then, I say, the Proportion of the Time in which A is moved, to the Time in which B is moved, is compounded of the *Ratio* of the Velocity T, to the Velocity V, and of the *Ratio* of the Space S, to the Space R. Let C be the Time of the Motion A ; and as the Velocity T is to the Velocity V, so let the Time C be to the Time E : And since C is the Time wherein A, with the Velocity V, passes the Space S, and since it is as the Velocity T, of the Moveable B, to the Velocity V, so the Time C to the Time E, E will be the Time wherein the Moveable B would pass thro' the same Space S.

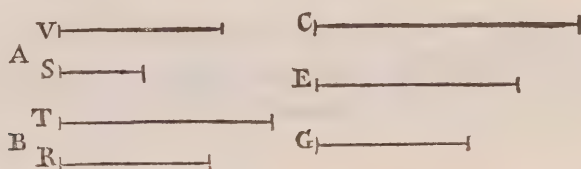


Again, as the Space S is to the Space R, so let the Time E be to the Time G : then 'tis manifest, that G is the Time wherein B would pass thro' the Space R : And because the Proportion of C to G is compounded of the *Ratios* of C to E, and of E to G ; and since the Proportion of C to E, is the same with that of the Velocities of the Moveables A and B reciprocally taken, that is, with that of T and V : and since the Proportion of E G is the same with the Proportion of the Spaces S and R ; therefore the Proposition is manifest.

PROP. VI. THEOR. VI.

If two Moveables move with an equable Motion, the Proportion of their Velocities will be compounded of the *Ratio* of the Spaces run thro', and of the *Ratio* of the Times reciprocally taken.

Let the two Moveables A and B move with an equable Motion ; and let the Spaces passed thro' by them be in Proportion of V to T, and let the Times be as S to R : then I say, that the Velocity of the Moveable A, to the Velocity of B, has a Proportion compounded of the *Ratio* of the Space V to the Space T, and of the Time R to the Time S.



Let C be the Velocity wherewith the Moveable A passes thro' the Space V, in the Time S ; and the *Ratio* the Space V has to the Space T, the same let the Velocity C have to another E : then E will be the Velocity wherewith the Moveable B passes thro' the Space T in the same Time S : But if it be made as the Time R to the Time S, so the Velocity E to another G, G will be the Velocity wherewith the Moveable B passes thro' the Space T in the Time R : We have therefore the Velocity C, wherewith the Moveable A passes thro' the Space V in the Time S ; and the Velocity G, wherewith the Moveable B passes the

Space

Space T in the Time R ; and the *Ratio* of C to G is compounded of the *Ratios* of C to E , and of E to G : But the *Ratio* of C to E , is supposed the same with the *Ratio* of the Space V to the Space T ; and the *Ratio* of E to G , is the same with the *Ratio* of R to S ; therefore the Proposition is manifest.

SALV. These Propositions are all our Author wrote concerning *Equable Motion*.

We will therefore pass on to a more subtile and new Contemplation, *viz.* That which treats of Motion *naturally accelerated* ; as is that which is generally observ'd by falling Bodies.

Of MOTION *naturally accelerated*.

I N T R O D U C T I O N.

IN the foregoing Book we considered the Accidents happening in the Equable Motion ; now we shall treat of Accelerated Motion : And first, it is necessary to find out, and lay down a Definition most suitable to that which Nature makes use of. For altho' it be not inconvenient to feign a Motion at Pleasure, and then contemplate its consequent Passions ; (as those have done who have feign'd *Helixes* or *Conchords*, *Lines* arising from certain Motions, tho' Nature makes use of no such ; and upon that Supposition have laudably demonstrated their Properties) yet seeing Nature makes use of a certain Kind of Acceleration in the Descent of heavy Bodies, we purpose to search out and contemplate their Passions, if-so-be the Definition we intend

intend is but agreeable to the Effence of Motion naturally accelerated And we believe, after a great deal of Trouble and Thought, we have found one that is so ; and we believe so the rather, because all the natural Experiments, which fall under the Observation of our Senses, do answer, and are exactly agreeable to the Properties we shall hereafter demonstrate of it. And in our Search concerning Motion naturally accelerated, we are, as it were, led by the Hand, by observing the usual Method and Proceeding of Nature herself, in all her other Operations ; wherein she commonly makes use of the first, simplest, and easiest Means : For I believe that no Man can think, that Swimming or Flying can be perform'd in a more simple or easy Manner than that very one which Fishes and Birds make use of by natural Instinct. When, therefore, I see a Stone falling from Rest from on high, to acquire continually new Increments of Velocity, why should I not believe such Additaments to be most simple, and to be made thus with more Reason than any other Way ? Now if we seriously consider the Matter, we shall find no Additament, no Increment more simple than that which makes its Additions after the same Manner ; as we may easily understand, by considering the great Affinity there is between Time and Motion. For as the Equability and Uniformity of Motion is defined and expressed by the Equability of the Times and Spaces, (for we call that Motion equable, by which equal Spaces are passed thro' in equal Times) so by the same Equality of the Parts of Time, we may denote or express the Increments of Celerity simply made, conceiving in our Mind that such Motion is continually accelerated uniformly, and at the same

same Rate, whilst equal Additions of Celerity are given them in equal Times, be the Times what they will : so that taking any equal Particles of Time from the first Instant, in which the Moveable departs from Rest, and begins its Descent, the Degree of Velocity in the first and second Particles of Time, is double the Degree of Velocity that the Moveable acquires in the first Particle : and the Degree which it acquires in three Particles, is triple, and that in four, quadruple of the same Degree of the first Time : so as (for our better Understanding) if a Moveable should continue its Motion according to the Degree or *Momentum* of Velocity acquired in the first Particle of Time, and should continue falling equably with that same Degree, this Motion would be twice as slow as that which it would acquire, according to the Degree of Velocity acquired in two Particles of Time ; so that we may reasonably enough understand the Intention of Velocity to be made according to the Extension of the Time.

From whence we may frame the Definition of Motion, which we are going to treat of, as follows :

D E F I N I T I O N.

Motion equably or uniformly accelerated, is that which, departing from Rest, super-addeth to itself equal *Momenta* of Velocity in equal Times.

SAGR. Tho' I can't with Reason contradict this, or any other Definition given by any other Authors, Definitions being all Arbitrary : Yet I may, without Offence, question whether this Definition, thus understood, and admitted in
Abstract,

Abstract, doth suit, agree, and hold true in that Sort of accelerate Motion which heavy Bodies naturally descending do exercise. And because the Author seems to promise us, that the natural Motion of heavy Bodies is such as he has defined it, I could wish that some Scruples, which perplex me, were removed, that so I may apply myself with greater Attention to the following Propositions, and their Demonstrations.

SALV. You and *Simplicius* are certainly in the Right to propose your Difficulties as they arise ; which I imagine will be the very same which I met with when I first read this Tract, and which were partly resolved and removed by weighing well the Author's Reasoning, and partly by my own considering them.

SAGR. For my Part, when I conceive a heavy Body to depart from Rest, *i. e.* from a Privation of all Velocity whatever, and to enter into Motion ; and in that to acquire continually a greater Velocity, in such Proportion wherein the Time increases from the first Instant of Motion ; so as, *e. g.* Eight Pulsations to acquire eight Degrees of Velocity, four Degrees whereof it had acquired at the fourth Pulsation, two at the second, one at the first : Since Time may be divided and subdivided infinitely, it seems to me to follow, since the antecedent Velocity always lessens in such Proportion, that there can be no Degree of Velocity so small, or if you will, no Degree of Tardity so great, in which the said Moveable is not found to be constituted after its Departure from infinite Tardity, *i. e.* from Rest. So that if that Degree of Velocity which it
had

had in four Pulsations of Time was such, as, continuing equable, it would have fell two Miles in an Hour, and with that Degree of Velocity which it had in two Pulsations of Time, it would have passed one Mile in an Hour, it must follow, that in the Instants of Time nearer and nearer to the first Instant wherein it passed from Rest to Motion, that it must be found so slow, that (continuing to move with so great Tardity) it would not have passed one Mile in an Hour, nor in a Day, nor in one, no nor in one thousand Years : Nor, indeed, would it have run one Foot in a much greater Time, which is what we shall hardly get our Imagination to assent to, since our Sense assures us, that a heavy Body is very quick in its Fall.

SALV. This is one of the Difficulties which I met with at my first thinking on this Subject, but I soon got rid of it : And that very Experiment which occasions your Doubt, at length clear'd mine. You say, that Experience seems to shew, that the Body hath no sooner departed from Rest, but it has a very notable Velocity : But I say, that Experience clearly shews us, that the first Imperus's of the falling Body, how heavy soever it be, are most easy and slow. For imagine a heavy Body to be laid on some yielding Matter, and let it alone till it hath pressed into it as much as by its simple Gravity it is able : 'tis manifest, that if you lift it up a Yard or two, and then let it fall upon the same yielding Matter, a new Pression will be made, greater than the former, which was made by its sole Weight, which Effect is caused from the Velocity acquired by the falling Body conjoin'd with its Fall ; and this Pression will be the greater, according as the Percussion

is made from the greater Height, *i. e.* according as the Velocity of the Percutient shall be greater. We may therefore, without Error, conjecture what the Velocity of a falling Body is, from the Quality and Quantity of the Percussion.

But tell me, Sirs, whether or no a Beetle, let fall upon a Stake, from a Height of four Yards, and drives it into the Ground, *e. g.* four Inches, doth not, when falling from an Height of but two Yards, drive it much less Way into the Ground, and less still from an Height of one Yard, and less still from one Foot high : And lastly, if it be elevated but one Inch, what will it do more than if it were laid upon that Stake without any Blow ? Certainly, but very little ; and if it were lifted up but the Thickness of a Leaf, the Operation would be wholly imperceptible.

And because the Effect of the Percussion depends on the Velocity of the Percutient, who will question but that the Motion is very slow, and the Velocity extreme small, where the Operation is altogether imperceptible ?

Pray observe now the Force of Truth, whilst the same Experiment, that, at first thinking, shews us one Thing, upon better Consideration, ascertains the contrary.

But not to have Recourse solely to this Experiment, (which yet is undoubtedly demonstrative) methinks we may come at this Truth by mere Discourse. Suppose a heavy Stone to be sustained at Rest in the Air : this Stone let go, being heavier than Air, falls downwards, and not with an equable Motion, but at first by a slow one, and continually afterwards with an accelerated one, or faster and faster : And since the Velocity may be increased and
diminished

diminished infinitely, what Reason have I to think that that Moveable, departing from infinite Tardity (for such *Rest* is) should immediately enter into ten Degrees of Velocity, more than into four; or into four rather than two, one, or one half, or one hundredth, or, in a Word, of all still less *ad infinitum*?

Pray hear me. You'll, I believe, easily grant, that a Stone departing from a State of Rest, acquires Degrees of Velocity in the same Order wherein those same Degrees are diminished, and lost, when by an impellent Power it is forced up again to the same Height it before fell from: and if this be so, 'tis plain, I think, that the ascending Stone, its Velocity growing less and less till the Whole is spent, can't be reduced to a State of Rest, till it has passed thro' all the Degrees of Tardity.

SIMP. But if the Degrees of Tardity be greater and greater *in infinitum*, they can never be all spent; and therefore such an ascending Body will never be brought to Rest, but will be mov'd constantly slower and slower *in infinitum*, a thing we see does not happen.

SALV. This would happen, *Simplicius*, if the Mover should stay any Time in each Degree; But it passes thro' them without staying any longer than an Instant or Point of Time in any of them: And because in every quantitative Time, tho' never so small, there are infinite Instants, they aptly enough answer to the infinite Degrees of Velocity diminished. Now that the ascending Body does not stay for any quantitative Time, in any Degree of Velocity, appears thus: Because if in any assign'd quantitative Time,

in the first Instant of Time, and likewise in the last, the Mover would be found to have the same Degree of Velocity ; from the second Degree, it might be likewise drawn upwards such another Space as from the first it was carried to the second ; and in the same Manner it would pass from the second to the third, and so, in short, its Motion would become uniform *ad infinitum*.

SAGR. From this Discourse methinks may be gather'd a Solution of a Question controverted amongst Philosophers, viz. What the Cause is of the Acceleration of the Natural Motion of heavy Bodies : For when I consider that the *Impetus* impressed by the Projicient, on a Body driven upwards, continually diminished, which *Impetus*, so long as it was greater than that other contrary one of Gravity, forc'd it upwards : but when both the *Impetus* and this Gravity come to an *Equilibrium*, the Mover ceases to rise higher, and is in a State of Rest, wherein the impressed *Impetus* is not annihilated, but only that Excess is spent, whereby it before exceeded the Gravity of the Mover, and therefore drove it upwards. Since now the external *Impetus* constantly lessening, and consequently the Gravity beginning to prevail, the Body begins to descend, tho' but slowly, on account of the Resistance or Opposition of the impressed *Impetus*, which in some measure still remains in the Mover : but because that *Impetus*, being more and more overcome by the Gravity, continually diminishes ; thence arises the continual Acceleration of the Motion.

SIMP. Your Reasoning I own is pleasant, but 'tis more subtiler than solid : For altho' it were conclusive, yet it
only
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only salves those natural Motions, which are preceded by a violent one, and wherein some Part of the external *Impetus* still remains in Force : but where there is no such remaining Impulse, but the Moveable departs from a long Quiescence, the Force of your whole Reasoning vanishes.

SAGR. I believe you are out, Sir, and that this Distinction of Cases, which you make, is superfluous, or, to be plain with you, of no Moment at all : For, pray answer me, Can't sometimes a greater, and sometimes a less *Impetus*, be impressed by the Projicient upon the Project, so that at one time it may be forced one hundred Yards high, at another twenty, or four, or one ?

SIMP. No doubt.

SAGR. Again ; this impressed *Impetus* may so little exceed the Resistance of the Gravity, as not to raise the Project above an Inch ; and at length the Power of the Projicient may be only so much as exactly to ballance the Resistance of the Gravity, and the Moveable is not driven upwards at all, but only sustain'd.

When therefore you hold a Stone in your Hand, what else do you do but impress on it an *Impetus*, which forces it as much upwards, as its Faculty of Gravity draws it downwards ? And do you not continue the same impress'd *Impetus* on it all the time you hold the Moveable in your Hand ? It is not surely lessen'd by holding it longer in your Hand ? And this holding it, which hinders the Descent of the Stone, is surely the very same, whether it be perform'd by
your

your Hand, or by a Table, or by a Rope, at which it is hung?

We may conclude therefore, *Simplicius*, that the Precedence of a long, a short, or of a momentary Rest, makes no Alteration at all, so as to hinder the Stone from always departing from Rest, affected with such an *Impetus*, contrary to its Gravity, as just suffices to keep it in Rest.

SALV. I don't think this a seasonable Time to enquire into the Cause of the Acceleration of Natural Motion ; concerning which Philosophers have greatly differed in Opinion : some reducing it to the approaching to the Centre ; others to the fewer Parts of the *Medium* successively remaining to be perforated : And others referring it to a certain Extrusion of the ambient *Medium*, which, upon conjoining upon the Back of, or behind the Moveable, presses upon, and continually thrusts it more ; which Conceits, and others of the like Nature, 'tis hardly worth while to examine and answer. For as to our Author, it is enough at present that we understand, that his Business is to search out and demonstrate to us some Passions of an accelerated Motion, (let the Cause of that Acceleration be what it will) so that the *Momenta* of that Acceleration may continually increase after the Departure from Rest, according to that most simple Proportion, wherewith the Continuation of the Time increases ; which is as much as to say, that in equal Times there are made equal Additions of Velocity. And if it shall be found that those Accidents which shall hereafter be demonstrated, are agreeable to Truth, in the Motion of heavy Bodies naturally descending and accelerated, we may conclude that our
assumed

assumed Definition comprehends or takes in such a Motion of heavy Bodies ; and that it is true that their Acceleration continually increases, as the Time and Duration of Motion does.

SAGR. In the Mind I am in at present, it may more clearly, and without altering your Meaning, be defined thus,

That Motion uniformly accelerated, is that wherein the Velocity constantly increases, as the Space passed thro' is increased.

So that, for Example, the Degrees of Velocity acquired by the Moveable, in a Descent of four Yards, should be double that which it had when it had descended two Yards, and this double to that acquired at the first Yard : For it can't be doubted sure, but that that Body which falls from an Height of six Yards, has, and strikes with an *Impetus* double to that which it had when it had fell but three Yards, and triple of that which it had at two Yards, and six times that it had at the End of one Yard's fall.

SALV. I'm not a little pleased to have such a Companion in Error as you are : Nay, I must tell you, there seems to be so much Likelihood and Probability in your Opinion, that our Author himself, when I proposed this to him, could not deny but that he for some time continued in the same Mistake. But that which I afterwards was greatly surprized at, was to see, in so few Words, two Propositions to be proved, not only to be false, but impossible ; which carry with them such seeming Truth, that, after having proposed them to many Persons, I never found one but what freely allowed them.

SIMP. For

SIMP. For my Part I should certainly be of the Number of those that allow, that a descending Body *vires acquirit eundo*, its Velocity increasing as the Space does ; and that the *Momentum* of the same Percutient is double, if it fall from a double Height : These seem to me to be Propositions to be granted without the least Hesitation or Doubt.

SALV. And yet they are as false and impossible, as 'tis for Motion to be made in an Instant ; as I will now, in the clearest Manner, demonstrate to you. If so be the Velocities have the same *Ratio* as the Spaces run thro', or to be run thro', those Spaces are passed thro' in equal Times : If therefore the Velocities with which the falling Body descends thro' the Space of four Yards, were double the Velocities with which it passed thro' the two first Yards, (like as the Space is double the Space) then the Times of such Transitions are equal : But for a Moveable to pass thro' four Yards, and two Yards in the same Time, this can have place only in instantaneous Motion : But we see that the falling Body performs its Motion in Time, and to pass thro' two Yards in less Time than it passes thro' four : Therefore its Velocity does not increase in the same *Ratio* with the Space run thro'.

And now, that the other Proposition is also false, may with the same Clearness be demonstrated : For since the Percutient is the same, the Difference and *Momentum* of the Percussions can be determin'd only by the Difference of the Velocities. If therefore a Percutient, falling from a double Height, makes a Percussion whose *Momentum* is double,

double, it must strike with a double Velocity : But the double Velocity runs thro' the double Space in the same Time : And the Time wherein it falls from a greater Height, we see is longer.

SAGR. 'Tis very evident and wonderful easy now : But this extreme Easiness wherewith you manifest the most abstruse Conclusions, will be apt to lessen the Value they had whilst they lay hid under contrary Appearances : Thus I dare say it is with the Generality of Mankind ; they have a much less Esteem for that Knowledge that's so very easily acquired, than they have for that about which more tedious and puzzling Debates are bandy'd to and fro.

SALV. To those who with great Brevity and Perspicuity detect the Fallacies of such Propositions as are received by the Generality for true, it would be tolerable, methinks, if, instead of Thanks, they met with only Contempt : But the Temper shewn by some Men, pretending to be, at least, upon a *Par* in the same Studies with another, is next to intolerable ; who, when they are sensible they've publish'd Conclusions as true ones, the Falsity whereof they afterwards see by a short and easy Disquisition, have by that other been detected and laid open. This Temper I would not call *Envy*, which usually in Time turns to Anger, if not to perfect Hatred of the Person that detects such Fallacies : But I chuse to call it an itching Desire rather to defend their inveterate Errors, than to admit the Reception of new-discover'd Truths ; which Humour sometimes carries them so far as to write against those Truths, which they are inwardly sufficiently

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convinced of, only to keep the Reputation of others under, in the Opinion of the numerous and ignorant Vulgar.

Of such false Conclusions, received indeed for true, tho' easily to be refuted, I have heard no small Number from our *Academic*: and I have kept Account of some of them.

SAGR. You must not deprive us of them, but impart them to us, when we shall appoint a particular Meeting for that Purpose. But to go on; I think by this Time we have establish'd the Definition of Motion uniformly accelerate, to be treated of in the following Discourses: and 'tis this;

DEFINITION.

Motion equably or uniformly accelerated (we say) is that which departing from Rest, super-adds to itself, in equal Times, equal Momenta of Celerity.

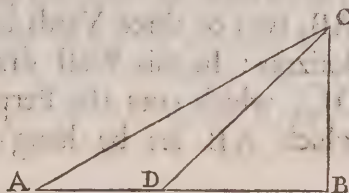
SALV. This Definition being allow'd, the Author asks and supposes but one Principle to be true; namely,

POSTULATUM.

He supposes that the Degrees of Velocity, acquired by the same Moveable, upon Planes of different Inclinations, are then equal, when the Elevations of the said Planes are equal.

By the Elevation of an inclin'd Plane, he means a Perpendicular let fall from the Top of the said Plane to its
Base,

Base, or horizontal Line produced : But that you may better understand this, let the Line AB be parallel to the Horizon, upon which let the two Planes CA and CD be inclin'd ; then the Perpendicular CB falling upon the horizontal Line AB, our Author calls the Elevation of the Planes CA and CD; and supposes that the Degrees of Velocity of the same Moveable, descending along the inclin'd

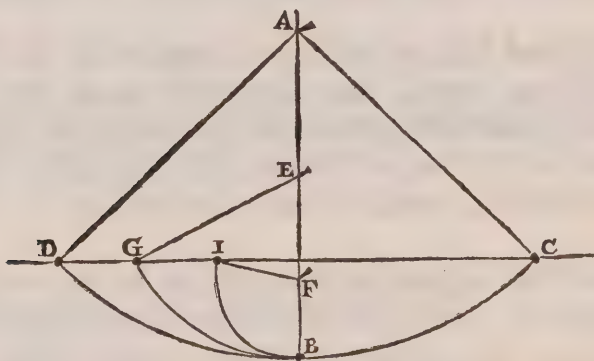


Planes CA and CD, which it hath acquired in the Terms A and D, are equal, their Elevations being the same, *viz.* CB. And such also must the Degree of Velocity be understood to be, which the same Moveable, falling from the Point C, would have in the Point B.

SAGR. This Supposition seems to me so very probable, that it must be surely granted, without any Dispute, supposing always that all accidental and external Impediments are removed, as also that the Planes be very hard and smooth, and that the Moveable be perfectly round, so that both Plane and Moveable may be free from all Roughness or Unevenness. These Obstacles and Impediments, I say, being removed, the Light of Nature dictates to me, that a Ball heavy and exactly round, descending by the Lines CA, CD, and CB, would necessarily come to the Terms A, D and B, with equal Impetus's.

SALV. What you say carries great Probability with it; but I will now increase this Probability by an Experiment, so as it shall amount to very little less than a real Demonstration.

Conceive this Leaf of Paper to be a Wall, perpendicular to the Horizon, and to a Nail fasten'd in it, imagine a Ball of Lead of an Ounce or two Weight, to hang by a fine Thread as AB, two or three Yards long, and perpendicular to the Horizon: In this Wall draw also an horizontal Line as DC, which cuts the Perpendicular AB at right Angles, which AB let be hang'd two Inches or



thereabouts from the Wall. Then transferring the String AB, with the Ball, to AC, let go the Ball, and you'll see it describe the Arch CBD, and to pass so far beyond the Term B, that running along the Arch BD, it will rise within a very little as high as the Parallel CD; it does not indeed quite reach it, being hinder'd by the Air and Thread: Whence we may safely conclude, that the *Impetus*

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of the Ball, acquired in the Point B, in its Descent thro' the Arch CB, is such as suffices to force it upwards to the same Height, along such another Arch BD: And now having often repeated this Experiment, drive into the Wall a Nail, just touching the Perpendicular AB, as in E or F, which let stand out five or six Inches, that the Thread AC returning as before, to carry back the Ball C, along the Arch CB; as soon as it is come to the Point B, striking upon the Nail E, it may be forc'd to proceed along the Arch BG, whose Centre is E; by which we shall see what the same *Impetus* is able to do, which before being conceiv'd in the same Term B, forc'd the same Moveable thro' the Arch BD, to the Height of the horizontal Line CD. Now, Sirs, you'll with Pleasure see the Ball to be carried to the Point G in the horizontal Line; and the same will happen if the Perpendicular strikes the Nail plac'd lower, as in F, in which Case the Ball would describe the Arch BI, always terminating its Ascent exactly in the Line CD: And if the Nail be plac'd, by Way of Check, so low, that the Part of the Thread below it, in its Motion, can't reach to the Height of the Line CD, (which would happen if the Nail be nearer to the Point B, than to the Intersection of the Line AB, with the horizontal Line CD) then it would whirl quite round the Nail. This Experiment takes away all Doubt of the Truth of the Supposition; for since the two Arches CB and DB are equal, and similarly situate, the *Momentum* acquired in the Descent thro' the Arch CB, is the same with that acquired in the Descent thro' the Arch DB: But the *Momentum* acquired in the Point B, in the Descent thro' the Arch CB, was sufficient to force the same Moveable up thro' the Arch

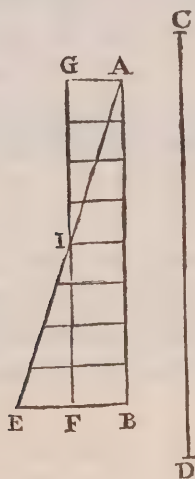
particular Manner make Use of this Assumption, in such Motions as are made upon plain Superficies, and not upon curv'd ones ; in which the Acceleration proceeds with Degrees very different from those wherewith we suppose it to proceed on plain Surfaces : Infomuch that altho' the Experiment I just mention'd, shews us that the Descent thro' the Arch CB, confers such a *Momentum* on the Moveable, as can force it back to the same Height, along any Arch, as BC, BG, or BI ; yet we can't shew, with like Evidence, that the same thing would happen in case a most perfect Ball were to descend along flat Planes, inclin'd according to the Inclinations of the *Chords* of these same Arches : But we may rather believe that Angles being form'd by those flat Planes, at the Point B, the Ball which descends along the Plane inclin'd, as is the Chord CB, finding an Obstacle in the Planes, ascending according to the Chords BD, BG, BI, by striking against them, will lose some Part of its *Impetus*, and could not rise again to the Height of the Line CD. But that Obstacle being removed which prejudiceth the Experiment, we may certainly conceive in our Mind, that the *Impetus* (which has its Force from the Quantity of the Descent) would be able to remount the Moveable to the same Height. Let us therefore take this at present as a *Postulatum*, the Truth whereof we shall hereafter find establish'd, when we shall see other Conclusions built upon this *Hypothesis*, to answer and most exactly to agree with Experience.

This single Principle being supposed, our Author goes on and demonstrates the following Propositions.

PROP. I. THEOR. I.

The Time wherein any Space is passed by a Moveable, with a Motion uniformly accelerate from Rest, is = the Time in which the same Space would be passed by the same Moveable, carried with an equable Motion, the Degree of whose Velocity is *subduple* to the greatest and ultimate Degree of the Velocity of the former uniformly accelerate Motion.

Let the Line AB represent the Time wherein the Space CD is passed by a Moveable, with a Motion uniformly



accelerate from Rest in C; and let the greatest and last Degree of Velocity acquired in the Instants of Time A, B, be represented by the Line EB, drawn at Pleasure to AB; and join AE: Divide AB into any Number of equal Parts, and

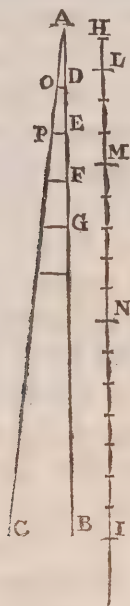
and thro' the Points of Division draw as many Lines to the Line AE, $\parallel$ EB; these will represent the increasing Degrees of Velocity after the first Instant A. Then divide BE into two equal Parts in F, and draw FG and AG, $\parallel$ BA and BF: then a Parallelogram AGFB will be constituted = the Triangle AEB, whose Side GF divides the Line AE into two equal Parts in (I). For if the Parallels of the Triangle AEB be extended unto GF, we shall have the Aggregate of all the Parallels contained in the quadrilateral Figure, equal to the Aggregate of those comprehended in $\triangle$ AEB; for those in $\triangle$ IEF are = those contained in $\triangle$ GIA, and those which are in the Trapezium AIFB are common: Now since all and singular the Instants of the Time AB, do answer to all and singular the Points of the Line AB; and since the Parallels drawn from these Points comprehended in $\triangle$ AEB, represent the increasing Degrees of augmented Velocity, and the Parallels contained within the Parallelogram, do in like manner represent as many Degrees of Velocity not augmented, but equable; it appears that as many *Momenta* of Velocity were spent in the accelerate Motion, according to the increasing Parallels of the $\triangle$ AEB, as in the equable Motion, according to the Parallels of the Parallelogram CB: For what is wanting of *Momenta* in the first half of the accelerate Motion (for the *Momenta* are deficient, represented by the Parallels of $\triangle$ AGI) is made up by the *Momenta* represented by the Parallels of $\triangle$ IEF. 'Tis manifest, therefore, that those Spaces will be equal, passed over in the same Time by two Moveables, one whereof is moved with a Motion uniformly accelerated from Rest, but the other with an equable Motion, according to a

Momentum subduple of the greatest *Momentum* of the Velocity of the accelerated Motion. Q. E. D.

PROP. II. THEOR. II.

If a Moveable descend from Rest, with a Motion uniformly accelerate, the Spaces passed by it in any Times whatsoever, are to each other in a *duplicate* Proportion of the same Times, *i. e.* as the Squares of the same Times.

Let the Line AB represent a Length of Time from any first Instant A, wherein take any two Times AD and AE;

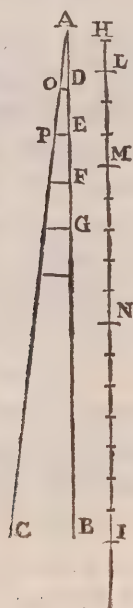


and let HI be a Line in which the Moveable from the Point H, as the first Beginning of Motion descends, accelerating

celerating uniformly ; and let the Space HL be passed over in the first Time AD, and let HM be the Space it shall descend in the Time AE : I say the Space MH is to the Space HL, in a *duplicate* Proportion of that which the Time EA has to the Time AD ; or, if you will, the Spaces MH and HL have the same Proportion to one another, as the Squares of EA and AD have to each other. Draw the Line AC, making any Angle with AB ; and from the Points D and E draw the Parallels DO, EP, of which DO will represent the greatest Degree of Velocity acquired in the Instant D, of the Time AD ; and PE the greatest Degree of Velocity acquired in the Instant E, of the Time AE : And since in the foregoing Proposition it was demonstrated, concerning Spaces passed over, that those are equal to one another, one of which is passed by a Moveable out of Rest, with a Motion uniformly accelerate ; and the other which is passed in the same Time, by a Moveable carried with an equable Motion, whose Velocity is *subduple* of the greatest acquired in the accelerate Motion ; it is manifest, that the Spaces MH and LH are the same which with equal Motions, whose Velocities would be as the Halves of PE and OD, would be passed over in the Times EA and DA. If therefore it be prov'd that these Spaces MH and LH are in a *duplicate Ratio* of the Times EA and DA, we have gain'd what we wanted.

Now in the fourth Proposition of the first Book it was demonstrated, that of Moveables carried with an equable Motion, the Spaces passed over are in a Proportion to one

another, compounded of the *Ratio* of the Velocities, and of that of the Times : Now here the *Ratio* of the Velocities is the same with the *Ratio* of the Times (for the same Proportion as the Half of PE has to the Half of OD, or the Whole PE to the Whole OD, the same AE has to AD) ; therefore the Spaces are in a *duplicate* Proportion of the Times, or the Spaces passed over are to each other as the Squares of the Times. *Q. E. D.*



Hence likewise 'tis manifest, that the same *Ratio* of the Spaces is *duplicate* to the *Ratio* of the greatest Degrees of Velocity, viz. of the Lines PE and OD, since it is $PE : OD :: EA : DA$.

COROL

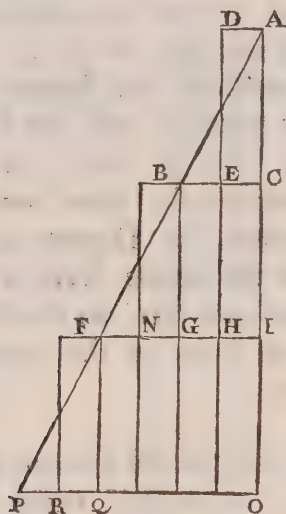
COROLLARY I.

Hence 'tis manifest, that if there were never so many equal Times, taken successively from the first Instant, or Beginning of Motion ; as, for Instance, AD, DE, EF, FG, in which are passed over the Spaces HL, LM, MN, NI, those Spaces will be one to another as the uneven Numbers are counted from the Unit, *viz.* as 1, 3, 5, 7 : For this is the *Ratio* of the Excesses of the Squares of the Lines exceeding one another equally, and the Excess of which Lines is equal to the least of them ; or of the Squares consequent, or that successively follow one another from Unity : Whilst, therefore, the Degrees of Velocity are increased, according to the simple *Series* of Numbers in equal Times, the Spaces run thro' in the same Times increase according to the *Series* of the uneven Numbers, counting from the Unit.

SAGR. Stay a little pray, whilst I mention a Thing just come into my Mind ; to explain which the better, I'll describe a Figure :

Let

Let then the Line AI represent the Continuation of Time, after the first Instant in A : Draw the right Line AF to the Point A, at any Angle with AI, and join the Points I and F ; then bisect AI in C, and draw the Line $CB \parallel IF$: Then considering BC as the greatest Degree of Velocity, which beginning from Rest, in the first Instant A, goeth on augmenting according as the Parallels to BC do increase, which are in the $\triangle ABC$ (which is really to in-



crease as the Time increases) by what has been said, I admit, without Dispute, that the Space run thro' by the falling Moveable, with the Velocity increased in the manner aforesaid, would be = the Space that the same Moveable would pass thro', in case it were in the same Time AC mov'd with an uniform Motion, whose Degree of Velocity should be = EC, the Half of BC : I now proceed farther, and imagine the Moveable having descended

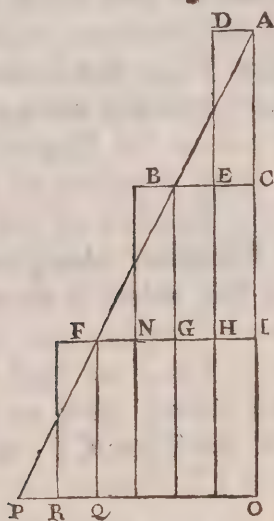
with

†

with an accelerate Motion, to have in the Instant C the Degree of Velocity BC ; 'tis manifest, that if it did continue to move with the same Degree of Velocity BC, without farther Acceleration, it will run thro' in the following Time CI, a Space double of that which it ran thro' in the equal Time AC, with the Degree of uniform Velocity EC, the Half of the Degree BC : But because the Moveable descends with a Velocity increased always uniformly in all equal Times, it will add to the Degree BC, in the following Time, those same *Momenta* of Velocity that increase according to the Parallels of the Triangle BFG, which is = Δ ABC : Wherefore adding to the Velocity GI the Half of the Degree FG, which is the greatest of all acquired by the accelerate Motion, and regulated by the Parallels of the Triangle BFG, we shall have the Degree of Velocity IN, with which, with an uniform Motion, it would have made in the Time CI ; which Degree IN being *triple* the Degree CE, proves that the Space run thro' in the second Time CI, ought to be *triple* of that it ran thro' in the first Time CA.

And if we conceive another equal Part of Time as IO, to be added to AI, and the Triangle to be enlarg'd to APO, it is manifest, since thro' all that Time IO the Motion is continued with the Degree of Velocity IF, acquired by the accelerate Motion in the Time AI, since the Degree IF is *quadruple* of the Degree EC, the Space run thro' in the Time IO, 'twill be *quadruple* of that which was run thro' in the first equal Time AC : But now if the Increase of the uniform Acceleration be continued in the Δ EPQ, like to that in Δ ABC, which being reduced to equable Motion, adds the Degree = EC, QR being added = EC, we

we shall have the whole equable Velocity exercised in the Time IO, *quintuple* of the equable Velocity of the first Time AC: Wherefore also the Space run thro' is *quintuple* that which was run thro' in the first Time AC.



'Tis manifest, therefore, from this Computation, that the Spaces run thro' in equal Times, by a Moveable, which, departing from Rest, acquires a Velocity conformable to the Increase of the Time, are to one another as the uneven Numbers, counting from the Unit, as 1, 3, 5, are: And that the Spaces run thro' being taken together, the Space run thro' in the *double* Time, is *quadruple* to that ran thro' in a *subduple* Time; that run thro' in the *triple* Time is *noncuple*; and, in a Word, that the Spaces run thro' are in a *duplicate Ratio* of the Times, *i. e.* as the *Squares* of the Times.

SIMP. This

SIMP. This simple and clear Discourse of *Sagredus*, pleases me beyond the Author's more obscure Demonstration ; so that I am thoroughly convinc'd that the thing must succeed, as has been said, the Definition of Motion uniformly accelerated being established and granted : But I yet question whether this be the Acceleration which Nature makes use of in the Motion of its descending heavy Bodies : Wherefore that I, and others like me, may the more clearly understand this, it would, methinks, be very seasonable, before we go on, to produce some of those Experiments, which are said to be many, which in divers Cases agree with the Conclusions demonstrated.

SALV. You, like a Person of great Skill, make a very reasonable Demand ; for so it is usual and convenient to do in Sciences that apply Mathematical Demonstrations to Physical Conclusions, as is seen in those that treat of *Perspective*, *Astronomy*, *Mechanics*, *Musick*, and others, who with sensible Experiments confirm those their Principles, which are the Foundation of the following Structure : And therefore I would not have you think it superfluous, to discourse with some Prolixity concerning this first and grand Fundamental, whereon lies the Weight of the immense Machine of infinite Conclusions, of which we have but a very small Part mentioned by our Author in this Book, who thought it enough to open the Door, hitherto shut, to speculative Genius's. As to Experiments, the Author made several ; and to assure us that the Acceleration of descending heavy Bodies observes the forementioned Proportion, I have often with him made the following Trial.

M m

We

We took a Piece of Wood, a Prism about twelve Yards long, half a Yard wide, and about three Inches thick, in which Thickness we made a Groove very strait, or direct, a little more than an Inch wide; and to make it very smooth and sleek, we glewed within it a Piece of Vellum polish'd as smooth as possible. In this Groove we let fall a Ball made of the hardest Brass, round, and well polish'd. Thus elevating this Prism at Pleasure, one or two Yards above the Plane of the Horizon, we let the Ball, (as I said) descend along the Groove, observing in the manner I shall tell you presently, the Time spent in its Fall from Top to Bottom.

We repeated this often, in order to be certain of the Quantity of Time spent in the Descent, and in these Times we never found any Difference worth mentioning, no not the tenth Part of a Second: This being done and thoroughly established, we let the same Ball descend but a fourth Part of the Length of the Groove, and measuring the Time of the Descent, we found it to be exactly the Half of the other: And then making Trial of the other Parts, by comparing the Time of its Descent thro' the whole Length, with that wherein it run thro', for Instance, $\frac{1}{2}$ the Length, or $\frac{2}{3}$, or $\frac{3}{4}$, or, in a Word, with the Time of its running thro' any other Part of the Length, by Experiments near an hundred times repeated, we always found the Spaces run thro' to be to one another, as the *Squares* of the Times: and this in all Inclinations of the Plane, *i. e.* of the Groove in which the Ball was made to descend: Where also we observ'd the Times of the Descents along different Inclinations, to retain the same Proportion to one another,

another, which anon we shall find assign'd them, and demonstrated by the Author.

Now as to the measuring of Time : We hung on high a large Bucket of Water, which, by a very small Hole thro' the Bottom, emitted a small Thread of Water, which was received by a small Cup, all the Time the Ball was falling thro' the Groove, or the several Parts of it : Then those Parcels of Water thus collected, were every Time weigh'd in a very exact Pair of Scales, and the Differences and Proportions of the Weight exhibited to us the Differences and Proportions of the Times, and that so accurately, that, as I before said, those Trials often repeated, never differ'd any thing worth speaking of.

SIMP. I should have been wonderfully pleased had I been present at those Experiments : but since I may be assured of your Care in making them, and Fidelity in relating them, I am satisfied, and admit them as true, and safely to be relied on.

SALV. Then we may resume our Reading, and go on :

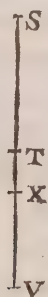
C O R O L L A R Y II.

It is inferred, secondly, that if from the Beginning of the Motion, any two Spaces are taken, passed thro' in any Times, those Times shall be to each other as either of those Spaces is to a Space which is a mean Proportional between the two Spaces passed thro' :

M m 2

Thus,

Thus, taking two Spaces ST and SV from the beginning of the Motion S , to which let SX be a mean Proportional ; then the Time of the Fall thro' ST , shall be to the Time of the Fall thro' SV , as ST to SX ; or again, the Time of the Fall thro' SV , to the Time thro' ST , is as VS to SX . For since it has been demonstrated that the



Spaces run thro' are in a *duplicate Ratio* of the Times, or (which is the same) as the Squares of the Times : But the *Ratio* of the Space VS to the Space ST , is *duplicate* of the *Ratio* of VS to SX , or as the *Squares* of VS and SX are ; 'tis manifest therefore that the Proportion of the Times of the Descents thro' SV , ST , is as that of the Spaces or Lines VS , SX .

SCHOLIUM.

Now that which has been demonstrated, in Motions perform'd perpendicularly, may be understood also to hold good in Motions made along Planes of any Inclination whatever ; for it is supposed that in them the Degree of

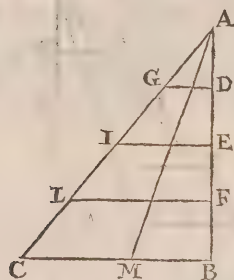
Acceleration

Acceleration increases in the same Proportion, *i. e.* according to the Increase of the Time, or, if you will, according to the simple and primary *Series* of Numbers.

[P R O P. III.] T H E O R. III.

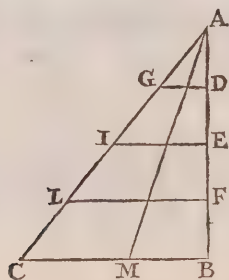
If the same Moveable, departing from Rest, do move along an inclin'd Plane, and also along the Perpendicular, whose Heights are the same, the Times of their Motions will be to one another as the Lengths of that Plane, and that Perpendicular.

Let the inclined Plane be AC, and the Perpendicular AB; both which have the same Heights above the Horizon CB, *viz.* the very same Line BA; I say the Time of the Descent of the same Moveable along the Plane AC, has



the same Proportion to the Time of its Descent along the Perpendicular AB, as the Length of the Plane AC to the Length of the Perpendicular AB: For let any Number of Lines DG, EI, FL, be drawn $\parallel$ the Horizon CB; 'tis manifest from the foregoing Assumption or Supposition, that

that the Degrees of the Velocity of the Moveable from A, the first Beginning of Motion acquired in the Points G and D, are equal, their Heights above the Horizon being equal ; And in like manner the Degrees in the Points (I) and E, will be the same, as also the Degrees in L and F : And if not only these Parallels, but others, be imagin'd drawn from all the Points of the Line AB to the Line AC, the *Momenta* or Degrees of the Velocities, at the Terms or Extremes of each Parallel, will be always = one another : therefore the two Spaces AC and AB are passed over with the same Degrees of Velocity : But it has been demonstrated, that if two Spaces are passed over by a Moveable, with the same Degrees of Velocity, the Times of the Motions have the same Proportion to each other as the Spaces have : therefore the Time of the Motion thro'



AC, to the Time thro' AB, is as the Length of the Plane AC to the Length of the Perpendicular AB. *Q. E. D.*

SAGR. Methinks this Trouble might have been spar'd, and we might have safely concluded the same thing, having before proved, that the Sum of the accelerated Motion of the

the Transitions thro' AC and AB, is = the equable Motion, whose Degree of Velocity is *subduple* the greatest Degree CB : Since therefore the two Spaces AC and AB are passed thro' with the same equable Motion, it hath already been shewn, by the first Proposition of the first, that the Times of the Transitions will be to one another as the said Spaces.

C O R O L L A R Y.

Hence is collected, that the Times of the Descents along Planes of different Inclinations, provided they have the same Elevation, are to one another as their Lengths.

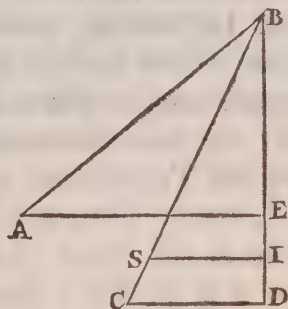
For if we suppose another Plane AM coming from A, and terminated by the same horizontal Line CB, 'tis demonstrated by the last Proposition, that the Time of the Descent thro' AM, to the Time thro' AB, is as the Line AM to AB; but as the Time AB to the Time thro' AC, so the Line AB to AC; therefore, *ex æquali*, as AM to AC, so is the Time along AM, to the Time thro' AC.

P R O P. IV. T H E O R. IV.

The Times of the Transitions along equal Planes, but unequally inclined, are to each other in a *subduple* Proportion of the Elevations of the same Planes, reciprocally taken.

Imagine

Imagine two equal Planes, but unequally inclin'd, as BA and BC, to proceed from the Point B; draw the horizontal Lines AE, CD, as far as the Perpendicular BD: Let BE be the Elevation of the Plane BA, and BD that of the Plane BC, and let BI be a mean Proportional between the Elevations DB and BE; then 'tis manifest, that the Proportion of DB to BI is *subduple* of the Proportion of DB to BE: Now I say, and must prove, that the Proportion of the Times of the Descents, or Transitions along the Planes BA and BC, is the same with the



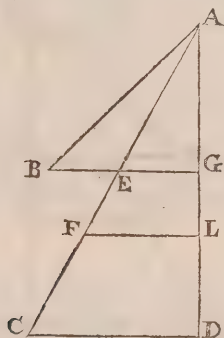
Proportion of DB to BI taken reciprocally: that is, I must prove that the Time along BA, is to the Time along BC, as DB is to BI: draw IS $\parallel$ DC: Now since it has been demonstrated, that the Time of the Descent thro' BA, to the Time of the Fall thro' the Perpendicular BE, is as BA to BE; and the Time thro' BE is to the Time thro' BD, as BE to BI; and the Time along BD to the Time along BC, as BD to BC, or as BI to BS: therefore, *ex æquali*, the Time along BA to the Time along BC, will be as BA to BS, or as CB to BS; but CB is to BS, as DB to BI, therefore the Proposition is manifest.

PROP. V.

PROP. V. THEOR. V.

The Proportion of the Times of the Descent along Planes that have different Inclinations and Lengths, as also their Elevations unequal, is compounded of the Proportion of the Lengths of those Planes, and of the *subduple* Proportion of their Elevations taken reciprocally.

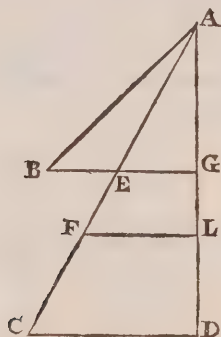
Let AB and AC be Planes inclined after different Manners, whose Lengths are unequal, and whose Elevations are also unequal : I say, that the Proportion of the Time of the Descent through AC, to the Time through AB, is composed of the *Ratio* of AC to AB, and of the *Subduple* of their Elevations taken reciprocally : For,



draw the Perpendicular AD, which let the horizontal Lines BG CD meet; and between the Elevations DA and AG, let AL be a mean Proportional; and from the Point L draw a Parallel to the Horizon, meeting the Plane AC in F, then AF will be a mean Proportional between CA and AE: And because the Time through

N n
AC

AC, to the Time thro' AE, is as the Line FA to AE ; and the Time along AE, to the Time along AB, as the same AE to the same AB ; 'tis manifest that the Time along AC, to the Time along AB, is as AF to AB : It remains, therefore, to be demonstrated, that the Proportion of AF to AB, is compos'd of the *Ratio* of CA to AB, and of the *Ratio* of GA to AL, which is a *Ratio subduple* of the Elevations DA and AG reciprocally taken. But that is manifest putting CA between FA and AB ; for the *Ratio* of FA to AC, is the same with that of LA to AD, or of GA to AL, which is *subduple* of the *Ratio* of the Elevations GA and AD : And the *Ratio* of CA to AB is the very *Ratio* of the Lengths ; therefore the Proposition is manifest.

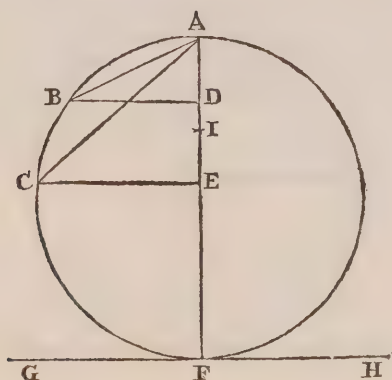


PROP. VI. THEOR. VI.

If from the uppermost and lowermost Points of a Circle, placed perpendicularly to the Horizon, certain inclined Planes be drawn to the Circumference, the Times of the Descents along these Planes, will be equal to each other.

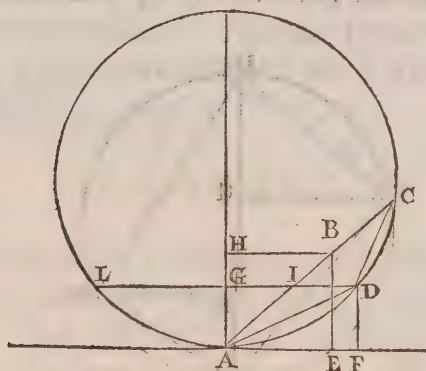
Let
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Let GH be the Horizon, and ACF a Circle erected upon it, whose Diameter FA, passing thro' its lowest Part, or Point of Contact, let be erected perpendicular to the horizontal Line GH; and from its uppermost Point A let any Number of inclin'd Planes, as AB and AC be drawn to the Circumference; I say, that the Times of the Descents along those Planes are equal to one another. Draw BD and CE perpendicular to the Diameter AF, and let AI be a mean Proportional between the Heights EA and AD of the Planes.



Now because the Rectangles FAE and FAD are = the Squares of AC and AB; and because it is as $\square$ FAE to $\square$ FAD, so AE to AD; therefore it is as the Square of CA to the Square of AB, so the Line EA to the Line AD. But as the Line EA to DA, so the Square of IA to the Square of AD; therefore the Squares of the Lines CA and AB are to one another as the Squares of the Lines IA and AD; and therefore as the Line CA to AB, so is IA to AD: But in the foregoing

mentum, is as DF to DA or BA ; therefore the *Momentum* of the same Weight upon the Plane, inclin'd according to DA , to the *Momentum* upon the Inclination, according to ABC , is as the Line DF to the Line BE : Wherefore the Spaces which the same Weight shall pass thro' in equal Times, upon the Inclinations CA and DA , will be to each other as the Lines BE and DF , by the second Proposition of the first Book : But as BE to DF , so it may be demonstrated, that AC is to DA ; therefore the same Moveable will pass thro' the Lines CA and DA in equal Times.



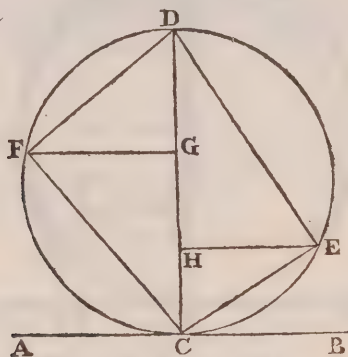
Now that BE is to DF , as CA to DA , is thus demonstrated. Join CD , and thro' D draw $DGL \parallel$ to AF , cutting CA in I , and thro' B draw BH also parallel to AF ; then $\angle ADI$ will be $= \angle DCA$; they standing upon the equal Arches LA and AD , and $\angle DAC$ is common : Therefore the Sides of the similar Triangles CAD and DAI , which are about the equal Angles, will be proportional ; and thence it will be, $CA : AD :: AD : AI$, *i.e.* as BA to AI , or as HA to AG , *i.e.* as BE to DF .
Q. E. D.

But

But the same Thing may be more expeditiously demonstrated thus ;

Let a Circle, as $DFCE$, be so erected upon the Horizon, as that its Diameter DC may be perpendicular thereto ; and from the uppermost Point D , let there be an inclin'd Plane, as DE , drawn to the Circumference : I say, the Descent thro' the Plane DE , and that thro' the Diameter DC of the same Moveable, are perform'd in equal Times : For,

Draw FG parallel to the Horizon AB , which will be perpendicular to the Diameter DC , and draw FC ; now



because the Time of the Fall along DC is to the Time of the Fall along DG , as a mean Proportional between CD and DG , is to DG ; and the mean Proportional between CD and DG , being DF ; $\angle DFC$ in the Semicircle being a Right one, and FG perpendicular to DC ; therefore the Time of the Fall through DC , is to the Time of the Fall through DG , as the Line FD to DG . But it has been demonstrated, that the Time of the Descent through DE , is to the Time of the Descent through DG , as the
same

same Line DF to DG; therefore the Times of the Descent thro' DF, and thro' DC, are to the Time of the Fall thro' DG, in the same Proportion; therefore they are equal. It may likewise be demonstrated, that if from the lowest Term C, be drawn the Chord CE, and EH being drawn $\parallel$ to the Horizon, and ED drawn; that the Time of the Descent thro' EC will be equal to the Time of the Fall thro' the Diameter DC.

C O R O L L A R Y I.

Hence we may conclude, that the Times of the Descents along all the Chords drawn from the Terms C or D, are equal to one another.

C O R O L L A R Y II.

Hence 'tis likewise manifest, that if the Perpendicular and inclin'd Plane descend from the same Point, along which Perpendicular and inclin'd Plane the Descents are made in equal Times, they are in a Semicircle, whose Diameter is that very Perpendicular.

C O R O L L A R Y III.

Hence again, 'tis manifest, that the Times of the Motions along inclin'd Planes, are then equal, when the Elevations of equal Parts of the same Planes shall be to one another, as the Lengths of those Planes.

For it has been shewn, that the Times thro' CA and DA, in the last Figure but one, are equal, the Elevation
of

of the Part AB being equal to AD, *i. e.* that BE shall be to the Elevation DF, as CA to DA.

SAGR. Pray, Sir, stop a little, 'till I am satisfied concerning something just come into my Head; which if it prove true, will be very pleasant, as are all such Conclusions as are natural or necessary.

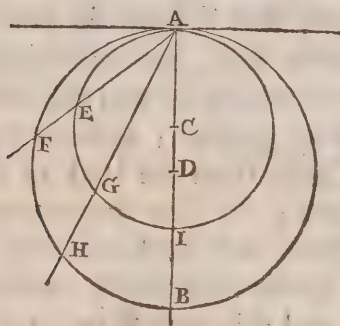
It is manifest, that if from a Point assigned in an horizontal Plane, be drawn an infinite Number of Lines every Way, upon each of which imagine a Point to move with an equable Motion; all these Points beginning to move in the same Instant of Time, if the Velocities of all of them be equal, those Points will consequently describe Circumferences of Circles always bigger and bigger, and all concentric about the first assign'd Point; just as we see done in the Undulations of standing Water, when a Stone is dropt from on high into it; the Percussion of which gives a Beginning of Motion every Way, and remains as the Centre of all the Circles; which are made successively bigger and bigger by those Undulations.

But now, if we imagine a Plane erected perpendicularly to the Horizon, and a Point in it assign'd or taken on high, from which imagine infinite Lines to proceed, inclin'd according to all Inclinations; upon, or along which, let heavy Moveables be imagined to descend, each with a Motion naturally accelerate, with such Velocities that agree with the several Inclinations: If now we suppose those Moveables to be continually visible, in what Kind of Lines pray should we see them constantly disposed?

For my Part, I am amazed, when I consider, that the preceding Demonstrations prove them all to be constantly
seen

seen in the same Circumference of Circles successively increasing, according as the Moveables in descending do more and more successively recede from the highest Point, from whence they begun their Fall.

And that I may explain myself the better, let A be the Point on high, from which, according to any Inclinations, let fall the Lines AF, AH, as also the Perpendicular AB, in which taking the Points C and D, describe Circles about them passing thro' the Point A and intersecting the inclin'd Lines in the Points F, H, B, and E, G, I. Then



by the foregoing Demonstrations, it is manifest, that if the Moveables falling thro' those Lines do depart from the Term A at the very same Time, that I say, when one is in E, another shall be in G, and the other in I: And thus continuing their Descent, in the very same Instant they will arrive in the Points F, H and B: And if these and infinite others continue still to move along infinitely different Inclinations, they shall always successively arrive at, or be in the very same Circumferences, which grow bigger and bigger *ad infinitum*.

From the two Kinds of Motion therefore which Nature uses, is produced, or arises with admirable harmonious Variety, the Generation of infinite Circles.

The former is placed as in its Seat and Original Beginning, in the Centre of infinite concentric Circles ; this latter resides in the lofty Contact of infinite Circumferences of Circles, all excentric to one another. Those arise from Motions which are all equal and equable ; these from Motions which are all always inequable to one another, and all unequal to each other, as being made at infinitely different Inclinations.

But I add farther, that if from the two Points assigned, we conceive Lines to flow, not only along two Surfaces, *viz.* the horizontal one, and the vertical or upright, but also every Way possible ; as from those beginning at one sole Point, Circles from the least to the greatest were successively produced ; so here beginning from one sole Point, infinite Spheres will be successively produced ; or, if you will, *one* Sphere continually increasing to an infinite Magnitude ; and this two Ways, *i.e.* let the Rise of it be supposed either in the Centre, or in the Circumference of those Spheres.

SALV. The Contemplation is really very ingenious, and suitable to *Sagredus's* Genius.

SIMP. Tho' I conceive these two Ways of producing Circles and Spheres, by two different natural Motions ; yet I don't thoroughly understand the Production, which depends on accelerate Motion : But yet because for the Place of Emanation may one while be assign'd the lowest Center,
and

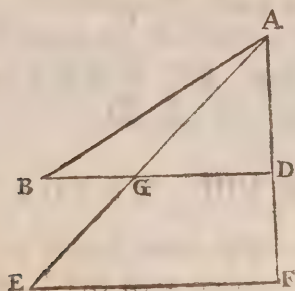
and another the highest Spherical Superficies; I can't but believe, that in these true and admirable Conclusions, there may be some great Mystery conceal'd; some Mystery, I say, in relation to the Creation of the Universe, whose Form is supposed Spherical, and concerning the Place of Residence of the first Cause.

SALV. I am pretty much of your Opinion: But such profound Speculations we'll leave to wiser Heads; and content ourselves to be such inferior Artificers, as discover and draw from the Quarry the Marbles, out of which the skilful Statuaries afterwards produce wonderful Images, which before lay hid under rude and misshapen Crusts. But now, if you please, we'll go on.

PROP. VII. THEOR. VII.

If the Elevations of two Planes shall have a Proportion *duplicate* of that of the Lengths of those Planes, two Bodies shall move from the same Point of Rest, the Length of those Planes in equal Times.

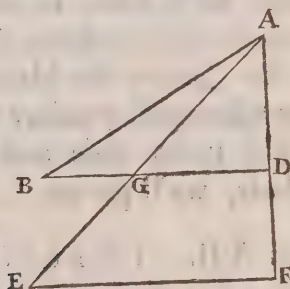
Let AE and AB be two unequal Planes, and unequally



inclined, and let the Elevation FA have a Proportion to

DA *duplicate* of AE to AB: Then, I say, that the Times of their being carried along the Planes AE and AB, from Rest in A, are equal.

For draw to the Line of Elevations AF, the horizontal Parallels EF and BD, which Line BD let cut AE in G: Now because the *Ratio* of FA to AD is *duplicate* of the *Ratio* of EA to AB, and because it is as FA to AD, so EA to AG; therefore the *Ratio* of EA to AG is *duplicate*



of that of EA to AB; and therefore AB is a mean Proportional between EA and AG; and because the Time of the Descent thro' AB, to the Time thro' AG, is as AB to AG; and the Time of the Descent thro' AG, to the Time thro' AE, is as AG to a mean Proportional between AG and AE, which is AB; therefore *ex æquali*, the Time thro' AB, to the Time thro' AE, is as AB to itself; therefore the Times are equal. Q. E. D.

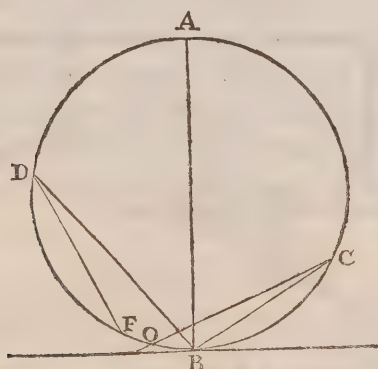
PROP. VIII. THEOR. VIII.

In Planes cut by the same Circle perpendicular to the Horizon in those that meet with the End of the erect Diameter, whether at Bottom or at Top of it, the Times of Bodies being carry'd thro' those Planes, are equal to the

Time

Time of the Fall along, and the Length of that Diameter ; and in those that fall short of the Diameter, the Times are shorter ; but in those that cut the Diameter, they are longer.

Let AB be the Diameter perpendicular to the Horizon of the Circle at Right Angles to the Horizon : Then that the Times of Bodies carried along Planes produc'd from the Terms A and B unto the Circumference of that Circle are equal, hath been already demonstrated : And that the Time of the Descent along the Plane DF falling short of

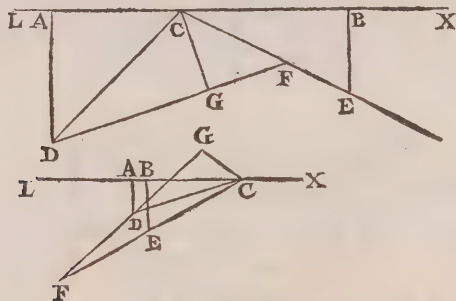


the Diameter, is shorter, is proved by drawing the Plane DB, which will be longer and less declining than DF ; therefore the Time thro' DF, is shorter than the Time thro' DB, *i. e.* thro' AB. But lastly, that the Time of the Descent along the Plane that cuts the Diameter, as CO, is longer, is in like Manner plain ; for it is longer and less declining than CB ; therefore the Proposition is manifest.

PROP. IX. THEOR. IX.

If two Planes be inclin'd at Pleasure from a Point in a Line parallel to the Horizon, and be cut by a Line which may make Angles alternately equal to the Angles contained between the said Planes and horizontal Parallel, then the Motions thro' the Parts cut off by the said Line, shall be performed in equal Times.

From the Point C of the horizontal Line LX, let any two Planes be inclin'd at Pleasure, as CD and CE; and



in any Point of the Line CD, make $\angle CDF = \angle XCE$: Then will the Line DF cut the Plane CE in F, in such Manner that $\angle$'s CDF and CFD will be $= \angle$'s LCD and XCE taken alternately, *i.e.* $\angle CDF$ will be $= \angle XCE$ and $\angle CFD = \angle LCD$. For the common Angle DCF being taken from the three Angles of the $\triangle CDF =$ two Right Angles, to which all the Angles made at the Point C, of the Line LX are equal, there remains in $\triangle CDF$ two Angles CDF and CFD $=$ the two Angles XCE and LCD: But by the Hypothesis $\angle CDF$ is $= \angle XCE$; therefore the remaining Angle CFD is $=$ the remaining Angle DCL.

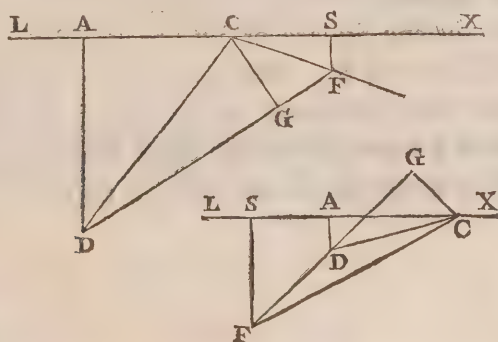
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This being done, I affirm, that the Descents thro' CD and CF are equal ; for let the Plane CE be suppos'd = the Plane CD, and from the Points D and E erect the Perpendiculars DA and EB unto the horizontal Parallel LX ; and from C unto DF, let fall the Perpendicular CG : Now because $\angle CDG$ is $= \angle ECB$; and $\angle$'s DGC and CBE are Right, the Triangles CDG and CBE will be equiangular, and it will be $DC : CG :: CE : EB$; but DC is $= CE$; therefore CG will be $= BE$. And since of Δ 's DAC and CGF $\angle$'s ACD and CAD are $= \angle$'s CFG and FGC ; therefore it will be $CD : DA :: FC : CG$, and alternately $CD : FC :: DA : CG$ or BE . The *Ratio* therefore of the Elevations of the equal Planes CD and CE, is the same with the *Ratio* of the Lengths DC and CF, therefore by the first Corollary of the foregoing sixth Proposition, the Times of the Descent along and thro' the same, shall be equal. *Q. E. D.*

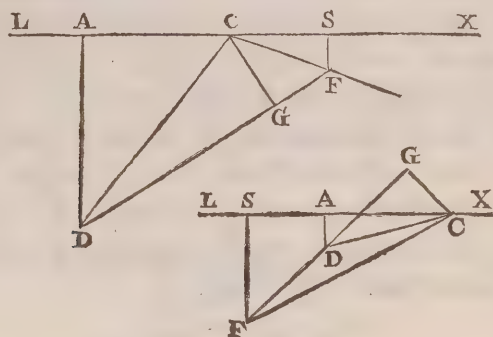
Otherwise thus,

Draw FS perpendicular to the horizontal Line AS ; then



because ΔCSF is similar to ΔDGC , it shall be $SF : FC :: GC : CD$. And because ΔCFG is similar to ΔDCA ,
it

it will be $FC : CG :: CD : DA$; therefore *ex æquali*, $SF : CG :: CG : DA$; therefore CG is a mean Proportional between SF and DA : And as DA to SF , so the Square of DA to the Square of CG . Again, since $\triangle ACD$ is similar to $\triangle CGF$, it will be $DA : DC :: CG : CF$, and alternately, $DA : CG :: DC : CF$; and as the Square of DA to the Square of CG , so is the Square of DC to the Square of CF . But it has been proved that the Square of



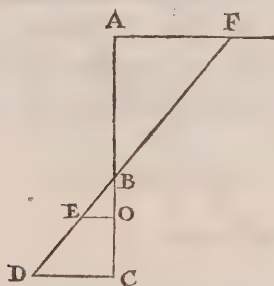
DA is to the Square of CG , as the Line DA to the Line FS ; therefore, as the Square of DC to the Square of CF , so the Line DA to FS ; therefore from the foregoing Seventh, since the Elevations of the Planes CD and CF have a *duplicate* Proportion of the same Planes, the Times of their Descents along the same will be equal.

PROP. X.

PROP. X. THEOR. X.

The Times of the Motions along different Inclinations of Planes whose Elevations are equal, are one to another as the Lengths of those Planes, whether the Motions be made from Rest, or a Motion from the same Height hath preceded them.

Let the Motions be made along ABC, and along ABD, to the Horizon DC; in such manner that the Lation thro'

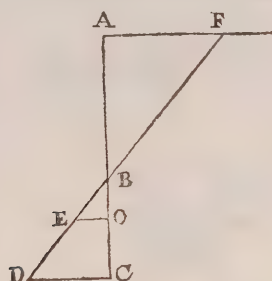


AB may precede the Motions thro' BD and thro' BC. Then, I say, the Time of the Lation thro' BD is to the Time thro' BC, as the Length BD to the Length BC. Draw AF parallel to the Horizon, to which continue out DB, meeting it in F, and let FE be a mean Proportional between DF and FB; draw also EO $\parallel$ DC, then will AO be a mean Proportional between CA and AB; but if we suppose the Time along AB to be as AB, the Time along FB will be as FB (by Corol. of Theor. 3.); and the Time along AB, to the Time along all AC, shall be as AB to AO, a mean Proportional between AB and AC;

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(by Cor. 2. of Theor. 2.) and the Time along FB, to the Time along all FD, shall be as FB to FE, a mean Proportional between FB and FD; wherefore the Time along the Remainder BC will be BO, and along the Remainder BD will be BE: But as BE is to BO, so is BD to BC; therefore the Times along BD and BC, after the Descent along AB and FB, or which is the same thing, along the common Part AB, shall be to one another as the Lengths BD and BC. But it has been already demonstrated, that

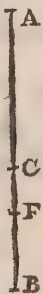


the Time along BD, is to the Time along BC, out of Rest in B, as the Length BD to BC; therefore the Times of the Motions along different Planes, whose Elevations are equal, are to one another as the Lengths of the said Planes, whether the Motion be made along the same, out of Rest, or whether another Motion from the same Height do precede those Motions. *Q. E. D.*

PROP. XI. THEOR. XI.

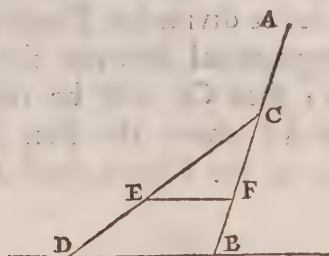
If a Plane, along which a Motion is made, out of Rest, be divided at Pleasure, the Time of the Motion along the first Part, to the Time of the Motion along the second, is as that first Part to the Excess by which the same Part is exceeded by a mean Proportional between the whole Plane, and the same first Part.

Let the Motion be along the whole Plane AB , from Rest in A , which let be divided at Pleasure in C ; and let AF be a mean Proportional between the whole BA and the first Part AC ; then CF will be the Excess of the mean Proportional AF above the Part AC . Then, I say, that the Time of the Motion thro' AC to the Time



of the following Motion thro' CB , is as AC to CF . This is manifest; for the Time along AC , to the Time along the whole AB , is as AC to the mean AF ; therefore by Division, the Time along AC to the Time thro' the remaining Part CB will be as AC to CF . If therefore the Time thro' AC be supposed to be the very AC , the Time along CB will be CF . *Q. E. D.*

But if the Motion be not made along the continu'd Plane ACB, but by the inflected ones AC and CD, 'till it reaches the horizontal Line BD, to which from F, the Parallel FE is drawn. It may, in like manner, be demonstrated, that the Time along AC, to the Time along CD, is as AC to CE : For the Time along AC to the Time along CB, is as AC to CF; but the Time along CB, after the Descent through AC to the Time through CD, after the same Descent through AC, has



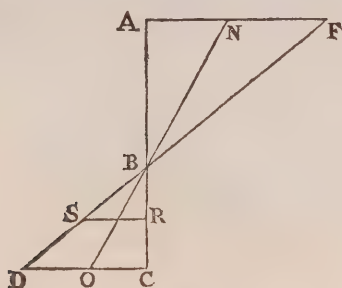
been demonstrated to be as CB to CD, *i.e.* as CF to CE; therefore, *ex æquali*, the Time along AC, to the Time along CD, will be as the Line AC to CE.

PROP. XII. THEOR. XII.

If the Perpendicular and Plane, at Pleasure inclin'd, be cut between the same horizontal Lines, and be taken mean Proportionals between them and the Parts of them contained between the common Section and upper horizontal Line; the Time of the Motion along the Perpendicular, to the Time of the Motion along the upper Part of the Perpendicular, and then along the lower Part of the cutting Plane, will have the same Proportion, as the Length
of

of the whole Perpendicular has to a Line compounded of the mean Proportional given in the Perpendicular, and of the Excess, by which the whole inclined Plane exceeds its mean Proportional.

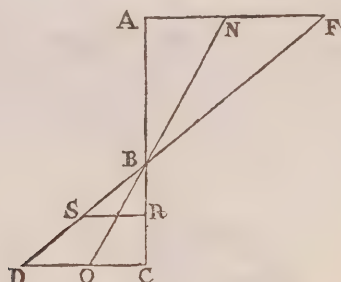
Let the horizontal Lines be AF the upper, and DC the lower ; between which let the Perpendicular AC, and the inclin'd Plane DF be cut in B ; and let AR be a mean Proportional between the whole Perpendicular CA, and the upper Part AB ; and let FS be a mean Proportional



between the whole inclin'd Plane DF, and its upper Part BF. Then, I say, that the Time of the Fall along the whole Perpendicular AC, to the Time along its upper Part AB, together with the lower Part of the Plane DF, *viz.* with BD, has the same Proportion as AC has to the mean Proportional of the Perpendicular, *i. e.* AR, together with SD, which is the Excess of the whole Plane DF above its mean Proportional FS : For let a Line be drawn from R to S, which will be $\parallel$ to the two horizontal Lines : and because the Time of the Fall along all AC, to the Time along the Part AB, is as CA to the mean Proportional AR, if we suppose AC to be the Time of
the

the Fall thro' AC, AR will be the Time of the Fall thro' AB, and RC that thro' the Remainder BC. For if the Time along AC be supposed, as was done to be AC itself, the Time along FD shall be FD; and in like manner DS may be concluded to be the Time along BD, after the Descent thro' FB, or thro' AB: The Time therefore along the whole AC, is AR with RC; and the Time along the inflected Planes AB and BD, will be AR with SD.

Q. E. D.



The same happens if instead of the Perpendicular another Plane be supposed or taken, as *v. g.* NO; and the Demonstration is the same.

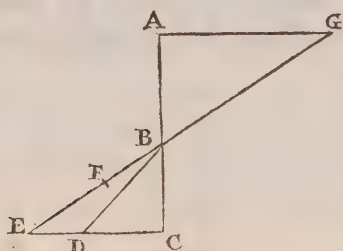
PROP. XIII. PROB. I.

A Perpendicular being given to inflect a Plane to it, along which, supposing it to have the same Elevation with the given Perpendicular, it may move after its Fall along the Perpendicular, in the same Time as it is falling from Rest along that Perpendicular.

Let the given Perpendicular be AB, to which extended to C, let the Part BC be equal, and draw the horizontal

Lines
†

Lines CE and AG; 'tis required from B to inflect a Plane reaching to the Horizon CE, along which a Motion may be made after a Descent from A, in the same Time as along AB from Rest in A. Make $CD = CB$, and drawing BD, let BE be drawn $= BD$ and DC taken together. Then, I say, that BE is the Plane sought. Produce EB meeting the horizontal Line AG in G, and let GF be a mean Proportional between EG and GB: Then EF will be to FB as EG to GF; and the Square of EF to the



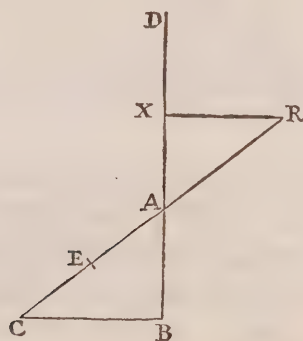
Square of FB, as the Square EG to the Square of GF, *i. e.* as the Line EG to GB; but EG is the Double of GB, therefore the Square of EF is the Double of the Square of FB; but the Square also of DB is double the Square of BC; therefore as the Line EF to FB, so is DB to BC; and by Composition and Permutation, as EB to $DB \times BC$, so is BF to BC; but BE is $= DB \times BC$; therefore BF is $= BC$ or to BA. If, therefore, AB be supposed to be the Time of the Fall along AB, GB will be the Time of the Fall along GB, and GF the Time along the whole Plane GE; therefore BF will be the Time of the Fall thro' the Remainder BE, after the Fall from G, or from A. Q. E. D.

PROP. XIV.

PROP XIV. PROB. II.

A Perpendicular and a Plane inclin'd to it, being given, to find a Part in the upper Perpendicular, which shall be pass'd thro' from Rest, in a Time = that wherein the inclin'd Plane is pass'd thro', after the Fall along the Part found in the Perpendicular.

Let the Perpendicular be DB, and the Plane inclined to it AC; it is required in the Perpendicular AD to find a Part, which shall be pass'd thro', *ex quiete*, in a Time = that in which the Plane AC is pass'd thro' after the Fall along the said Part.



Draw the horizontal Line CB; and as BA $\times$ twice AC to AC, so make CA to AE; and as BA is to AC, so let EA be made to AR; and from R let fall the Perpendicular RX, to the Line DB; then, I say, that X is the Point sought: And because as BA $\times$ twice AC is to AC, so CA is to AE, by Division it will be as BA $\times$ AC to AC, so CE to EA; and because as BA to AC, so EA

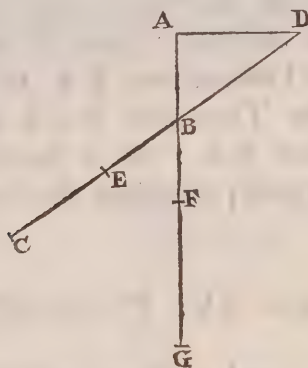
EA to AR ; it will be by Composition as $BA \times AC$ to AC, so ER to RA : But as $BA \times AC$ to AC, so is CE to EA ; therefore as CE to EA, so ER to RA, and both the Antecedents to both the Consequents, *viz.* CR to RE ; therefore CR, RE and RA are Proportionals : Furthermore, because as BA to AC, so EA is supposed to be to AR, and because of the Similarness of Triangles, as BA to AC, so XA to AR ; therefore as EA to AR, so XA to AR, therefore EA and XA are equal.

And now if we understand the Time along RA to be RA, the Time along RC will be RE, a mean Proportional between CR and RA : And AE will be the Time along AC, after its Descent thro' RA, or after its Descent thro' XA ; but the Time thro' XA is XA, so long as RA is the Time thro' RA : But it has been shewn, that XA and AE are equal ; therefore the Proposition is manifest.

PROP. XV. PROB. III.

A Perpendicular, and a Plane inflected to it, being given, to find a Part in the Perpendicular extended downwards, which shall be pass'd thro' in the same Time as the inflected Plane is, after a Descent along the given Perpendicular.

Let the Perpendicular be AB , and the Plane inflected to it BC : It is required in the Perpendicular extended downwards, to find a Part, which from the Fall from A shall be passed thro' in the same Time as BC is passed thro' from the same Fall from A : Draw the horizontal Line AD , which let CB meet, extended to D , and let DE be a mean Proportional between CD and DB , and let $BF = BE$; and lastly, let AG be a third Proportional to BA and AF ; then, I say, BG is the Space that, after the Fall AB , shall be passed thro' in the same Time



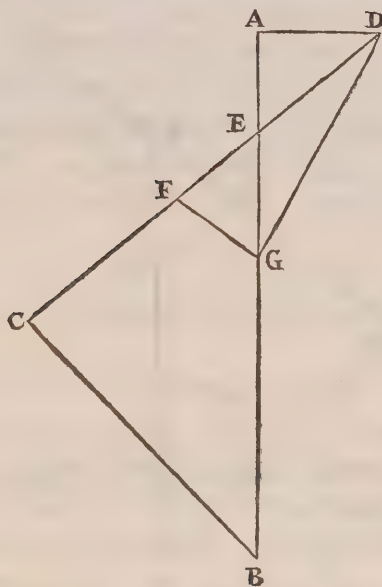
as the Plane BC shall be passed thro', after the same Fall: For if we suppose the Time along AB to be as AB , the Time along DB will be as DB ; and because DE is a mean Proportional between BD and DC , the same DE shall be the Time along the Whole DC , and BE the Time along the Remainder BC , from Rest in D , or after the Fall thro' AB ; and in like manner it may be concluded, that BF is the Time of the Fall thro' BG , after the same Fall thro' AB ; but $BF = BE$; therefore the Proposition is manifest.

PROP. XVI.

PROP. XVI. THEOR. XIII.

If the Parts of an inclin'd Plane and Perpendicular, the Times of whose Motions from Rest are equal, meet at the same Point; a Moveable coming from any greater Height, will sooner run thro' the said Parts of the inclined Plane, than that Part of the Perpendicular.

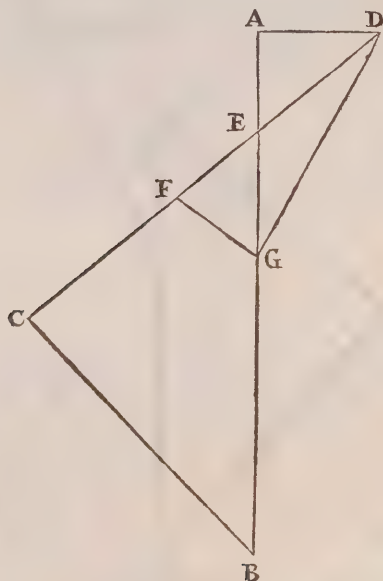
Let the Perpendicular be EB, and the inclin'd Plane CE meeting each other in the Point E, the Times of whose



Motions from Rest in E are equal, and in the Perpendicular extended, take any higher Point at Pleasure, as A; from which let the Moveable fall; I say, that the inclined Plane

EC will be sooner passed over than the Perpendicular EB will after the Fall AE.

Join C and B, and having drawn the horizontal Line AD, produce CE 'till it meets it in D; and make DF a mean Proportional between CD and DE; and make AG a mean Proportional between BA and AE, and draw FG and DG: Now because the Times of the Moveables being carried thro' EC and EB, from Rest at E, are equal, $\angle C$ will be a Right one, by *Cor. 2. of Prop. 6.* and $\angle A$ is Right, and $\angle$'s at the Vertex are equal, therefore $\triangle$'s



AED and CEB are Equiangular, and the Sides about the equal Angles proportional, therefore $BE : EC :: DE : EA$, $\square BEA$ is $= \square CED$: And because $\square CDE$ exceeds $\square CED$

by
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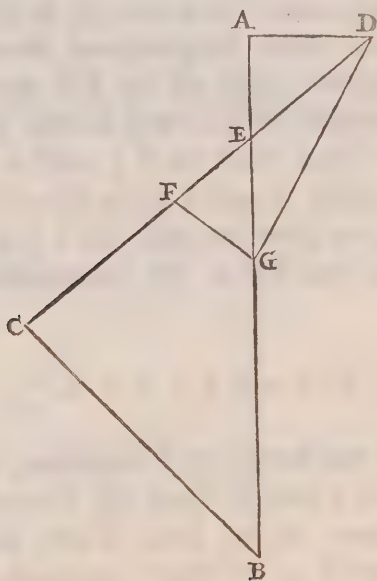
by the $\square$ of ED, and $\square$ BAE exceeds $\square$ BEA, by $\square$ of EA, the Excess of $\square$ CDE above $\square$ BAE, *i. e.* of $\square$ FD above $\square$ AG, will be the same with the Excess of $\square$ of DE above the Square of AE, which Excess is $\square$ of DA ; therefore the $\square$ of FD is = the two Squares of GA and AD, to which $\square$ of GD is also equal ; therefore the Line DF is = DG, and $\angle$ DGF = $\angle$ DFG, and $\angle$ EGF less than $\angle$ EFG, and the opposite Side EF less than the Side EG : Now if we suppose the Time of the Fall thro' AE to be as AE, the Time thro' DE will be as DE ; and since AG is a mean Proportional between BA and AE, AG will be the Time of the Fall thro' the whole AB, and the Remainder EG will be the Time thro' the Remainder EB, from Rest in A ; and it may in like manner be concluded, that EF is the Time thro' EC, after the Descent DE, or after the Fall AE ; but EF has been proved to be less than EG ; the Proposition therefore is manifest.

C O R O L L A R Y.

From this and the foregoing Proposition, it is manifest, that the Space that is passed along the Perpendicular, after the Fall from above, in the same Time wherein the inclin'd Plane is pass'd, is less than that which is pass'd in the same Time as in the inclined, no Fall from above preceding, yet greater than the inclined Plane.

For it having but just now been proved, that of the Moveables coming from the sublime Term A, the Time of its passing thro' EC is shorter than that of its running along EB ; it is manifest, that the Space that is passed
along

along EB, in a Time = the Time along EC, is less than the whole Space EB; but that the same Space along the Perpendicular is greater than EC, is manifest, by reassuming the Figure of the foregoing Proposition, wherein the Part of the Perpendicular BG hath been demonstrated to be moved over in the same Time as BC is, after the Fall AB: but now that this BG is greater than BC, is thus collected; since BE and FB are equal, and BA less than BD, FB has a greater Proportion to BA, than EB has to



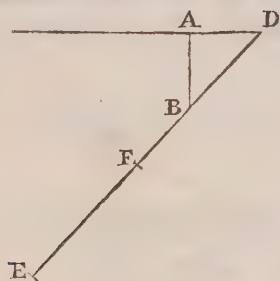
BD, and, by Composition, FA has a greater Proportion to AB than ED has to DB; but it is $FA : AB :: GF : FB$ (for AF is a Mean between BA and AG); and in like manner $ED : BD :: CE : EB$; therefore by Composition, GB has a greater Ratio to BF than CB has to BE; therefore GB is greater than BC.

PROP. XVII.

PROP XVII. PROB. IV.

A Perpendicular and a Plane inclin'd to it, being given, to assign a Part in the given Plane, in which, after the Fall along the Perpendicular, the Motion may be made in a Time = that wherein the Moveable from Rest passed thro' the given Perpendicular.

Let the Perpendicular be AB , and let the Plane inclined to it be BE : It is required in BE to assign a Space along which the Moveable, after the Fall thro' AB , may move in a Time = that wherein it passed from Rest thro' the Perpendicular AB .



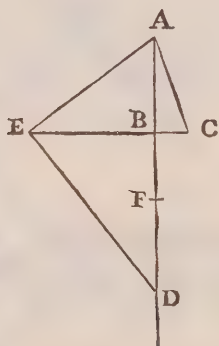
Let AD be an horizontal Line, which let the Plane produced meet in D , and make $FB = BA$, and make $BD : DF :: FD : DE$: Then, I say, the Time thro' BE , after the Fall along AB , is = the Time thro' AB , from Rest in A : For if we suppose AB to be the Time along AB , DB will be the Time thro' DB : And because it is $BD : DF :: FD : DE$, DF will be the Time thro' the whole Plane DE , and BF will be that thro' the Part BE from D ; but the Time thro' BE , after DB , is the same as after AB ; therefore the Time thro' BE , after AB , will be BF ; *i. e.* = the Time AB , from Rest in A . $\mathcal{Q}. E. F.$

PROP. XVIII.

PROP. XVIII. PROB. V.

Any Space in the Perpendicular being given, from the assigned Beginning of Motion, that is passed in a given Time, and any other lesser Time being also given; to find another Length in the said Perpendicular, that may be passed thro' in the given lesser Time:

Let the Perpendicular be AD , in which let the Space given be AB , whose Time from the Beginning A , let be AB , and let CBE represent the Horizon, and let a Time be given less than AB , to which make BC equal in the horizontal Line EC ;



'Tis required in the same Perpendicular to find a Space = the same AB , that may be passed in the Time BC .

Draw a Line from A to C ; then because BC is less than BA , $\angle BAC$ will be less than BCA . Make $CAE =$ to it, and let the Line AE meet the Horizon in the Point E , to which suppose ED perpendicular, cutting the Perpendicular

pendicular in D; and make $DF = BA$; then, I say, that FD is that Part of the Perpendicular, along which the Lation from the Beginning of Motion in A, is performed in the given Time BC.

For since in the right-angled Triangle AED, from the right Angle E, a Perpendicular EB is drawn to the opposite Side AD, AE will be a mean Proportional between DA and AB, and BE a mean between DB and BA, or between FA and AB (for FA is $= DB$) and since AB is supposed to be the Time thro' AB, AE or EC will be the Time thro' the whole AD, and EB the Time thro' AF; therefore the remaining Part BC will be the Time thro' the remaining Line FD. Q. E. F.

P R O P. XIX. P R O B. VI.

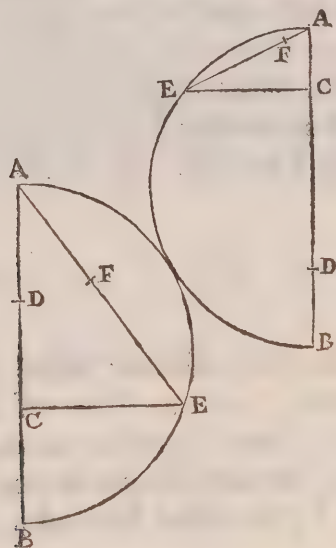
Any Space in the Perpendicular passed over, from the Beginning of Motion, with the Time of the Fall, being given, to find the Time in which another equal Space, taken at Pleasure in the same Perpendicular, shall be afterwards passed thro' by the same Moveable.

R r

In

In the Perpendicular AB, let AC be any Space taken from the Beginning of the Fall in A, to which let another Space, as DB, taken at Pleasure, be equal, and let the Time of the Fall thro', or Motion along AC be given; and let it be AC. Now it is required to find the Time of the Fall thro', or Motion along DB, after its Fall from A.

About the whole Line AB, as a Diameter, describe the Semicircle AEB, and from C let fall CE perpendicular to



AB, and draw the Line AE, which will be longer than EC. Make $EF = EC$: Then, I say, that the Remainder FA is the Time of the Fall thro', or Motion along DB, after the Fall from A; For,

DEMONSTRATION.

Because AE is a mean Proportional between BA and AC, and because AC is the Time of the Fall thro' AC,
AE

AE will be the Time of the Fall thro' the whole Line AB. And since CE is a Mean between DA and AC, (for DA is = BC) CE, *i.e.* EF, will be the Time of the Fall thro' AD; therefore what remains AF, is the Time of the Fall thro' the remaining Space DB. Q. E. D.

C O R O L L A R Y

Hence is gathered, that if the Time of any Space, *ex quiete*, be as the said Space; the Time thereof, after another Space is added, will be the Excess of the mean Proportional betwixt the Space added, and former Space taken together, and the said Space above the mean Proportional, betwixt the first Space and the Space added.

As for Example.

Supposing that the Time thro' AB, from Rest in A, be AB; AS being another Space added, the Time along AB,



after the Fall thro' SA, will be the Excess of the mean Proportional between SB and BA above the mean Proportional between BA and AS.

PROP. XX. PROB. VII.

Any Space, and a Part therein, after the Beginning of Motion, being given, to find another Part towards the End, that shall be passed thro' in the same Time as the first Part given is passed thro' :

Let the Space be CB, and let CD be the Part in it given, after the Beginning of the Motion in C ; now 'tis required to find another Part towards the End B, which may be passed in the same Time as the given Part CD is.



Take a mean Proportional between BC and CD, to which suppose BA equal ; and let CE be a third Proportional between BC and CA ; I say, EB is the Space that after the Fall out of C shall be passed over in the same Time as CD is.

For

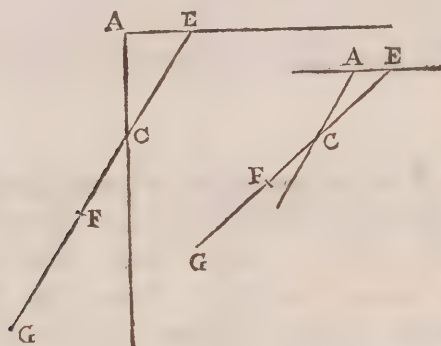
For if we suppose the Time along the whole CB, to be as CB ; BA (the mean Proportional between BC and CD) shall be the Time along CD : And since CA is a Mean betwixt BC and CE, CA will be the Time thro' CE ; but the whole BC is the Time thro' the whole CB, therefore the remaining Part BA, will be the Time thro' the remaining Part EB, after the Fall thro' C ; but the same BA was the Time thro' CD, therefore CD and EB will be passed over in equal Times from Rest in A.
Q. E. F.

PROP. XXI. THEOR. XIV.

If from Rest a Fall be made perpendicularly, in which from the Beginning of the Motion, a Part be taken at Pleasure, passed over in any Time, after which let an inflex Motion follow, along any Plane inclin'd at Pleasure ; the Space, which in such a Plane is passed in a Time = the Time of the Fall already made in the Perpendicular, will be more than double, and less than triple the Space already passed in the Perpendicular.

Under

Under the Horizon AE , let AB be a Perpendicular to it; along which, from the Beginning A , let a Fall be made, of which take a Part at Pleasure as AC ; then from C let a Plane CG be inclin'd at Pleasure, along which, after the Fall from A to C , let the Motion be continued; then, I say, that the Space pass'd by such Motion along CG , in a Time equal to the Time of the Fall thro' AC , is more than double, and less than triple that same Space AC : For make $CF = AC$, and extend the Plane GC to the Horizon in E , and make as CE to EF , so FE to EG ;



i.e. let EF be continu'd to G , so that EF may be a mean Proportional between EC and EG : If then we suppose the Time of the Fall thro' AC , to be as the Line AC , CE shall be the Time thro' EC , and CF or CA the Time of the Motion thro' CG : Then it is to be proved, that the Space CG is more than double, and less than triple the said CA . Thus then since it is as CE to EF , so is EF to EG , it will also be, so is CF to FG : but EC is less than EF , wherefore also CF will be lesser than FG , and consequently GC more than double FC or AC : and again,
since
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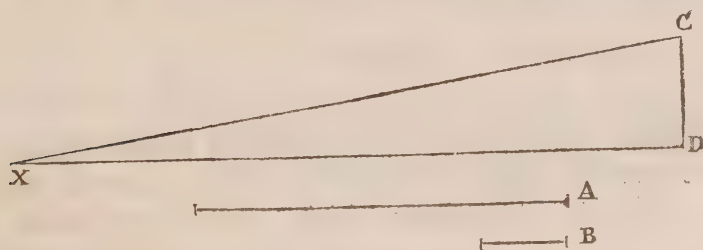
since FE is less than double EC (for EC is greater than CA or CF) GF also will be less than double FC, and consequently GC less than triple CF or CA. *Q. E. D.*

But the same thing may be more universally proposed; for that which has been proved as to the Perpendicular and inclined Plane, holds true also, if after the Motion along any inclin'd Plane, it moves along a Plane more inclined, as you may see by Fig. 2. and the Demonstration is the same.

PROP. XXII. PROB. VIII.

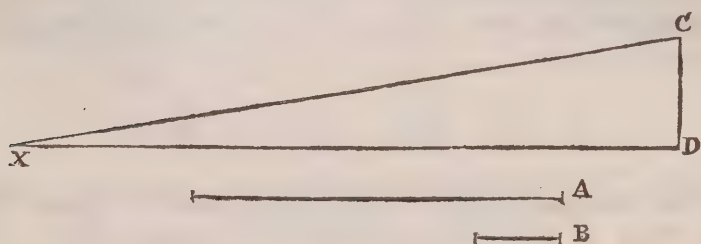
Two unequal Times being given, and the Space that from Rest is passed over along the Perpendicular in the shorter of the given Times; to inflect a Plane from the highest Point of the Perpendicular unto the Horizon, thro' which the Moveable may descend, in a Time = the longer of the given Times:

Let the greater of the unequal Times be A, the lesser B; and let the Space that from Rest is passed along the



Perpendicular, in the Time B, be CD; then it is requir'd from the Term C to inflect a Plane 'till it reach the Horizon, that shall be passed over in the Time A. As B is
to

to A, so let CD be to a fourth Line, to which let CX, drawn from C to the Horizon, be equal ; then 'tis manifest, that CX is the Plane thro' which the Moveable will pass in the given Time A : For it has been demonstrated, that the Time along, or thro' the inclined Plane has the same Proportion to the Time along its Elevation (or the



Perpendicular) as the Length of the Plane has to the Length of its Elevation; the Time therefore thro' CX, to the Time thro' CD, is as CX to CD, *i. e.* as the Time A to the Time B ; but the Time B is that wherein the Perpendicular is passed thro' from Rest ; therefore the Time A is that wherein the Plane CX is passed thro'. Q. E. F.

PROP. XXIII. PROB. IX.

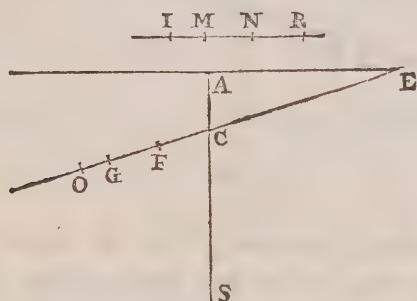
A Space passed from Rest, along the Perpendicular, in any Time being given, to inflect a Plane from the lowest Term of that Space, along which, after the Fall along the Perpendicular, a Space may be passed in the same Time = any given Space, that is more than double, and less than triple the Space passed along the Perpendicular :

Along

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Along the Perpendicular AS, in the Time AC, let the Space AC be passed, *ex quiete* in A; to which let IR be more than double and less than triple: It is required from the Term C to inflect a Plane, along which a Moveable, after the Fall along AC, may in the same Time AC pass a Space = the said IR.

Let RN and NM be each equal to AC, and what Proportion the Remainder IM has to MN, the same let the Line AC have to a fourth Line, equal to which draw

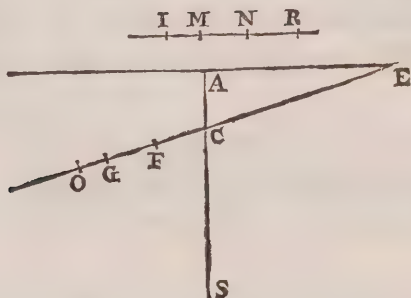


CE from C to the Horizon AE; continue CE towards O, and take CF, FG, and GO = the said RN, NM, and MI; then, I say, that the Time along the inflected Plane CO, after the Fall AC, is = the Time AC, out of Rest in A: For since it is $OG : GF :: FC : CE$; it will be by Composition $OF : FG$ or $FC :: FE : EC$; and as one of the Antecedents to one of the Consequents, so all the Antecedents to all the Consequents, *i. e.* the whole $OE : EF :: FC : EC$; therefore OE, EF, and EC are continual Proportionals: And since it was supposed that the Time along AC was as AC, CE will be the Time thro' EC; and EF the Time along the whole EO; and the Remain-

S f

der

der CF that along the Remainder CO; but CF is = the said CA; therefore what was required is done: For the Time CA is the Time of the Fall along AC, *ex quiete*, in A; and CF (which is = CA) is the Time along CO, after the Descent thro' AC, or after the Fall thro' AC. Q.E.F.



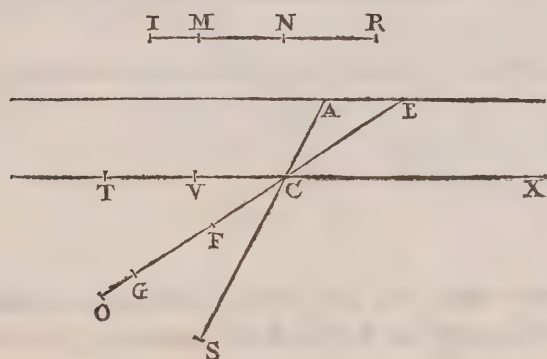
But here it must be noted, that the same will happen, if the preceding Motion be not made along the Perpendicular, but along an inclined Plane; as in the following Figure, wherein let the preceding Motion be made along the inclined Plane AS, below the Horizon AE; and the Demonstration is the very same.

SCHOLIUM.

Upon due Consideration, it will be manifest, that the less the given Line IR wants of being triple AC, the nearer the inflected Plane, along which the second Motion is to be made, which suppose to be CO, approaches to the Perpendicular, along which, in a Time = AC, a Space shall be passed triple to AC.

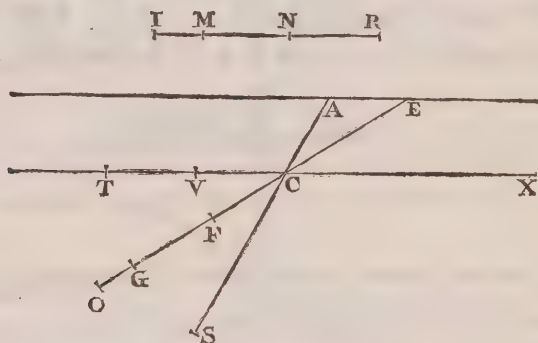
For

For in case IR were very near the triple of AC, IM would be very nearly = MN; and since CE has by Construction the same Proportion to AC, as MN has to IM, 'tis manifest, that the said CE will be found but very little bigger than CA, and consequently the Point E will be very near the Point A, and the Lines CO and CS will make but a very acute Angle, they almost concurring in one Line. But, on the contrary, if the given Line IR were but a very little more than double the said AC, IM would be but a very short Line, whence it will follow also, that AC

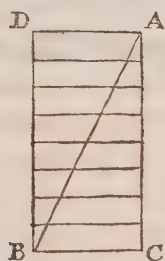


will be very short in respect of CE, which will be very long, it approaching, in such Case, very near to the horizontal Parallel drawn thro' C; Whence we may gather, that in the present Figure, if after the Descent thro' the inclined Plane AC, a Reflection be made along the horizontal Line, as along CT; the Space thro' which the Moveable wou'd of Consequence be carried in a Time = the Time of the Descent thro' AC, would be exactly double the Space AC. And the same Way of Reasoning seems to be applicable here: For it appears from what was but now said,

since it is $OE : EF :: FE : EC$, that FC determines the Time thro' CO . And if a Part of the horizontal Line TC , double to CA , be divided equally in V , and extended towards X , it will be infinitely produced, whilst it seeks to meet the prolonged Line AE . And the *Ratio* of the infinite Line TX to the infinite Line VX , will be no other than the *Ratio* of the infinite Line VX , to the infinite Line XC .



We may come at the same Conclusion another Way, viz. by resuming the Method of Reasoning, us'd in the De-



monstration of the first Proposition, thus ; the Triangle ABC represents, by the Parallels to its Base BC , the Degrees of Velocity continually increasing, according to the Increments of

of the Times; from which, since they are infinite as the Points are infinite in the Line AC, and the Instants in any Time will result the Area of the Triangle: And if we conceive the Motion to be continued for just such another Time, but no longer accelerately but equably, according to the greatest Degree of Velocity, by the former Motion acquired, which Degree of Velocity is represented by BC; then of such Degrees will be made the Aggregate or Parallelogram ADBC, which is double $\triangle ABC$. Wherefore the Space which with such Degrees shall in the same Time be passed over, will be double the Space passed with the Degrees of Velocity represented by $\triangle ABC$; but along the horizontal Plane the Motion, is equable there being no Cause of Acceleration or Retardation: We therefore conclude, that the Space * CT, passed in a Time = the Time AC, is double to the Space AC; for this is made, *ex quiete*, by an accelerate Motion, according to the Parallels of the Triangle; and that according to the Parallels of the Parallelogram, which being infinite, are double to the infinite Parallels of the Triangle.

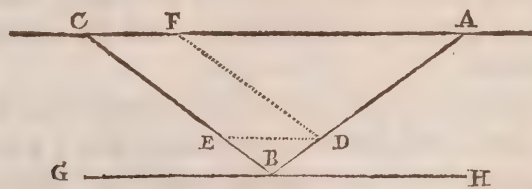
It may be yet farther observed, that whatever Degree of Velocity is found in the Moveable, is indelibly impressed on it of its own Nature; since all external Causes of Acceleration or Retardation are removed, which happens in the horizontal Plane only: For in declining Planes, there is a Cause of greater Acceleration; and in the rising ones of Retardation.

From whence in like Manner it follows, that the Motion along the horizontal Plane is perpetual: For if it be equable, it is neither weaken'd or retarded, much less destroy'd. Farther the Degree of Celerity acquired by the Moveable, by a

* Vid. Fig. ad Schol. præced.

Natural Descent, being of its own Nature indelible and eternal ; what offers itself to be considered, is, that if after the Descent along a declining Plane, a Reflexion be made along another Plane that is rising ; in this latter there is an immediate Cause of Retardation ; for in such a Plane, the same Moveable naturally descends ; whereupon there arises a certain Mixture of contrary Affections, *viz.* that Degree of Celerity acquired in the precedent Descent, which would of itself carry the Moveable uniformly, *in infinitum*, and of natural Propension, to the Motion of Descent, according to that same Proportion of Acceleration wherewith it always moves. Wherefore it will be but reasonable, if, enquiring what Accidents may happen when the Moveable, after the Descent along any inclin'd Plane, is reflected along some rising Plane ; it will be but reasonable, I say, if we take that greatest Degree acquired in the Descent, to be kept perpetually the same in the ascending Plane ; but that in the Ascent there is superadded a natural Inclination downwards, *i. e.* the Motion from Rest, accelerated according to the received Proportion. But lest this should not, without Difficulty, be understood, we will endeavour to explain it by a Scheme.

Let the Descent therefore be supposed to be made along the declining Plane AB ; from which along another rising

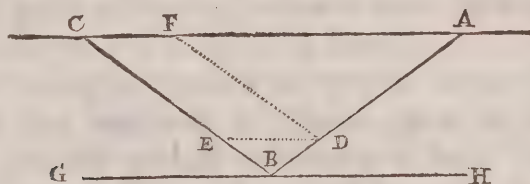


Plane BC, let be continued a reflex Motion ; and, first, let the Planes be equal, and elevated at equal Angles above

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the Horizon GH. Now 'tis manifest, that the Moveable from Rest in A, descending along AB, acquires Degrees of Velocity according to the Increase of its Time; and that the Degree in B is the greatest of those acquir'd, and by Nature immutably impressed; I mean, the Causes of new Acceleration or Retardation being removed: Of Acceleration, I say, which it would get if it shou'd move any farther along the Plane extended; and of Retardation, whilst the Reflection is making along the rising Plane BC; but along the horizontal Plane GH, the equable Motion, according to the Degree of Velocity acquired in its Passage from A to B, would be continual and extend *in infinitum*: And it would be such a Velocity, as in a Time = the Time of Descent thro' AB, wou'd in the Horizon pass over a Space double the Space AB: Now let us imagine the same Moveable, with the same Degree of Swiftness, to be equally moved thro' the Plane BC, so that also in this Time = the Time of the Descent thro' AB, it may pass over along BC extended a Space the double of AB. But let us understand, that as soon as it begins to ascend there naturally happens the same Thing to it, as happens to it in falling from A along AB, *viz.* a certain Descent *ex quiete* according to those Degrees of Acceleration, by Virtue whereof, as it happens in AB, it may descend as much in the same Time along the reflected Plane, as it does along AB: Now it's manifest, that by this Mixture of an equable Motion of Ascent, and of an accelerated one of Descent, the Moveable will be carried along the Plane BC (not a Space double the Space AB, as with only an equable Motion of Ascent we supposed it to do; but the accelerated Motion of Descent lessening the
Moveable's

Moveable's Ascent by one half ; it will be carried, I say) by the equable Motion of Ascent, or according to those Degrees of Velocity which shall be equal, but to the Term C: And that two Points D and E being taken at Pleasure, but equidistant from the Angle B, the *Transit* thro' DB is made in a Time = the Time of the Reflexion thro' BE, we may collect from hence. Draw DF $\parallel$ BC ; then it's

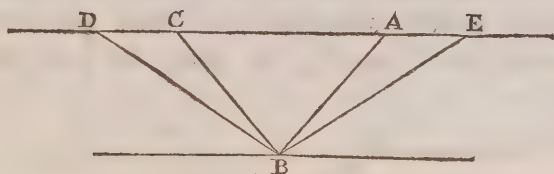


manifest, that the Descent along AD is reflected thro' DF. And if when the Moveable be got to D, it pass along the horizontal Plane DE, the *Impetus* in E will be the same with the *Impetus* in D ; therefore it will ascend from E to C ; and therefore the Degree of Velocity in D is = the Degree in E. And therefore the *Transit* thro' DB is made in a Time = the Time of the Reflexion thro' BE.

From hence we may rationally assert, that if a Descent be made along any inclin'd Plane, after which a Reflexion follows along an elevated Plane, the Moveable, by the conceived *Impetus* will ascend to the same Height or Elevation above the Horizon, as if a Descent be made thro' AB, the Moveable will be carried thro' the reflected Plane BC, up to the horizontal Line ACD ; and this not only when the Planes are equally inclined, but also when they are unequally so, as is the Plane BD : For by the

Postulatum

Postulatum, preceding, the Degrees of Velocity are equal, which are acquired upon Planes unequally inclined, provided the Elevation of those Planes above the Horizon be the same. But if there being the same Inclination of the Planes EB and BD, the Descent thro' EB be sufficient to force the Moveable along the Plane BD to D; seeing this Impulse is made by the *Impetus* of Velocity conceived in the Point B: And if the *Impetus* be the same in B, whether the Moveable descend along AB, or along EB, 'tis manifest, that the Moveable shall be in the same Manner

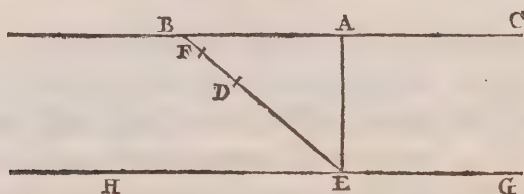


driven along BD, after its Descent along AB, and after that along EB; but it will happen that the Time of the Ascent thro' BD will be longer than that thro' BC, as the Descent thro' EB is made in a longer Time than that thro' AB; but the Proportion of those Times has been before demonstrated to be the same with that of the Lengths of the Planes. Now it follows, that we seek the Proportion of the Spaces passed over in equal Times, along Planes whose Inclinations are different, but whose Elevations are still the same, *i. e.* which are comprehended between the same horizontal Parallels, and this will be found by what follows.

PROP. XXIV. THEOR. XV.

There being given between the same horizontal Parallels, a Perpendicular, and a Plane elevated from its lowest Term, the Space that a Moveable, after the Fall along the Perpendicular passeth along the elevated Plane in a Time = the Time of the Fall thro' the Perpendicular, is greater than that Perpendicular, but less than double the same.

Between the same horizontal Parallels BC and HG, let there be a Perpendicular, as AE, and an elevated Plane EB, along which, after the Fall thro' the Perpendicular AE; from the Term E, let there be a Reflexion made



towards B: I say, the Space thro' which the Moveable ascends, in a Time = the Time of the Descent AE, is greater than AE, but less than double the same AE.

Make $ED = AE$, and as EB is to BD, so let DB be to BF.

First it shall be shewn, that the Point F is the Place to which the Moveable, with a reflex Motion along EB, shall arrive in a Time = the Time AE; and then that EF is greater than AE, but less than double the same AE.

First,

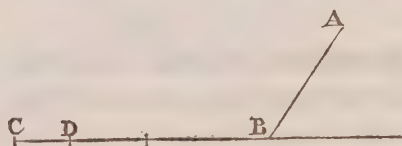
First, then, If we suppose the Time of the Descent thro' AE to be as AE ; the Time of the Descent thro' BE, or of the Ascent thro' EB, will be as the Line BE ; and since DB is a mean Proportional between EB and BF ; and since BE is the Time of the Descent thro' the whole BE, BD will be the Time of the Descent thro' BF, and the remaining Part DE will be the Time of the Descent thro' the remaining Part FE ; but the Time thro' FE *ex quiete* in B, and the Time of the Ascent thro' EF is the same, since the Degree of Velocity in E was acquired by the Descent BE or AE ; therefore the same Time DE shall be that wherein the Moveable, after its Fall from A thro' AE, with a reflex Motion along EB, arrives at the Point F : But ED was supposed or taken = AE, therefore, &c. Which was first to be proved.

Secondly, Because as the whole Line EB is to the whole Line BD, so is the Part taken away DB to the Part taken away BF ; it will be as the whole EB to the whole BD, so the Remainder ED to DF ; but EB is greater than BD, therefore also ED is greater than DF, and EF less than double DE or AE. *Q. E. D.* And the same Thing will happen, if the precedent Motion be not made along the Perpendicular, but along an inclin'd Plane ; and the Demonstration is the same, provided that the reflex Plane be less rising, *i. e.* longer than the declining one.

PROP. XXV. THEOR. XVI.

If after the Descent along any inclin'd Plane, a Motion follow along the Plane of the Horizon, the Time of the Descent along the inclin'd Plane will be to the Time of the Motion along any horizontal Line, as the double Length of the inclin'd Plane to the horizontal Line taken.

Let the horizontal Line be CB, the inclin'd Plane AB, and after the Fall thro' AB, let follow a Motion along the Horizon, in which take any Space BD; I say, the Time of the Fall thro' AB, to the Time of the Motion thro' BD, is as the double of AB to BD.



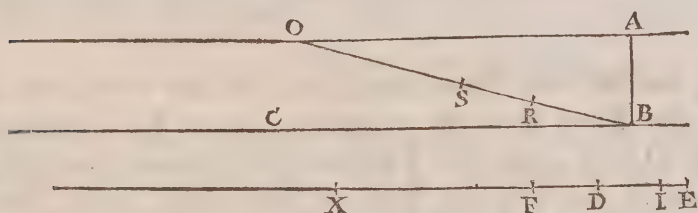
For, make BC double AB, then 'tis plain from what has been before demonstrated, that the Time of the Fall thro' AB is = the Time of the Motion thro' BC; but the Time of the Motion thro' BC, to the Time of the Motion thro' BD, is as the Line BC to the Line BD; therefore the Time of the Motion thro' AB, to the Time thro' BD, is as the double of AB to BD. *Q. E. D.*

PROP. XXVI.

PROP. XXVI. PROB. X.

A Perpendicular between two horizontal Lines being given, as also a Space greater than the Perpendicular, but less than double the same, to elevate a Plane between the said Parallels, from the lowest Term of the Perpendicular, along which the Moveable may with a reflex Motion, after its Fall thro' the Perpendicular, pass a Space = the Space given, and in a Time = the Time of the Descent along the Perpendicular.

Let AB be a Perpendicular between the horizontal Parallels AO and BC; and let FE be greater than BA, but

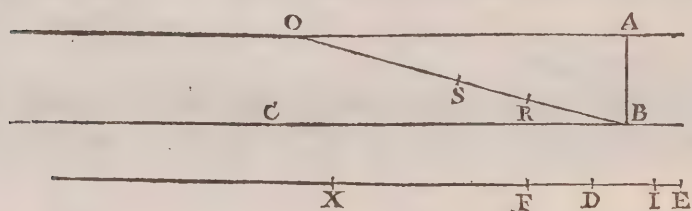


less than double the same. It is required from the Point B to elevate a Plane between the said Parallels, along which the Moveable, after its Fall from A to B, may, with a reflex Motion, in a Time = the Time of the Fall thro' AB, ascend a Space = the said FE.

Suppose $ED = AB$, then the remaining Part DF is less, since the whole Line EF is less than the double of AB : Let $DI = DF$, and as EI to ID , so make DF to a fourth FX ; and from B draw the reflex Line $BO = EX$; then, I say, that a Plane along BO , is that along which, after

after the Fall AB, a Moveable in a Time = the Time of the Fall thro' AB passes, by ascending a Space = the given Space EF. For,

Suppose BR and RS to be = ED and DF; then because it is EI : ID :: DF : FX, it will be by Composition ED : DI :: DX : XF; *i.e.* ED : DF :: DX : XF, and :: EX : XD; *i.e.* BO : OR :: RO : OS; but now if we suppose the Time along AB to be AB, the Time along OB will be



OB, and RO the Time along OS; and the remaining Part BR the Time thro' the remaining Space SB, descending from O to B; but the Time of the Descent thro' SB, *ex quiete* in O, is = the Time of the Ascent from B to S, after the Fall AB; therefore BO is the Plane raised from B, along which, after the Fall through AB in the Time BR or BA, is passed the Space BS = the given Space EF. Q. E. F.

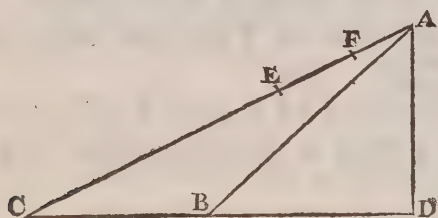
PROP. XXVII. THEOR. XVII.

If a Moveable descend along unequal Planes whose Elevation is the same, the Space that's passed over in the lowest Part of the longer, in a Time = that wherein the whole shorter Plane is passed, is = the Space that is composed of

the
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the said shorter Plane, and of the Part to which the said shorter Plane hath the same Proportion that the longer Plane hath to the Excess, by which the longer Plane exceeds the shorter.

Let AC be the longer Plane, and AB the shorter, whose Elevations are the same; from the lowest Part of AC (viz. from C) set off CE = AB; then make EF of the same Proportion to CE, as AE (*i. e.* as the Excess of the Plane CA above AB) has to the whole Line AC: Then, I say, the

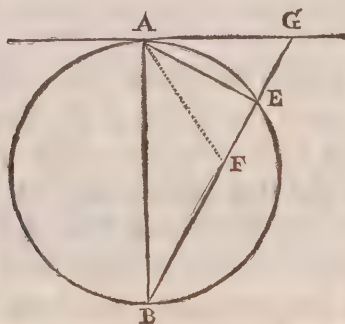


Space FC is that which is passed thro' after the Descent from A, in a Time = the Time of the Descent thro' AB: For since the whole CA is to the whole AE, as the Part taken away CE to the Part taken away EF; the remaining Part EA will be to the remaining Part AF, as the whole CA to the whole AE; therefore these three CA, AE, and AF, are continual Proportionals: but if the Time thro' AB be supposed to be as AB, the Time thro' AC shall be as AC; and the Time thro' AF shall be as AE, and that thro' the remaining Part FC shall be as EC: but EC is = the said AB, therefore, &c. *Q.E.D.*

PROP. XXVIII. THEOR. XVIII.

Let the horizontal Line AG be a Tangent to a Circle, and from the Point of Contact draw the Diameter AB; draw also the two Chords AE and EB at Pleasure, and let the Proportion of the Time of the Fall thro' AB, to the Time of the Descent through both AE and EB be required: Produce BE 'till it meets the Tangent Line in G, draw AF so as to bisect $\angle BAE$; then, I say, that the Time thro' AB is to the Time thro' both AE and EB, as AE is to $\overline{AE \times EF}$. For,

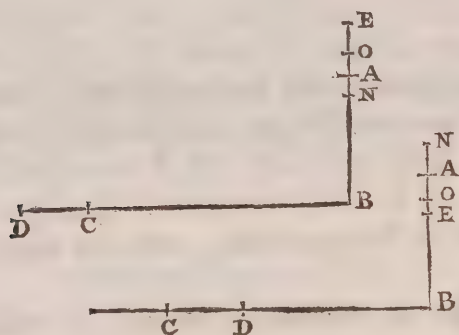
Since $\angle FAB = \angle FAE$, and $\angle EAG = \angle ABF$, the whole Angle GAF will be = the two Angles FAB and ABF; to



which Angles, $\angle GFA$ is also equal; therefore the Line GF is = GA; and because $\square BGE$ is = $\square$ of GA, it will be also = $\square$ of GF, and the three Lines BG, GF, GE Proportionals: but now if we suppose AE to be the Time thro' AE, GE will be the Time thro' GE, and GF the Time thro' the whole GB, and EF the Time thro' EB, after the Descent from

G,
†

that the Time in which a Moveable, let fall from A, passes both the Spaces AB and BC, is the shortest of all the Times wherein the same Space BC, with a Part of the Perpendicular, whether greater or less than the Part AB can be passed. Let a greater be taken, as EB, in Fig. 1. or a lesser as EB in Fig. 2 ; then we must prove, that the Time in which the Spaces EB and BC are passed, is longer than the Time in which AB and BC are passed.



Suppose the Time thro' AB to be as AB, then the same AB shall be the Time of the Motion along the horizontal Space BC; because BC is double to AB, and therefore the double of BA shall be the Time thro' both Spaces AB and BC. Make BO a mean Proportional between EB and BA; then will BO be the Time of the Fall thro' EB. Again, let the horizontal Space BD be double the said EB; then it's manifest, that the Time of it after the Fall EB is the same BO; for as DB is to BC, or as EB to BA, so make OB to BN; and since the Motion along the horizontal Plane is equable, and since OB is the Time thro' BD, after the Fall from E, therefore NB will be the Time thro' BC, after

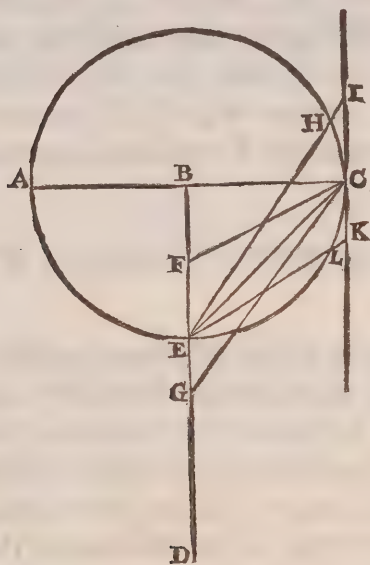
after the Fall from the same Height E : Whence 'tis manifest, that O A, together with B N, are the Time along $\overline{EB \times BC}$. And since the double of B A is also the Time thro' $\overline{AB \times BC}$; it remains to be proved, that O B, together with B N, are greater than the double of B A. And now because O B is a mean Proportional between E B and B A, the *Ratio* of E B to B A is double that of O B to B A : And since E B is to B A, as O B is to B N, therefore the *Ratio* of O B to B N, shall also be double that of O B to B A. But now the *Ratio* of O B to B N is compounded of the *Ratios* of O B to B A, and of A B to B N ; therefore the *Ratio* of A B to B N is the same with that of O B to B A ; therefore B O, B A, and B N, are three continual Proportionals, and O B, together with B N, greater than the double of B A. Whence the Proposition is manifest.

PROP. XXX. THEOR. XIX.

If a Perpendicular be let fall from any Point of the horizontal Line, and if from another Point in the same horizontal Line, a Plane be drawn 'till it meet the Perpendicular ; thro' which a Moveable may descend in the shortest Time to the said Perpendicular ; such a Plane shall cut off from the Perpendicular a Part = the Distance of the Point taken in the horizontal Line from the Point the Perpendicular was let fall from.

Let the Perpendicular BD be let fall from the Point B of the horizontal Line AC, in which let there be any Point C, taken at Pleasure; and suppose the Distance BE, in the Perpendicular, to be = the Distance BC, and draw CE; then, I say, that of all Planes that can be drawn from the Point C to the Perpendicular CE, is that along which, in the shortest of all Times, a Descent is made to the Perpendicular. For,

Let the Planes CF and CG be inclined, the one above, the other below the Plane CE, and draw IK touching the



Circle, whose Radius is BC, in the Point C; this Tangent IK will be $\parallel$ to the Perpendicular BD; draw EK $\parallel$ FC, 'till it meet the Tangent in K, and cutting the Circumference of the Circle in L; then 'tis manifest, that the

Time
†

Time of the Fall thro' LE is = the Time of the Fall thro' CE; but the Time thro' KE is longer than thro' LE, therefore the Time thro' KE is longer than thro' CE, but the Time thro' KE is = the Time thro' CF (KE and CF being equal, and equally inclined): In like manner, since CG and IE are equal and equally inclined, the Times of the Motions through them will be equal; but the Time thro' HE shorter than IE, is shorter than the Time thro' IE; therefore also the Time thro' CE (which is = the Time thro' HE) will be shorter than the Time thro' IE; therefore the Proposition is manifest.

P R O P. XXXI. T H E O R. XX.

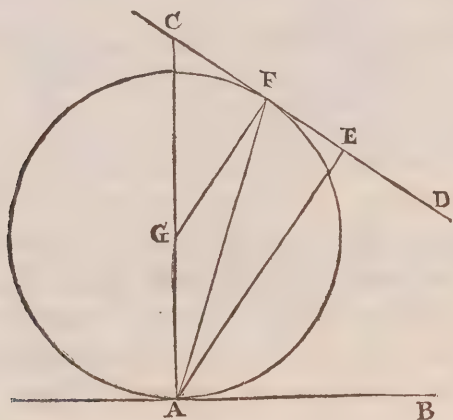
If a right Line be at Pleasure inclin'd upon the horizontal Line; the Plane produced from a given Point in the horizontal Line, 'till it meet the inclin'd Plane along which the Descent is made in the shortest of all Times; such a Plane, I say, is that which bisects $\angle$ made by two Lines drawn from the given Point, the one perpendicular to the horizontal Line, the other to the inclin'd Line.

Let

Let CD be a Line inclin'd at Pleasure to the horizontal Line AB ; and in the Line AB , let any Point as A be given, from which erect AC perpendicular to AB , and AE perpendicular to CD , and bisect the Angle CAE by the Line FA ; then, I say, that of all Planes inclined to the Plane CD , out of any Point of CD , to the Point A , the Plane extended thro' FA , is that along which the Descent is made in the shortest of all Times.

Draw $FG \parallel AE$, then the alternate Angles GFA and FAE will be equal; but $\angle EAF = \angle FAG$, therefore FG and GA will be equal.

If therefore on G as a Center with the Radius GA , you describe a Circle, it will pass through F , and will touch



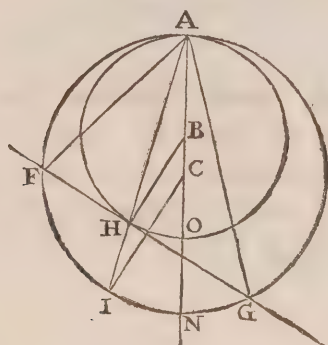
the horizontal Line AB in A , and the inclined Line CD in F ; For $\angle GFC$ is a Right one, the Lines GF and AE being parallel to each other: Whence it's manifest, that all the Lines drawn from the Point A , to the inclined Plane CD , are extended beyond the Circumference, and
confe-

consequently that the Motion thro' them must take up longer Time than that thro' FA requires: Wherefore, &c.
Q. E. D.

L E M M A.

If two Circles touch one another within, the innermost of which some right Line toucheth, and the outermost of which the same right Line cuts; three Lines drawn from the Point of Contact of the two Circles, to three Points of the Tangent right Line, that is one to the Point of its Contact with the interior Circle, and the other two to the two Points where it cuts the exterior Circle, shall contain equal Angles, at the Point of Contact of the two Circles.

Let two Circles touch one another inwardly in the Point A; let B be the Center of the lesser, and C that of the

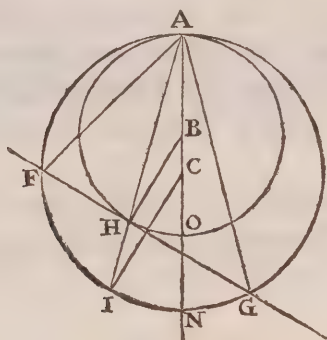


greater: And let any right Line, as FG touch the inner Circle in the Point H, and cut the outermost in the Points F and G; and draw the three Lines AF, AH, and AG; then, I
say;

say, that the Angles by them contained, *viz.* FAH and GAH are equal.

Produce AH to the Circumference in I, and from the Centers draw BH and CI, and thro' the said Centers let BC be drawn, which continued, will meet with the Point of Contact in A, and with the Circumferences of the Circles in O and N.

Then because $\angle$'s ICN and HBO are equal, each of them being the double of the Angle IAN, the Lines BH

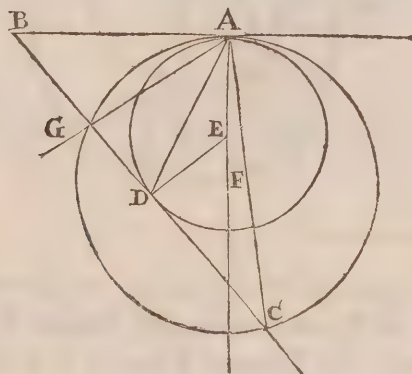


and CI are parallel. And since the Line BH from the Centre to the Point of Contact, is perpendicular to FG, CI will also be perpendicular to FG, and the Arch FI will be = the Arch IG; and consequently the Angle FAI to the Angle IAG. *Q.E.D.*

PROP. XXXII. THEOR. XXI.

If two Points be taken in the Horizon, and any Line be inclined from one of them towards the other, out of which a right Line is drawn unto the inclin'd Line, cutting off a Part thereof = that which is included between the Points of the Horizon; the Descent thro' this Line shall be sooner performed than along any other right Line drawn from the same Point to the said inclin'd Line. And along other Lines which are at equal Angles on each Side this, the Descents are made in equal Times.

In the Horizon let there be two Points A and B, and from B incline the right Line BC; in which from the End B, take



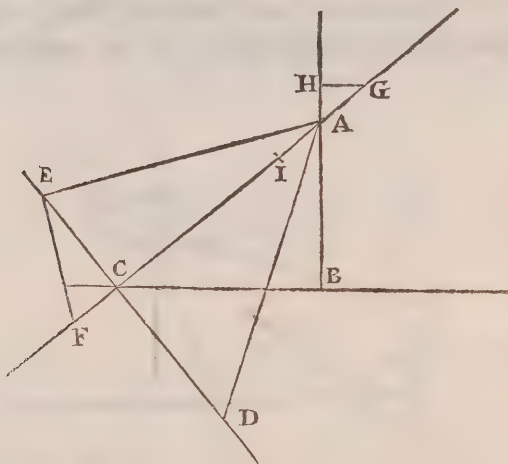
BD = BA, and draw the Line AD; then, I say, that the Descent along AD, is more swiftly made than along any other Line whatsoever, drawn from the Point A to the inclined Line BC: For from the Points A and D, unto BA and BD, raise the Perpendiculars AE and DE, intersecting each other

X x

in

right Angles to AC, and make $CD = AB$, and join AD; then $\angle ADC$ will be greater than $\angle CAD$ (for CA is greater than AB or CD): Make $\angle DAE = \angle ADE$; make EF perpendicular to AE, and produce it till it meets the inclin'd Plane produced in F; and let AI and AG be each made $= CF$, and thro' G draw GH parallel to the Horizon; then I say, that H is the Point sought. For,

Supposing the Time of the Fall along the Perpendicular AB to be AB, the Time of the Descent along AC *ex quiete* in A, will be AC; and since in the right-angled

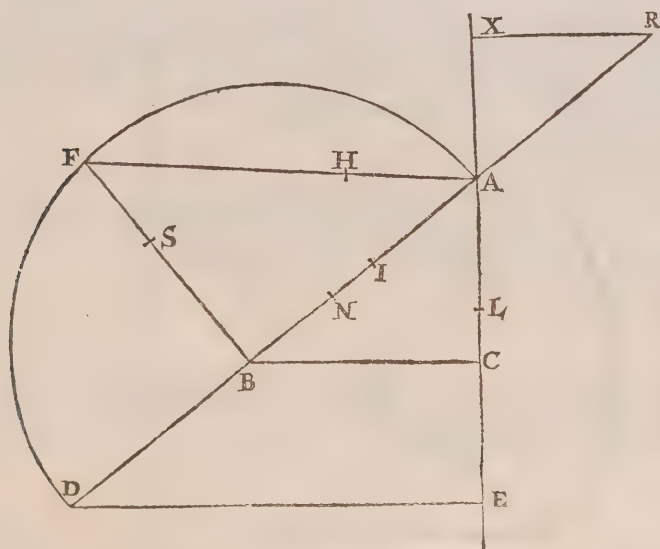


Triangle AEF, from the right Angle E, unto the Base AF, EC is perpendicular, AE will be a mean Proportional between FA and AC, and CE a mean between AC and CF, *i. e.* between CA and AI; and since the Time of AC out of A, is AC, AE will be the Time of the whole AF, and AC the Time of AI; and because in the equicrural Triangle AED, the Side AE is $=$ the Side ED, ED will be the Time thro' AF, and EC that thro' AI; therefore

fore CD, *i.e.* AB will be the Time thro' IF *ex quiete* in A, which is the same as if we say AB is the Time thro' AC out of G, or out of H. *Q. E. F.*

PROP. XXXIV. PROB. XIII.

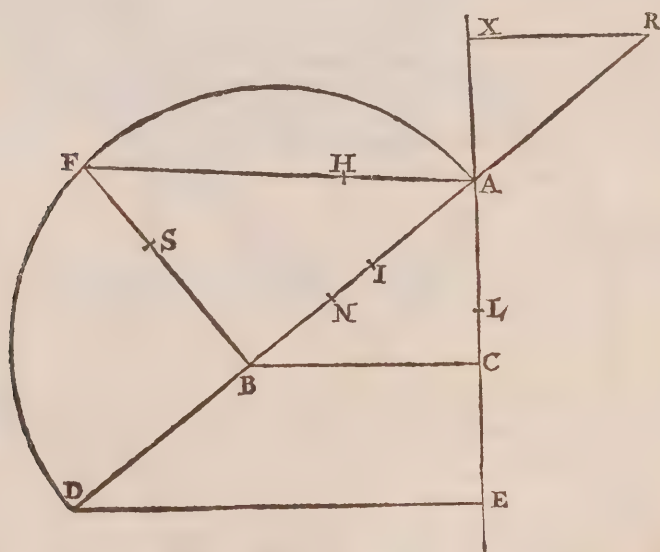
An inclin'd Plane, and a Perpendicular being given, which have the same Term above, to find a higher Point in the Perpendicular produced, from which a Moveable descending and turning along the inclin'd Plane, may pass them both in the same Time as it wou'd pass through the inclined Plane only, were it let go *ex quiete* from its highest Term.



Let the inclin'd Plane and Perpendicular be AB and AC, whose highest Term A is the same. It is required in the Perpendicular produced from A, to find a Point above,

above, from which a Moveable descending, and turning along the Plane AB, may pass the assigned Part of the Perpendicular, together with the Plane AB, in the same Time as it would the sole Plane AB out of Rest in A.

Let BC be the horizontal Line, and make $AN = AC$; and make as AB to BN, so AL to LC: And make $AI = AL$, and let CE set off in the Perpendicular AC produced, be a third Proportional to AC and BI; then I say, CE is the Space sought; so that the Perpendicular being extended above A, and taking the Part $AX = CE$, a Moveable from X will pass both the Spaces XA and AB, in the same Time as it would pass only the Space AB from A.



Draw the Line $XR \parallel$ the horizontal Line BC, which let BA extended meet in R, produce AB to D, and draw ED

ED $\parallel$ CB, and upon AD as a Diameter, describe a Semi-circle AFD, and from the Center B erect a Perpendicular to DA as BF, and produce it 'till it meet the Circumference in F; then 'tis manifest, that FB is a mean Proportional between AB and BD, and that a Line drawn from F to A will be a mean Proportional between DA and AB. Suppose BS = BI, and FH = FB; then because it is AB : BD :: AC : CE; and because BF is a mean Proportional between AB and BD; and BI a Mean between AC and CE; it will be BA : AC :: FB : BS; and because it is BA : AC or AN :: FB : BS; it will be by Conversion BF : FS :: AB : BN, *i.e.* :: AL : LC; therefore $\square$ under FB and CL is = $\square$ under AL and SF; but this Rectangle under AL and SF is the Excess of $\square$ under AL and FB, or AI and BF above $\square$ under AI and SB, or $\square$ AIB; but $\square$ under FB and LC is the Excess of $\square$ under AC and BF above $\square$ under AL and BF; but $\square$ under AC and BF is = $\square$ ABI (for it is BA : AC :: FB : BI); therefore the Excess of $\square$ ABI above $\square$ under AI and BF, or AI and FH, is = the Excess of the Rectangle under AI and FH, above $\square$ AIB; therefore twice the Rectangle under AI and FH, is = the two Rectangles ABI and AIB, *viz.* to twice $\square$ AIB, together with $\square$ of BI.

Let the Square of AI be common to both, then twice $\square$ AIB, with the two Squares of AI and IB (*i.e.* $\square$ of AB) will be = twice $\square$ AI and FH, with $\square$ of AI: Again, taking the Square of BF common on both Sides, the two Squares of AB and BF, that is the Square AF, will be = twice the Rectangle under AI and FH, with the two Squares of AI and FB, *i.e.* of AI and FH; but the same Square of AF is = twice $\square$ AHF, with the two Squares of

AH

between RB and BA ; and since $BF = FH$, is a Mean between AB and BD, *i. e.* RA and AB, therefore from what has been before * proved, the Excess AH shall be the Time along AB out of Rest in R, or after the Fall out of X : Since the Time along the said AB, out of Rest in A, shall be AB ; therefore the Time along XA is IB, and along AB after RA, or after XA is AI ; Therefore the Time along XAB shall be as AB, namely, the self-same with the Time along AB, only out of Rest in A, which was the Proposition.

PROP. XXXV. PROB. XIV.

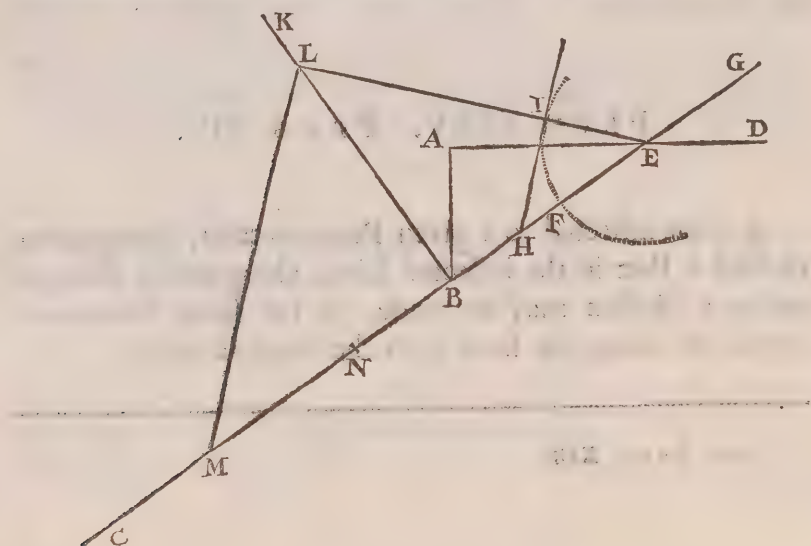
A Line inflected to a given Perpendicular, being given, to find a Part in the inflected Line, along which alone *ex quiete* a Motion may be made, in the same Time as it would be along the same with the Perpendicular.

* COR. PROP. XIX.

Y y Lec

Let the Perpendicular be AB , and the Line inflected to it BC . It is required in BC to determine a Part along which alone, out of Rest, a Motion may be made in the same Time, as it would along the same, together with the Perpendicular AB .

Draw the horizontal Line AD , which let the inclin'd Line CB produced, meet in E ; and suppose $BF = BA$; and on the Center E , with the Radius EF , describe the Circle

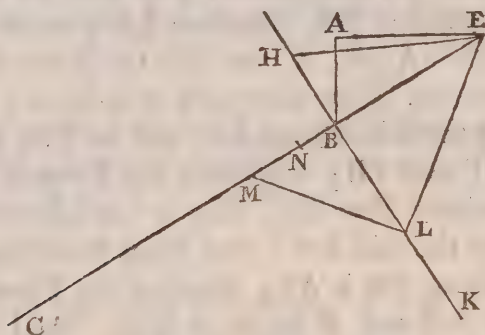


FIG, and produce FE to the Circumference in G ; and cut the Line BF in H , so that as BG is to BF , so may BH be to HF ; and let HI touch the Circle in I : then to BC from B erect the Perpendicular BK , which let the Line EIL meet in L ; lastly, to EL from L , raise the Perpendicular LM , meeting BC in M ; then, I say, that along
the

the Line BM, from Rest in B, a Motion is made in the same Time as it would be out of Rest in A, along both AB and BM. Make $EN = EL$: And since it is $GB : BF :: BH : HF$; it will be by Permutation $GB : BH :: BF : FH$; and by Division, $GH : HB :: BH : HF$. Wherefore $= GHE$ will be $= HB$ *q*; But the same Rect-angle is also $= HI$ *q*; therefore BH is $= HI$: And since in the four-sided Figure $ILBH$, the Sides HB and HI are equal, and $\angle$'s B and I right, the Side BL will also be $= LI$; but EL is $= EF$, therefore the whole Line LE or NE is $=$ the two Lines LB and EF ; take away the common Line EF , and the Remainder FN will be $= LB$; but FB was supposed $= BA$, therefore LB is $=$ the two Lines AB and BN . Again, if you take the Time along AB to be AB , then the Time along EB will be $= EB$; but the Time along the whole Line EM will be EN , *viz.* a Mean between ME and EB ; wherefore the Time of the Descent of the remaining Part BM , after it has fallen thro' EB or AB , will be the said BN : But it has been supposed, that the Time along AB is AB ; therefore the Time of the Fall along both AB and BM , is ABN . And because the Time along EB , out of Rest in E , is EB ; the Time along BM , out of Rest in B , will be a mean Proportional between BE and BM ; but that is BL ; therefore the Time along both AB and BM , out of Rest in A , is ABN ; but the Time along only BM , out of Rest in B , is BL ; but BL has been proved to be $= AB$ and BN ; therefore the Proposition is manifest.

More expeditiously thus ;

Let BC be the inclin'd Plane, and BA the Perpendicular (as before) ; produce CB till it meet the horizontal Line AE in E ; upon EC , and from the Point B , erect a Perpendicular, which extend both Ways ; then take $BH =$ the Excess of BE above BA ; make $\angle HEL = \angle BHE$; and let EL produced meet BK in L ; and unto EL , and from the Point L erect the Perpendicular LM , meeting BC in M ; then, I say, that BM is the Space in the Plane BC sought. For because $\angle MLE$ is right, BL

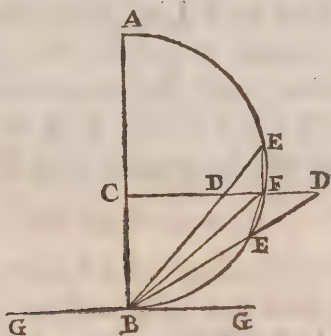


will be a mean Proportional between MB and BE , and LE a Mean between ME and EB . Make $EN = LE$, and the three Lines NE , EL , LH will be equal ; and HB will be the Excess of NE above BL : but the same HB is also the Excess of NE above NB and BA ; therefore the two Lines NB and BA are $= BL$: but now if we suppose EB to be the Time along EB , BL will be the Time along BM , out of Rest in B ; and BN will be the Time of the same after EB , or after AB ; and AB will be the

the Time along AB, therefore the Times along ABM, viz. ABN are = the Time thro' only BM, out of Rest in B. *Q. E. F.*

L E M M A I.

Let FC be perpendicular to the Diameter BA, and from the Term B, draw BED at Pleasure, and join FB ; then, I say, that FB is a mean Proportional between DB and BE. Draw a Line from E to F ; and thro' B draw the Tan-



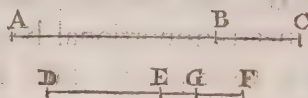
gent BG, which will be $\parallel$ CD : Wherefore $\angle$ DBG will be $= \angle$ FDB ; but $\angle$ EFB, in the alternate Segment, will also be $=$ the same DBG ; therefore $\triangle$'s FBD and FEB are similar ; and therefore it will be $BD : BF :: BF : BE$; Therefore, *Q. E. D.*

L E M M A II.

LEMMA II.

Let the Line AC be greater than DF ; and let AB have a greater Proportion to BC, than DE has to EF ; then, I say, that AB is greater than DE.

For, because the *Ratio* of AB to BC is greater than that of DE to EF ; therefore what *Ratio* AB has to BC, the



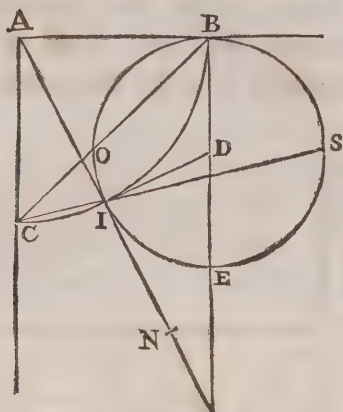
same shall DE have to a Line lesser than EF ; let it have it to EG : And because it is $AB : BC :: DE : EG$, it will be by Composition, and by Conversion of Proportion, $CA : AB :: GD : DE$; but CA is greater than GD, therefore also BA will be greater than DE.

LEMMA III.

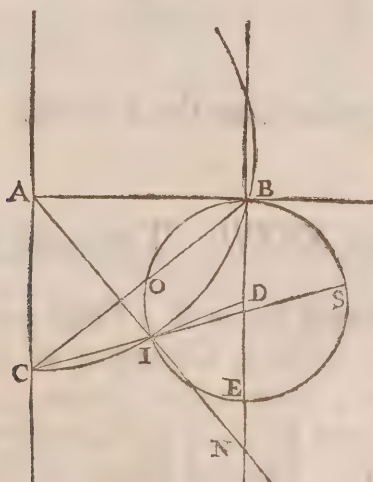
Let ACIB be a Quadrant of a Circle, and from B draw $BE \parallel AC$, and from any Point taken in BE, as a Center, describe the Circle BOES touching AB in B, and cutting the Arch of the Quadrant in I ; and draw a Line from C to B ; and another from C to I ; and produce it to S ; then, I say, that the Line CI is always less than CO. Draw a Line from A to I, which shall touch the Circle BOE ; for if DI be drawn it will be $= DB$; but since DB touches the Quadrant, it shall also touch DI, and be perpendicular to the Diameter AI : Wherefore also AI shall touch the Circle BOE in I : And because $\angle AIC$ is greater than $\angle ABC$, as it stands upon a bigger Arch, therefore also $\angle SIN$ will be greater than the said ABC ; wherefore the Arch IES is greater than the Arch BO ; and the Line

CS,
t

CS, nearer the Center, greater than CB : Wherefore also CO is greater than CI, since 'tis $SC : CB :: OC : CI$.

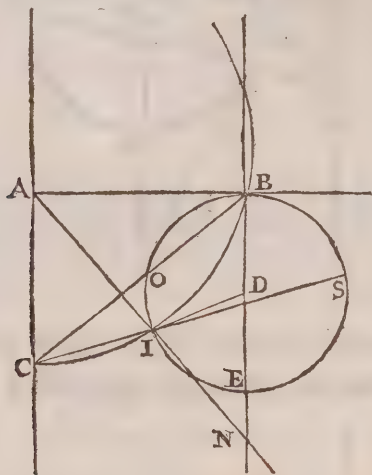


And this would be more so, if (as in the following Figure) BIC were less than the Arch of a Quadrant : For



the Perpendicular DB will then cut the Circle CIB ; wherefore

fore DI also will cut the same Circle, since it is $= DB$: And $\angle DIA$ will be obtuse ; and therefore AIN will also cut BIN : And since $\angle ABC$ is less than $\angle AIC$, which is $= \angle SIN$; and this now still less than that which would be made at the Contact in I by the Line SI ; therefore the



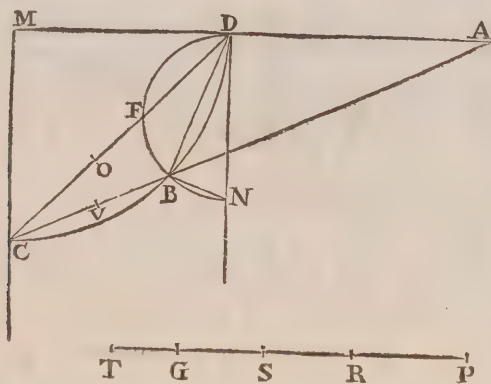
Arch SEI is far greater than the Arch BO : Whence, &c.
Q.E.D.

PROP. XXXVI. THEOR. XXII.

If from the lowest Point of a Circle erect to the Horizon, be elevated a Plane, subtending an Arch not greater than a Quadrant ; from whose Terms, two other Planes are inflected to any Point of the Circumference ; the Descent along both the inflected Planes will be performed in a shorter Time than along the former elevated Plane alone,

alone, or than along one only of the other two, namely, the lower.

Let CBD be the Arch, not greater than that of a Quadrant of the Circle erect unto the Horizon, from the lowest Point C, in which let CD be an elevated Plane, and let the two Planes CB and DB be inflected from the Terms D and C, to any Point in the Arch CBD taken at Pleasure, as B; then, I say, that the Times of the Descent along both these Planes DB and BC, is shorter than the Time of the Descent along DC alone, or than that along only BC *ex quiete* in B.

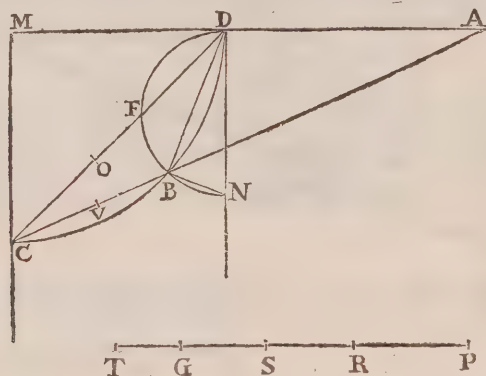


Thro' D draw the horizontal Line MDA, which let CB produc'd meet in A; and let DN and MC be perpendicular to MD, and BN to BD; and about the right-angled Triangle DBN circumscribe a Semicircle as DFBN, cutting DC in F, and let DO be a mean Proportional between CD and DF, and let AV be a Mean between CA and AB; and let PS be the Time wherein the whole DC or BC shall be passed (for it is manifest, that they shall each

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be

be passed in the same Time) and what Proportion CD has to DO, the same let the Time SP have to the Time PR; then the Time PR will be that wherein a Moveable from D passes DF; and RS that wherein it passes the Remainder FC: but since PS is also the Time wherein the Moveable from B, passes BC, if it be made $BC : CD :: SP : PT$; PT will be the Time of the Descent from A to C; since DC is a Mean between AC and CB, from what has been before proved. Lastly, make $CA : AV :: TP : PG$, then will PG be the Time wherein the Moveable from A comes to B; and the Remainder GT the Time



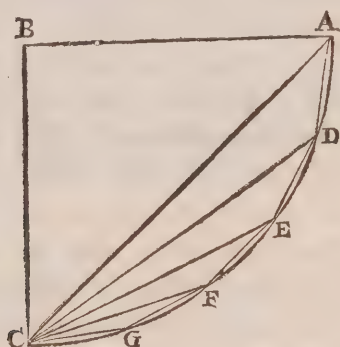
of the Motion thro' BC consequent after the Motion from A to B. But since DN the Diameter of the Circle DFN is perpendicular to the Horizon, the Lines DF and DB shall be passed in equal Times. Wherefore, if it shall be demonstrated, that the Moveable will sooner pass along BC, after the Descent DB, than along FC, after the Descent DF, we shall have what we wanted. But now the Moveable will, with the same Celerity of Time, pass BC,
coming
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coming out of D along DB, as if it came out of A along AB ; since in either Descent DB or AB, it requires equal Moments of Velocity : Therefore it remains to be demonstrated, that BC after AB is passed in a shorter Time than FC after DF is. But it has been proved, that the Time wherein BC, after AB is passed, is GT ; and the Time wherein FC, after DF is passed, is RS ; therefore we have now to prove, that RS is greater than GT. Which is thus done ; because it is $SP : PR :: CD : DO$, therefore first by Conversion, and then by Inversion of Proportion it will be $RS : SP :: OC : CD$. But $SP : PT :: DC : CA$; and because it is $TP : PG :: CA : AV$, by Conversion of Proportion it will be also $PT : TG :: AC : CV$; therefore *ex æquali* $RS : GT :: OC : CV$; but OC is greater than CV, as shall be demonstrated anon ; therefore the Time RS is greater than the Time GT ; which was required to be demonstrated. But since CF is greater than CB, and FD less than BA, CD will have a greater Proportion to DF than CA has to AB ; but $CD : DF :: COq : OFq$, since CD, DO, and DF are Proportionals. And $CA : AB :: CVq : VBq$; therefore CO has a greater Proportion to OF, than CV has to VB ; therefore from the foregoing Lemma, CO is greater than CV. 'Tis moreover manifest, that the Time along DC to the Time along DBC, is as DOC to DO, together with CV.

SCHOLIUM.

From what has been demonstrated, may be gathered, that the swiftest of all Motions from Term to Term, is not made along the shortest Line, *viz.* a right Line, but by or along the Arch of a Circle.

For in the Quadrant B A E C whose Side BC is perpendicular to the Horizon, let the Arch AC be divided into any Number of equal Parts, AD, DE, EF, FG, GC; and draw right Lines from C to the Points A, D, E, F, G; draw also the right Lines AD, DE, EF, FG, and GC: Then



'tis manifest, that the Motion along the two Lines AD and DC, is sooner perform'd than that along the one Line AC, or along DC, out of Rest in D: But out of Rest in A, DC is sooner passed over than the two AD and DC; but along the two DE and EC, out of Rest in A, the Descent is likewise sooner made than along CD only; therefore the Descent along the three Lines AD, DE, and EC is made sooner than along the two AD and DC. And, in like Manner,

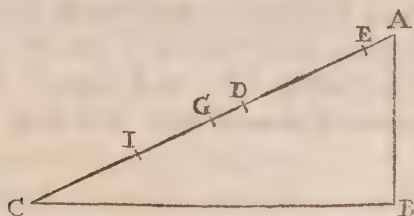
Manner, after the Descent along AD and DE, the Motion is sooner made along the two EF and FC, than along FC alone ; therefore the Motion is sooner performed along the four AD, DE, EF, and FC, than along the three AD, DE, and EC. And lastly, along the two FG and GC, after a preceding Descent thro' AD, DE, and EF, the Motion is sooner accomplished than along FC alone ; therefore along the five AD, DE, EF, FG, and GC, the Descent is made in a yet shorter Time than along the four AD, DE, EF, and FC ; the nearer therefore we approach'd by inscribed Polygons to the Circumference, the sooner will the Motion be performed between the two assign'd Terms A and C.

And that which is explained in the Arch of a Quadrant, holds true likewise in an Arch less than that of a Quadrant ; and the Reasoning is the very same.

PROP. XXXVII. PROB. XV.

A Perpendicular and an inclined Plane of the same Elevation being given, to find a Part in the inclined Plane = the Perpendicular, and that may be pass'd over in the same Time as the said Perpendicular.

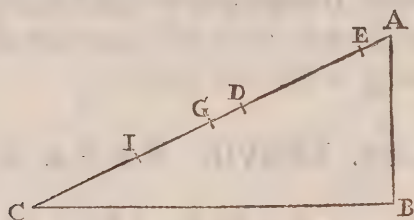
Let AB be the Perpendicular, and AC the inclined Plane.



Now we must find in the inclin'd Plane, a Part = the Perpendicular

pendicular AB, that after Rest in A, may be moved thro' in a Time = the Time wherein the Perpendicular is passed.

Make $AD = AB$, and bisect the Remainder DC in I ; and make $AC : CI :: CI : AE$; to which AE make GD equal; then it is manifest, that EG is $= AD = AB$. I say, moreover, that this EG is that Part of the inclined Plane which is passed over by a Moveable coming out of Rest in A, in a Time = the Time wherein a Moveable falls thro' AB . For because it is $AC : CI :: CI : AE$; or $:: ID : DG$; therefore by Conversion it is $AC : AI :: DI : IG$. And because as the whole CA to the whole AI ,



so is the Part taken away CI , to the Part taken away IG ; therefore the Remainder AI is to the Remainder AG , as the whole AC is to the whole AI ; therefore AI is a mean Proportional between AG and AC , and CI a Mean between CA and AE ; if therefore we suppose the Time along AB to be as AB , AC shall be the Time along AC , and CI or ID the Time along AE ; and because AI is a mean Proportional between CA and AG , and CA is the Time along the whole AC ; therefore AI shall be the Time along AG , and the Remainder IC that along the Remainder GC ; but DI was the Time along AE , therefore DI and IC are the Times along both the Spaces AE and CG , therefore the

the Remainder DA shall be the Time along EG, and = the Time along AB. *Q. E. F.*

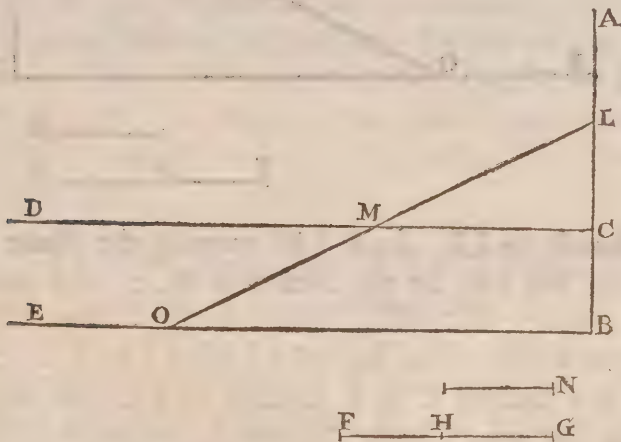
C O R O L L A R Y.

Hence it is manifest, that the Space required is that which lies betwixt the upper and lower Parts (*viz.* AE and CG) that are passed in equal Times.

P R O P. XXXVIII. P R O B. XVI.

Two horizontal Planes cut by a Perpendicular being given, to find a Point above in the Perpendicular, out of which Moveables falling and turning along the horizontal Planes, may, in Times = the Times of the Descents, pass along the said horizontal Planes, namely, along the upper and along the lower Spaces that have to each other any given Proportion of the lesser to the greater.

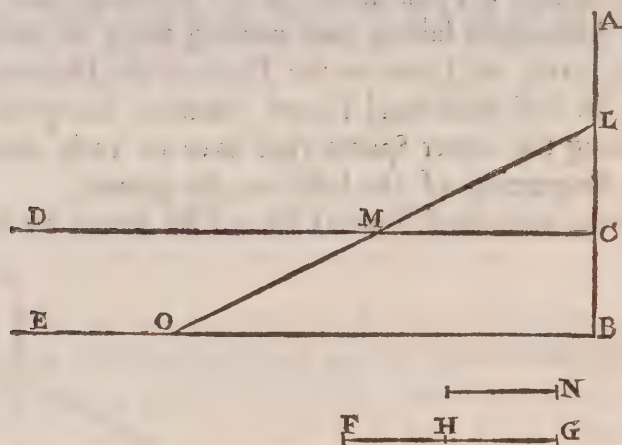
Let the horizontal Planes CD and BE be cut by the Per-



pendicular ACB, and let the given Proportion of the less
to

to the greater, be that of N to FG . It is required in the Perpendicular AB , to find a Point on high, out of which a Moveable falling, and turning along CD , may, in a Time = the Time of its Fall, pass a Space that shall have unto the Space passed by the other Moveable falling from the same sublime Point, in a Time = the Time of its Fall with a reflex Motion along the Plane BE , the same Proportion as the given Line N has to FG .

Make $GH =$ the said N ; and make $FH : HG :: BC : CL$; then, I say, L is the Point on high required : For taking



CM double to CL , draw LM meeting the Plane BE in O ; then BO will be the double of BL : And because it is $FH : HG :: BC : CL$; it will be by Composition and Conversion $HG : (i.e. N) GF :: CL : LB, i.e. :: CM : BO$. But because CM is double to LC , let the Space CM be that which by the Moveable coming from L , after the Fall LC , is passed along the Plane CD ; and by the same Reason BO is that which is passed after the Fall LB ,

in
†

in a Time = the Time of the Fall along LB, since BO is double to BL ; therefore the Proposition is manifest.

SAGR. Most certainly we may grant to our *Academic*, that he hath done what, without Arrogance, he promised at the Beginning of this Treatise, viz. that he hath shewed us a *New Science* concerning a *very Old Subject* : And to see with what Ease and Clearness from one sole most simple Principle he deduces the Demonstrations of so many Propositions, I can't sufficiently admire how this Business escap'd unhandled by *Archimedes*, *Apollonius*, *Euclid*, and so many other excellent Mathematicians and Philosophers : Especially since we meet with many large Volumes of Motion.

SALV. There is extant a small Fragment of *Euclid* of *Motion*, but there appear no Steps of his taken towards the Discovery of the Proportion of *Acceleration*, and of its Varieties along different Inclinations : So that we may truly say, that no Door, 'till now, was ever open'd to a Contemplation new and fraught with infinite and admirable Conclusions, wherein in future Ages other Wits may exercise themselves.

SAGR. I verily believe, that as those few Properties of the *Circle*, for Instance, demonstrated by *Euclid* in the third Book of his *Elements*, are an Introduction to innumerable others more abstruse ; so those produced and demonstrated in this short Treatise, if other speculative Genius's apply to them and consider them well, will point out the Way to infinite others still more admirable : And

A a a

this

this I the rather imagine, on Account of the Nobleness of the Subject, whereby it so vastly exceeds all other Physical Subjects.

This Day's Conference has been very long and laborious, wherein I have been delighted more with the bare Propositions than with their Demonstrations; several of which will take more than an Hour a Piece, I dare say, to understand them well: A Task that I reserve to myself to perform at Leisure, if you please to lend me the Book, after we have seen the remaining Part concerning the Motion of *Projects*, which, if you will, shall be the Subject of To-morrow's Conference.

SALV. I'll not fail to be with you.

The END of the THIRD DAY'S CONFERENCE.



GALILÆUS



G A L I L Æ U S

H I S

DIALOGUES.

DIALOGUE IV.

SALVIATUS, SAGREDO, and SIMPLICIUS.

SALV.  INCE *Simplicius* is also come, let us;
without Delay, proceed to *Motion*; and
see here the *Text* of our *Author* :

Of the MOTION of PROJECTS.

What Properties belong to *Equable Motion*, as also to
Motion naturally accelerate, upon all whatever Inclinations
of Planes, we have already considered : In this Contem-
plation we are now entering upon, I'll endeavour to shew
you some of the chief Properties most worthy your Notice,
and to establish the same by solid Demonstrations, belong-
ing to a Moveable, whilst it is carried by a Motion
compounded of a twofold Lation, viz. the Equable and

the Naturally Accelerate : And this is that Motion which we call *The Motion of Projects* : The Generation of which Motion may be understood by what follows.

Conceive a Moveable projected or thrown along an horizontal Plane, all Impediments secluded : Now 'tis manifest, by what we have before said at large, that its Motion will be equable and perpetual along the said Plane, if the Plane be extended *in infinitum* : but if we suppose this horizontal Plane terminated and placed on high, the Moveable which I conceive endued with Gravity, being come to the End of the Plane, proceeding still forward, there is added to its equable and former indelible Lation, that Propension downwards which it has from its Gravity, and thence a certain Motion will be produced, compounded of the equable horizontal one, and of the descending naturally accelerate one : Which compound Motion I call *Projection*, some of whose Properties we will demonstrate ; the first of which shall be this.

PROP. I. THEOR. I.

A *Project*, when it is carried by a Motion compounded of the horizontal Equable, and of the naturally accelerate one downwards, describes by its Lation or Tract a *Semi-parabolical* Line.

SAGR. It's requisite, *Salviatus*, for my own, and, I believe, for *Simplicius's* sake also, to stop a little ; for I am not so far vers'd in Geometry as to have studied *Apollonius* any farther than just to know that he treats of these *Parabolas*, and of the other Conic Sections ; with-

out

out understanding which with their Properties, I don't think one can understand the Demonstrations of the other Propositions depending on them: And because already, in the very first Proposition, it is propos'd by the Author, to prove the Line described by a *Project* to be parabolical, I imagine, since we are to treat of these Lines only, that it must be absolutely necessary to have a perfect Knowledge, if not of all the Properties which *Apollonius* has demonstrated of those Figures, yet, at least, of such as are necessary to the Science in Hand.

SALV. You think too meanly of yourself to make strange of that. the Truth whereof, not long since, you made no Dispute about; thus, when in treating of Resistance, I told you that a certain Proposition of *Apollonius* shou'd be understood, you, without the least Difficulty, admitted of it.

SAGR. Perhaps I might then either accidentally conceive it, or for once might take for granted what serv'd my Turn in that Treatise: But here, where I imagine we are to hear all the Demonstrations concerning those Lines, I can't think it right to take any Fundamental for granted, for this would be going on to no Purpose.

SIMP. Tho' *Sagredus* may be, as I believe he is, well vers'd in what may prove necessary to him, yet, for my Part, I am a Stranger to the very first Terms; for altho' our Philosophers have treated of the Motion of *Projects*, yet I don't remember that they have defin'd those Lines which they describe; any otherwise than to be curv'd Lines,
except

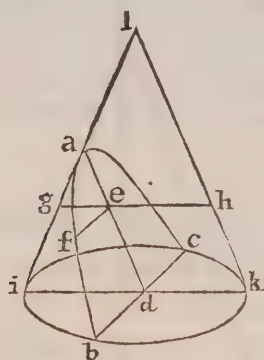
except the Projections be perpendicularly upwards : But of what kind those Curves are they have not told us. Therefore in case that little Geometry which I have learnt from *Euclid*, within the Time of the foregoing Conferences, be not sufficient to make me capable of understanding the following Demonstrations, I must content myself with an implicit Belief of them.

SALV. No, you shan't, for the Author of this Work himself will instruct you, as heretofore he did me, when he granted me a Sight of this his Work ; for finding that I at that Time was not ready in the Books of *Apollonius*, he demonstrated to me two of the chief Properties of the *Parabola*, without any other *Precognitum*, which two only we have Need of in the present Treatise ; which are both indeed proved by *Apollonius*, but after many Propositions proved before, which to consider wou'd take up too much of our Time ; for I would willingly shorten our Trouble as much as possible ; deducing the first of these immediately from the pure and simple Generation of the *Parabola* itself ; and from this without any Trouble, the Demonstration of the other. To the first then : Conceive a *Right-Cone*, whose Base let be the Circle $ibkc$, and *Vertex* the Point l , in which, cut by a Plane parallel to the Side lk , there arises the Section bac , called a *Parabola*, whose Base bc let cut the Diameter ik of the Circle $ibkc$ at right Angles, and let the *Axis* ad be $\parallel$ the Side lk ; then taking any Point f in the Line bfa , draw the right Line $fe \parallel bd$; I say, the Square of bd has to the Square of fe the same *Ratio* as the *Axis* da has to the *Axis* ea .

Conceive

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Conceive a Plane to pass thro' the Point e $\parallel$ the Circle $ibkc$, which makes in the Cone a circular Section, whose Diameter let be the Line geb . And because to the Diameter ik of the Circle $ibkc$, the Line bd is perpendicular, the Square of bd will be $= \square$ made of the Parts id and dk . And likewise in the upper Circle, which is conceived to pass thro' the Points gfb ; the Square of the

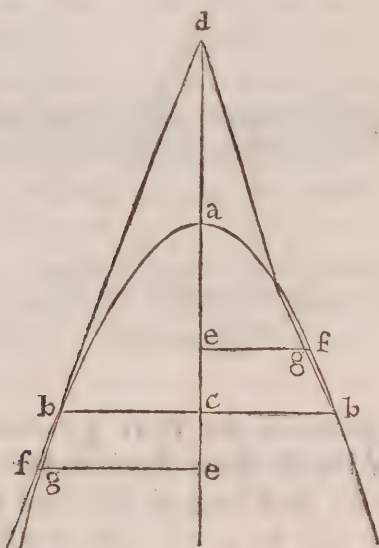


Line fe is $= \square$ made of the Parts ge and eb ; therefore the Square of bd has the same *Ratio* to the Square of fe as $\square idk$ to $\square geb$: And because the Line ed is $\parallel$ the Line bk , the Line eb will be $= dk$, which also are Parallels; and therefore $\square idk$ will have the same *Ratio* to $\square geb$ as id has to ge ; i.e. as da to ae ; therefore $\square idk$ to $\square geb$; i.e. $\square$ of bd has the same *Ratio* to $\square$ of fe as the *Axis* da has to the *Axis* ea . *Q. E. D.*

The other Proposition necessary to the present Treatise, we will demonstrate as follows.

Describe

Describe a *Parabola*, whose *Axis* ca produce outwards to d , and taking any Point b , thro' it conceive a Line as bc to be drawn $\parallel$ to the Base of the *Parabola*. Suppose $da =$ the Part ca of the *Axis*; then, I say, that a right Line drawn thro' the Points d and b will not fall within the *Parabola*, but without it, so as to touch it only in that very Point b .



If it be possible, let it fall within the *Parabola*, it will then cut it above, or being prolong'd it will cut it below the Point b . And in that Line take any Point as g , thro' which draw the right Line fge ; and because the $\square$ of fe is greater than $\square$ of ge , the same Square of fe will have a greater *Ratio* to $\square$ of bc than $\square$ of ge has to the same Square of bc . And because by the preceding Proposition $\square$ of

of fe is to $\square$ of bc , as ea to ca , therefore ea has a greater Ratio to ca , than $\square$ of ge has to $\square$ of bc ; i.e. than the Square of ed to $\square$ of cd , (since in $\triangle dge$, it is as ge to its Parallel bc , so is ed to cd ; but the Line ea is to the Line ca , i.e. to the Line ad as four Rectangles ead to four Squares of ad , i.e. to $\square$ of cd (which is = four Squares of ad); therefore four Rectangles ead will have a greater Ratio to $\square$ of cd than $\square$ of ed has to the same Square of cd , therefore four Rectangles ead will be greater than $\square$ of ed , which is false, since they are less, because the Parts ea , ad of the Line ed are not equal. Therefore the Line db touches the Parabola in b , and does not cut it.

Q. E. D.

SIMP. Your Reasoning is too lofty for me; you seem all along to presuppose that all the Propositions of *Euclid* are as familiar as the first Axioms themselves; but the Case is otherwise. And your asserting to me but just now that four Rectangles ead are less than $\square$ of ed , because the Parts ea , ad are unequal, doth not satisfy me, but leaves me in Doubt.

SALV. The Truth is, all Mathematicians of Note, do suppose the Reader to have the Elements of *Euclid* at his Fingers Ends; but to supply your Want, let it suffice to put you in Mind of a Proposition in the Second Book, by which is demonstrated, that, *When a Line is cut into equal Parts, and into unequal, the Rectangle of the unequal Parts is less than the Rectangle of the equal (i. e. than the Square of the Half) by the Square of the Line comprehended between the Sections.* Whence it is manifest, that

B b b

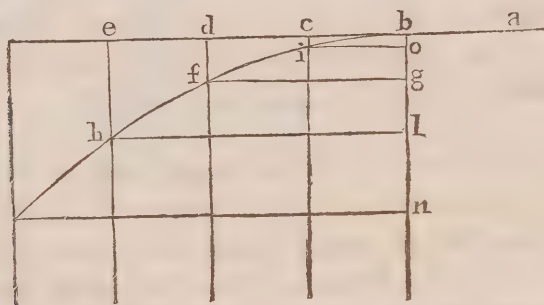
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the Square of the Whole, which contains four Squares of the Half, is greater than four Rectangles made of the unequal Parts.

Now 'tis necessary that we bear in Mind these two Propositions just now demonstrated, taken from the Conic Elements, in order to the better understanding what follows in this Tract, for of these two, and these only, the Author makes Use.

Now we may reassume the Text, and see in what Manner he demonstrates his first Proposition, by which he intends to prove, that the Line described by the moveable Body, whilst it descends with a Motion compounded of the equable horizontal, and of the naturally descending, is a *Semi-Parabola*.

Suppose the horizontal Line or Plane ab to be plac'd on high, upon or along which let the Moveable pass with an equable Motion out of a to b : imagine the Support of

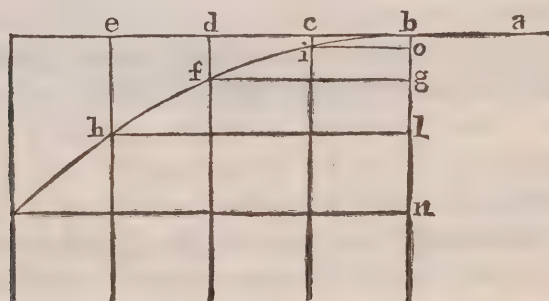


the Plane to be taken away in b ; suppose the Moveable to acquire by its own Gravity a natural Motion downwards, according to the Perpendicular bn . Imagine a right Line, as be , placed upon the Plane ab , representing the Flux, or
Measure

Measure of Time, upon which set off at Pleasure what Number of equal Parts you please, as bc , cd , de . And from the Points b , c , d , e , imagine Lines drawn parallel to the Perpendicular bn : In the first of which take any Part ci , whose Quadruple set off in the next df nine times, which set off from e to h , and so on in the rest, according to the Proportion of the Squares of cb , db , eb , or, which is the same Thing, let the Lines set off be in a duplicate Proportion of the Lines or Times cb , db , eb , &c. And if unto the Moveable moving beyond b towards c with an equable Lation, we imagine a perpendicular Descent at the same Time to be added, according to the Quantity ci . In the Time bc the Moveable will be found in the Point or Term i . And proceeding farther, in the Time db , viz. in double the Time bc the Line of Descent shall be quadruple the first Line ci : For it has been demonstrated in Prop. 2. of the last Dialogue, that the Spaces passed over by a heavy Body, with a Motion naturally accelerate, are in a duplicate Proportion of the Times. It consequently follows, that the Space eb passed over in the Time be , will be nine times ci ; so that it manifestly appears, that the Spaces eb , df , ci are one to another as the Squares of the Lines eb , db , cb . Now from the Points i , f , h , draw the right Lines io , fg , hl , $\parallel eb$; then the Lines hl , fg , and io will be = the Lines eb , db , and cb , each to each; as also bc , bg , and bi , to ci , df , and eb . And the Square of hl will be to the Square of fg , as the Line lb to bg ; and the Square of fg to the Square of io , as gb to bo ; therefore the Points i , f , and h , are in one and the same parabolick Line. And in like manner it may be demonstrated, that taking any equal Particles of Time, of

what Magnitude you will, that the Places of the Moveable carried with the like compound Motion, in the same Times are found in the same parabolick Line; therefore the Proposition is manifest.

SALV. This Conclusion is gathered from the Converse of the former of the two foregoing Propositions: Because the *Parabola* being, for Example, described thro' the Points *b* and *h*, if one of the two *f* and *i* was not in the describ'd



parabolick Line, it must be either without it or within it, and consequently the Line *fg* would be either lesser or greater than that which should terminate in the parabolick Line: And therefore the Square of *hl* would have, not to the Square *fg*, but to another greater or lesser, the same Proportion as the Line *lb* has to the Line *bg*: But it hath the same Proportion to the Square of *fg*, therefore the Point *f* is in the parabolick Line: And so of all the Rest.

SAGR. I can't deny but the Discourse is new, subtile, and conclusive, supposing the transverse Motion to continue always equable, and that the natural Motion downwards do likewise keep its Tenor, and is continually accelerate, in a duplicate Proportion of the Times ; and that those Motions and their Velocities mix'd with one another do not alter, disturb, or hinder one another ; so that the Line of the *Project* in the Continuation of the Motion degenerates into some other kind : Which indeed seems to me to be impossible ; because the Axis of our *Parabola*, which we say a grave Body describes by its Motion, being perpendicular to the Horizon, terminates in the Center of the Earth : Since the parabolic Line doth continually depart from its Axis, no Project could so go on as ever to terminate in the Center ; or if it should tend thither, as it seems necessary it must, the Line of the Project must degenerate into another most different from that of a *Parabola*.

SIMP. To this Difficulty I add others also : One of which is this, that we suppose the horizontal Plane, which has neither Acclivity nor Declivity, to be a right Line ; as if such a Line was in all its Parts equally distant from the Earth's Center ; which is not true ; for by being produced from its Middle towards the Extremes, it constantly more and more recedes from the Center, and therefore continually ascends ; the Impossibility, which consequently follows, is, that its Motion cannot be perpetual ; nay, that it does not so much as remain equable thro' any Space, but continually grows weaker and weaker : Besides, as I imagine, it is impossible

impossible to avoid the Impediment of the *Medium*, but that it will destroy the Equability of the transverse Motion, and the Law of Acceleration in falling Bodies. All which Difficulties render it highly improbable, that Things demonstrated from such uncertain Suppositions, should afterwards hold true in Practice.

SALV. All these alledged Instances and Difficulties are so well grounded, that I think it impossible to remove them : And, for my Part, I admit them all, in the manner I believe our Author would have admitted them. And I grant, that Conclusions thus demonstrated in the Abstract are alter'd, and so far disclose their Falsity in the Concrete, that neither is a transverse Motion equable, nor is the Acceleration of a Body naturally descending, made according to the supposed Proportion, nor is the Line of a Project parabolical, &c. But then again I desire, that you will not dispute with our Author, for supposing that which other famous Men have supposed ; altho' it be not strictly true : I'll mention but one of these Men, and that is *Archimedes*, who, in his *Mechanicks*, and first Quadrature of the *Parabola* assumes it as a true Principle that the Iron Rod of the Ballance, or of the * *Statéra Romana*, is a right Line equally distant in all its Points from the Center of the Earth ; and that the Lines whereby the Weights are hung, are parallel to one another. Which Liberty, indeed, these Men excuse, saying, that the Instruments we use in Practice, and the Distances we take, are so small in Comparison of our great Distance from the Earth's Center,

* Steel-yard.

we may very well be allow'd to take one Minute of a Degree of a great Circle for a right Line, and also two Perpendiculars hanging at its Ends to be parallel Lines : And if that in practical Operations there was any Regard to be had to such very minute Quantities, the Architects, before all others, ought to be blam'd, who by Help of the Plumb-Line pretend to erect the highest Buildings between parallel Lines. To these I add, that we may affirm, that *Archimedes* and others have in their Contemplations supposed themselves to be removed to an infinite Distance from the Center ; in which Cases their Assumptions were just, and their Conclusions absolutely true.

Now if we would reduce to Practice Conclusions demonstrated at a limited but immense Distance, from the Truth of such Conclusions must be abated what is sufficient, on Account of our Distance from the Earth's Center not being really infinite ; but indeed this Distance is such, as may be said to be infinite, in respect of the Littleness of our practical Artifices, the greatest of which are the Paths or Ranges of Projects, especially those that are thrown out of Ordnance, which, be it of what Size soever, will not throw a Ball four of such Measures, as we are distant from the Center of the Earth near four thousand : Wherefore since this Path terminates on the Superficies of the Earth, the parabolic Form cannot be sensibly alter'd ; which we allow would be greatly trasform'd, if it should terminate in the Earth's Center.

Again, as to the Irregularity proceeding from the Impediment of the *Medium*, this, we grant, is more considerable, and by Reason of its so manifold Varieties can't possibly be reduced to certain Laws : For if we should only
consider

consider the Impediment which the Air causes in such Motions as we have throughly examin'd into, we should plainly find it to disorder them all, and that after infinite Ways, according as the Figures, Gravities and Velocities of the Moveables are infinitely varied. For as to the Velocity, the greater it is, the greater will be the Impediment it shall meet with from the Air; and also the lighter the Moveables are, the more will they be hindered by the Air; so that tho' a falling heavy Body ought to be continually accelerated in a duplicate *Ratio* of the Duration of its Motion, however great the Moveable be, and let it fall from never so great a Height, yet such will be the Impediment of the Air, that the Body will be deprived of any further Increase of its Velocity, and reduc'd to an uniform and equable Motion; which Equability will be so much the sooner obtained, and in lesser Heights, by how much the Moveable is the higher.

Nay, that Motion also, which in an horizontal Plane ought continually to be equable, all other Obstacles being removed, shall, by the Impediment of the Air be altered, and at length quite stopped, and that the sooner by how much the Moveable Body is the lighter: And inasmuch as these Accidents of Gravities and Figures are subject to infinite Mutations, we can come at no certain Knowledge concerning them: Wherefore to treat of this Matter scientifically, 'tis requisite to abstract from them; and having found and demonstrated the Conclusions, abstracted from all Impediments, to make Use of them in Practice, with such Limitations as Experience shall teach and direct us: Nor shall these Conclusions so abstracted from, be of small Advantage; because we'll chuse Bodies and their Figures,

which

which are less subject to the Impediments of the *Medium*, such are the very heavy and round ones ; and their Spaces and Velocities at the greatest shall not be so great, but that their Exorbitances may, by an easy Diminution, be reduced to our Purpose. But among those Projects, which we make Use of, if they are of an heavy Matter, and of a round Form ; nay, if they are of a lighter Matter, and have a cylindric Form, such as are Arrows thrown from Bows, their Track or Path will not sensibly deviate from the Curve of a *Parabola*.

Nay, (to go further) among those Artifices which we make Use of in our Practice, I can prove, by two Experiments, that their Exiguity renders their external and accidental Impediments very inconsiderable, of which that of the *Medium* is of the greatest Moment ; considering the Motions to be made in the Air, since such are chiefly the Subject of our Discourse ; against which the Air exerts its Power two Ways ; first in obstructing the lighter Moveables more than the heavier ; and in resisting the greater more than the lesser Velocity of one and the same Moveable.

As to the first, we may learn by Experiment, that two Balls, equal in Magnitude, but one ten or twelve times heavier than the other ; as for Instance, one of Lead, and the other of Oak, would by falling from an Height of 150 or 200 Yards, arrive at the Earth with very little Difference of Velocity ; from whence we are plainly convinced, that the Impediment and Retardation of the Air is but very small in both ; if the Leaden Ball falling from the same Height, and at the same Moment of Time with the Wooden one, should be but little retarded, and the Wooden one much ; in such Case the Leaden Ball, at its coming to the Ground, would leave the Wooden one a considerable

ble Distance behind it, because the Leaden one is ten Times heavier ; which, nevertheless, does not happen, for it outgoes it not indeed above an hundredth Part of the whole Height. And between a Ball of Lead and another of Stone, the Weight of this being one third, or one half that, the Difference of Time wherein they come to the Ground would be scarce observable. Now because the *Impetus*, which the Leaden Ball acquires, whilst it is falling from an Height of 200 Yards (which is so much as, continuing on with an equable Motion, it would in the same Time fall 400 Yards) is very considerable in respect of the Velocities, which we give to our Projects by Bows or other Machines (that *Impetus* which is given by Fire excepted); we may, without any considerable Error, conclude those Propositions to be absolutely true, which are demonstrated without any Consideration of the Alteration of the *Medium*.

But now, Secondly, we are to shew, that the Impediment which the said Moveable receives from the Air, whilst it moves swiftly, is not much greater than that which the same Body receives, when it moves slowly : And of this we are very well assured by the following Experiment : Let two equal Balls of Lead be hung to two Threads of equal Length, *viz.* each four or five Yards long ; and these two Threads being fasten'd on high, let each Ball be removed from its perpendicular Situation, the one to 80 Degrees Distance or more, the other not to more than four or five ; so that one being let go may descend, and passing the perpendicular Line, describe very large Arches of 160, 150, 140, &c. Degrees, diminishing them by little and little ; but that the other playing freely may describe small Arches of 10, 8, 6, &c. Degrees, diminishing them

in like manner by little and little. Here, first, I assert, that the first Ball will pass over its 160° , 150° , &c. in the same Time that the other passes over its 10° , 8° , &c. Whence it becomes manifest, that the Velocity of the first Ball will be 16, and above 18, &c. times greater than the Velocity of the second; and in case the greater Velocity were to be more impeded by the Air than the lesser, the Vibrations ought to be fewer in the greater Arches of 160 or 150 Degrees, &c. than in the lesser of 10 , 8 , 4 ; nay even of 2 and of 1 . But Experience contradicts this: For if two Persons shall count one the greater Vibrations, the other the lesser, they'll see that in counting of not only ten, but a hundred of them, there will not be the Difference of one single Vibration, nay, not of the least Part of one. And this Experiment confirms to us two Propositions at the same Time, namely, that the greatest and least single Vibrations are made in the same Time: And that the Impediment and Retardation of the Air is not more in the swiftest Motions than in the slowest; contrary to that which before it seem'd that we ourselves would in common have judged.

SAGR. Rather, because it can't be denied but that the Air impedes both those swiftest and slowest Motions, since they both continually grow more languid, and at last cease moving, we should say that these Retardations are made in the same Proportion in both Cases. But how? Whence proceeds its making at one Time a greater Resistance, at another, less, but that it at one Time is assaulted with a greater *Impetus* and Velocity, and at another Time less? And if this be so, the same Quantity of the Velocity of the Moveable is both the Cause, and also the Measure of

the Quantity of the Resistance. Therefore all Motions, be they slow or swift, are retarded and impeded in the same Proportion ; a Notion, in my Judgment, not contemptible.

SALV. We may therefore farther, in this second Case, conclude, that the Fallacies in those Conclusions which are demonstrated, abstracting from outward Accidents, are in our Machines but of little Moment, in respect of Motions made with great Velocity, of which we are chiefly speaking ; and of the Distances, which are but very small in respect of the Semidiameter of a great Circle of the Terrestrial Globe.

SIMP. I would gladly hear, why you separate the Projects thrown by the *Impetus* of the first, *i. e.* as I conceive by the Force of the Gunpowder ; from those which are thrown from Slings, Bows, or Cross-Bows, as touching their not being after the same Manner subject to the Alteration and Impediment of the Air.

SALV. I am induc'd thereto by the excessive, and, as I may say, supernatural Violence, with which such Projects are driven out. And sure, without speaking hyperbolically, that Velocity with which a Ball is flung from a Musket or Piece of Ordnance, I may call supernatural, inasmuch as that if such a Ball descend naturally in the Air from some great Height, its Velocity, on Account of the Air's Resistance, will not increase continually ; but what we see happen to light Bodies falling no very great Way, I mean, their being reduced in the End to an equable Motion, will also happen after thousands of Yards Descent to

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an Iron or a Leaden Ball : And this determinate and ultimate Velocity, may be called the greatest which such a heavy Body can naturally obtain in the Air : But this Velocity I imagine to be much less than that which is given to the same Ball flung by fired Powder : As this may be proved by the following Experiment : From an hundred Yards high, or more, fire a Musket, loaden with a Leaden Bullet, perpendicularly down against a Stone Pavement ; and fire the same Musket, with the like Charge, against such another Stone, at a Yard or two's Distance ; and then observe which of these two Bullets is most flattened. Now if that fired from on high be found less flattened than the other, we must conclude, that the Air has hinder'd and diminished the Velocity given it by the Force of the Powder at the Beginning of its Motion : And consequently, that the Air would not permit it to acquire so great a Velocity, let it come from what Height it will : For if the Velocity impress'd upon it by the Powder, did not exceed that which it could acquire of itself by a natural Descent, the Blow downwards, after such natural Descent, ought rather to be more valid than less. I have not indeed made this Experiment ; but am inclin'd to believe, that a Cannon or Musket Ball, falling from never so great an Height, will not give so great a Stroke as such another Ball gives to a Wall when fir'd at a few Yards Distance, *i. e.* so few that the short Perforation or Scissure which it makes in the Air, is not sufficient to obviate the Excess of the supernatural Impetuosity impressed on it by the Powder. This excessive *Impetus* of Projects, thrown with such Violence, may cause some Irregularity in the Path of the Project ; by making the parabolic Lines less inclin'd,

or

or curv'd at its Beginning than at its End ; but this can be of little or no Prejudice to our Author in practical Operations : Amongst which the Chief is the Composition of a Table of Ranges, which contains the Distances Balls are flung to at every different Elevation ; and because such Projections or Shots are made with Mortars, by Help of a small Quantity of Powder ; in these, the *Impetus* not being supernatural, the Projects describe their Paths exactly enough.

But let us now go on with the Treatise, where the Author endeavours to lead us to the Contemplation and Investigation of the *Impetus* of the Moveable, whilst it is mov'd with a Motion compounded of two Motions : And first of that compounded of two equable Motions ; the one horizontal and the other perpendicular.

PROP. II. THEOR. II.

If a Moveable be moved with a twofold equable Motion, *i. e.* an horizontal and a perpendicular one, the *Impetus* or *Momentum* of the Lation compounded of both Motions, shall be in *Power* both the *Momenta* of the former Motions.

For let any Moveable be moved equably with a twofold Lation, and let ab represent the perpendicular one,



and let bc answer to the horizontal one made in the same Time. Forasmuch then as the Spaces ab and bc are passed over by equable Motions in the same Time, their

Momenta
†

Momenta shall be to each other as the said ab and bc . But the Moveable which is carried by a Motion compounded of these two different Motions, describes the Diagonal ac , and the *Momentum* of its Velocity shall be as ac ; but ac in *Power* is $= ab$ and bc , therefore the *Momentum* compounded of both the *Momenta* ab and bc is in *Power* $=$ them both taken together. Q. E. D.

SIMP. I beg you'll remove one Scruple I have as to this Matter; 'tis this, This present Conclusion seems to me to contradict another in the former Treatise; where it is affirmed, that the * *Impetus* of the Moveable coming from a to b is $=$ that coming from a to c ; and now you conclude, that the *Impetus* in c is greater than that in b .

SALV. The Propositions, *Simplicius*, are both true, but very different one to another. Here we speak of one sole Moveable, mov'd with one only Motion, but compounded of two, and those both equable; but there two Moveables were spoken of, mov'd with Motions naturally accelerated, one along the Perpendicular ab , and the other along the inclin'd Plane ac : And, besides, the Times there † are not supposed to be equal, but the Time along the inclin'd Plane ac to be greater than the Time along the Perpendicular ab : But in the Motion we are now speaking of, the Motions thro' ab , bc , ac , are supposed equable, and made in the very same Time.

SIMP. Excuse me; pray go on, I am satisfy'd.

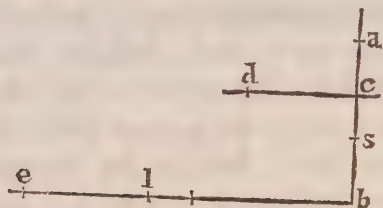
* See *Postulat.* before Prop. I. of Dial. III.

† Dial. III. Prop. III.

SALV. The Author proceeds to shew us what happens concerning the *Impetus* of any Moveable, which is mov'd, in like manner, by a Motion compounded of two, viz. the one horizontal and equable, the other perpendicular, but naturally accelerated, of which, in fine, the Motion of the Project is compounded, and thence the parabolic Line is described : In every Point of which he endeavours to determine, what the *Impetus* of the Project is : For our Understanding whereof, the Author shews us the Manner or Method of regulating and measuring such *Impetus* upon that Line along which the Motion of a Body departing from Rest, and falling with a Motion naturally accelerate, is made, as follows.

PROP. III. THEOR. III.

Let a Motion be made along the Line ab from Rest in a , and take in it any Point as c ; and suppose ac to be



the Time or the Measure of the Time of the said Fall thro' the Space ac , as also the Measure of the *Impetus* or *Momentum* in the Point c acquired by the Fall thro' ac : Now take in the same Line ab , any other Point,

Point, as b . In which we are to determine concerning the *Impetus* acquired by the Moveable by its Fall ab , in Proportion to the *Impetus* which it had in c , whose Measure is supposed to be ac .

Let as be a mean Proportional between ba and ac . Now we'll demonstrate that the *Impetus* in b to the *Impetus* in c is as the Line sa to ac . Let the horizontal Line cd be double ac , and be the double of ab : Then, from what has been before proved, the Cadent through ac turning along the horizontal Line cd , and according to the *Impetus* acquired in c , carry'd with an equable Motion, passes the Space cd in the same Time as it passed thro' ac , with an accelerate Motion; and in like Manner be is passed in the same Time as ab . But the Time of the Fall thro' ab is as , therefore the horizontal Line be is passed in the Time as .

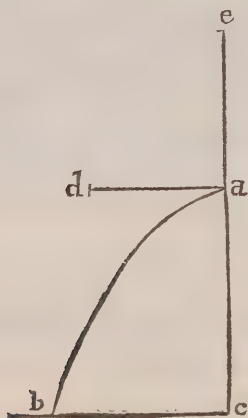
Now as the Time sa is to the Time ac , so let eb be to bl : And since the Motion thro' be is equable, the Space bl shall be passed in the Time ac , according to the *Momentum* of Velocity in b . But in the same Time ac the Space cd is passed according to the *Momentum* of Velocity in c , but the *Momenta* of Velocity are one to another as the Spaces which, according to those *Momenta* are passed in the same Time; therefore the *Momentum* of Velocity in c is to the *Momentum* of Velocity in b as dc is to bl . But because as dc is to be , so are their halves, viz. ca and ab ; but as eb is to bl so is ba to as , therefore, *ex æquali*, as $dc : bl :: ca : as$, that is, as the *Momentum* of Velocity in c is to the *Momentum* of Velocity in b , so is ca to as ; i. e. the Time along ca to the Time along ab . The Manner therefore of measuring the *Impetus* or

the *Momentum* of Velocity upon a Line wherein the Motion of Descent is made, is manifest; which *Impetus* is indeed supposed to encrease according to the Proportion of the Time.

But here, before we proceed, we must pre-advise, that since hereafter we are to speak of a Motion compounded of an equable horizontal one, and of a naturally accelerate one downwards (for from this Composition results, and is made the Line of the *Project*, that is a *Parabola*;) it is necessary that we define some common Measure, according to which we may measure the Velocity, *Impetus*, or *Momentum* of both Motions.

And since the Degrees of Velocity of the equable Motion are innumerable, of which not any one at Pleasure may be taken; but one certain one only is to be compared and conjoined to the Degree of Velocity acquired by the Motion naturally accelerate; I can think of no easier Way for that Choice and Determination, than by assuming another of the same Kind. But to explain myself the better; Let ac be perpendicular to the horizontal Line cb ; let ac be the Height, and cb the Amplitude of the *Semi-parabola* ab , which is describ'd by the Composition of the two Motions, one of which is that of the Moveable falling thro' ac , by a Motion naturally accelerate from Rest in a ; the other is the equable transverse Motion along the horizontal Line ad . The *Impetus* acquired in e , by the Descent ac , is determined by the Quantity of the said Height ac ; for the *Impetus* of a Moveable falling from the same Height, is always one and the same: but in the horizontal Line, not only one, but innumerable Degrees of Velocities of equable Motions may

may be assigned, out of which Number, that I may single or point out that for my Purpose, I extend the Altitude $c a$ upwards, as was shewn before, and as is necessary: I'll settle the Height $a e$; from which if I conceive in my Mind a Moveable to fall *ex quiete* in e , it will appear,



that its *Impetus* acquired at the Point a , is the same where-with I conceive the same Moveable turning along the horizontal Line $a d$ to be carried; and its Degree of Velocity to be that, with which, in the Time of the Descent thro' $e a$, it will pass a Space in the horizontal Line double to the said $e a$. This Premonition I judg'd necessary.

You must know also, that I call the horizontal Line $c b$ the Amplitude of the *Parabola*; the Axis the Altitude of the said *Parabola*; and the Line $e a$, by whose Descent the horizontal *Impetus* is determined, I call the Sublimity. These Things declar'd and defin'd, I proceed to the Demonstration.

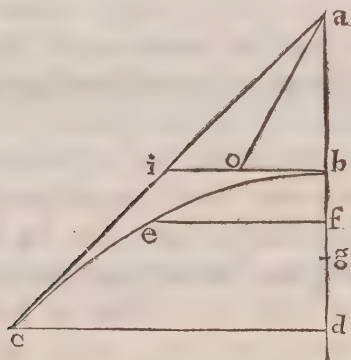
SAGR. First give me leave to take Notice, that this Thought of our Author's is like that of *Plato's*, concerning the Determination of the different Velocities of the equable Motions, in the Revolutions of the different Celestial Bodies; who, peradventure, thinking that no Moveable could pass from Rest to any determinate Degree of Velocity, in which it ought afterwards perpetually and equably to move, unless it first pass thro' all the other lesser Degrees of Velocities, or, if you will, greater Degrees of Tardity, which are between the assign'd Degree of Velocity, and the highest Degree of Tardity, that is, *Rest*; said, that God, after having created the moveable Celestial Bodies, in order to assign them those Velocities wherewith they were afterwards perpetually to move with an equable circular Motion, made them, departing from Rest, to move thro' determinate Spaces with that natural Motion, and in a right Line, according to which we sensibly perceive our Moveables to move from a State of Rest successively accelerating; and adds, that having made them acquire that Degree of Velocity which it pleased him, since they were afterwards perpetually to preserve it, he turn'd their direct Motion into a circular one; the only one fit to keep equable, always revolving, without receding from or approaching to any prefix'd Term by them sought. This Conceit is truly worthy of *Plato*; and is the more to be praised, in that the Foundations or Grounds by *Plato* silently passed by, and discover'd by our Author, taking off the Mask or Poetic Representation, do clearly shew it as a true Account. And I can't but believe, since by the Doctrine of Astronomy we have a competent
Knowledge

Knowledge of the Magnitudes of the Orbs of the Planets, and of their Distances from the Centre about which they turn, as also of their Velocities ; that our Author (who was no Stranger to *Plato's* Conjecture) out of Curiosity, may sometimes have had some Thought of attempting to investigate, whether any determinate Height could be assigned, from which, as from a State of Rest, the Bodies of the Planets, departing and moving thro' certain Spaces with a right and naturally accelerate Motion, the acquir'd Velocity afterwards being chang'd into an equable one, they would be found to answer to the Magnitudes of their Orbs, and the Times of their Revolutions.

SALV. I now remember he has told me, that he made the Computation, and found it answer the Observations with sufficient Exactness ; but that he had no Mind to publish his Thoughts, fearing that the too many Novelties discover'd by him, by which he had already drawn the *Odium* of many upon him, might raise a new Flame. But if any one have the like Desire, he may of himself, with the Instructions the present Tract will afford him, be fully satisfied. But to return to our Business, which is to demonstrate

PROP. IV. PROB. I.

How to determine the *Impetus* in every Point of a given *parabolic* Curve describ'd by a *Project*. Let *bec* be a *Semi-parabola*, whose Amplitude is *cd*, and Altitude *db*, which produced upwards, meets the Tangent of the *Parabola ca* in *a*; Thro' the *Vertex b*, draw *bi* parallel to



the horizontal Line *cd*. Now if the Amplitude *cd* be = the whole Height *da*, *bi* shall be = *ba*, as also to *bd*. And if we put *ab* for the Measure of the Time of the Fall thro' *ab*, and of the *Momentum* of Velocity acquir'd in *b*, by the Descent *ab* from Rest in *a*, then *dc* (*viz.* the Double of *bi*) will be the Space which it shall pass in the same Time, by the *Impetus ab*, when turned along the horizontal Line *bi*: But a Cadent thro' *bd*, from Rest in *b*, passes the Depth *bd*; in the same Time, therefore, a Moveable falling from Rest in *a*, thro' *ab*, with the *Impetus ab* being turned along the horizontal Parallel *bi*, shall

shall pass a Space $= dc$; and the Fall along bd supervening, it passes the Depth bd by these two Motions, one horizontal the Distance of cd , the other perpendicular the Distance bd , in the same Time is describ'd the *Parabola bc*, whose *Impetus* in the Term c is compounded of the equable Transverse, whose *Momentum* is ab , and of another *Momentum* acquired in the Fall bd , in the Term d or c , which *Momenta* are equal. If therefore we suppose ab to be the Measure of one of them, as for Instance, of the transverse equable one; and bi , which is $= bd$, to be the Measure of the *Impetus* acquired in d or c ; then the Subtense ia will be the Quantity of the *Momentum* compounded of both: It therefore will be the Quantity or Measure of the whole *Momentum* which the Project, describing the *Parabola bc*, strikes with in c . These Things being premised, take in the *Parabola* any Point e , in which we are to determine the *Impetus* of the Project. Draw the horizontal Line ef , and take bg a mean Proportional between bd and bf : Then since ab or bd is supposed to be the Measure of Time, and of the *Momentum* of Velocity in the Fall bd , from Rest in b , bg will be the Time or Measure of the Time, and of the *Impetus* in f , coming from b . If therefore bo be supposed $= bg$; then a Line drawn from a to o will be the Quantity of the *Impetus* in the Point e ; for ab is the supposed Determiner of the Time, and of the *Impetus* in b , which turn'd along the horizontal Parallel, always continues the same: But bo determines the *Impetus* in f or e thro' the Descent bf , from Rest in b ; but ao is in Power $=$ these two ab and bo : Therefore what was sought is manifest.

SAGR. The

SAGR. The Contemplation of the Composition of these different Forces, and of the Quantity of that *Impetus* which results from this Mixture, is so new to me, that I'm confounded. I don't speak of the Mixture of two equable Motions, tho' unequal to one another, one of which is made along the horizontal Line, and the other along the perpendicular one; for I easily perceive that thence there arises a Motion, equal in Power to both the compounding Motions: But my Difficulty arises in the Mixture of the horizontal, equable, and perpendicular naturally accelerate Motion: Wherefore I could wish we might together consider this Matter a little more fully.

SIMP. I am much more at a Loss than you are, for I'm not yet wholly satisfied concerning the Fundamental Propositions on which the others are built; what I mean is, I want better to understand the Conclusion of the second Proposition, *viz.* what, in the Mixture of two equable Motions, the one horizontal, the other perpendicular, is meant by the *Potentia* of their Compound.

SALV. Your Request is very reasonable, and I'll endeavour to explain this to your Satisfaction; but you must excuse my repeating several Things already said. And to discourse determinately about Motions, and their Velocities or *Impetus's*, whether they be equable or naturally accelerate, we must first pitch upon a Measure to be used in the Dimensions of such Velocities; as also a Measure of the Time. As to the Measure of the Time, we have what all agree in, I mean Hours, Minutes,
Seconds,

Seconds, &c. Now as this Measure is received in common by all, so we must allot one to the Velocities, that is commonly understood and received by all, *i. e.* which is the very same every where. The Author, as has been said, thought the Velocity of heavy Bodies naturally descending fit for this Purpose, whose increasing Velocities observe the same Laws in all Parts of the World : So that that Degree of Velocity, which (*e. g.*) a Leaden Ball of one Pound Weight departing from Rest, hath acquired in a Descent perpendicularly a certain Distance, is always and in all Places the same, and therefore most commodious to explain the Quantity of the *Impetus* arising from a natural Descent : It remains then, that we find out a Manner of determining also the Quantity of the *Impetus* in an equable Motion, and that in such a Manner, that all they who reason about it may form the very same Conception of both its Magnitude and Velocity ; and that one mayn't imagine it swifter, and the other slower, which may occasion afterwards, in compounding and mingling this equable Motion, imagin'd by them, with the establish'd accelerate Motion, that different Men may form different Conceptions of different Greatnesses of Forces.

To determine and exhibit this *Impetus* and particular Velocity, our Author thought it the best way to make Use of the *Impetus* which the Moveable acquires, in a Motion naturally accelerate, whose every acquired *Momentum*, turn'd into an equable Motion retaining its Velocity exactly limited, and such, that in an equal Time to that wherein it did descend, it passes thro' double the Space of the Height from which it fell : But because this is the main Article in the Business we are upon, it may be

worth while, that you may see it the clearer, to explain it by some particular Example.

Re-assuming therefore the Velocity and *Impetus* which a heavy Body, as we have said, hath acquired falling a certain Distance; which Velocity we will make use of for a Measure of other Velocities and *Impetus*'s upon other Occasions; and supposing, for Example, the Time of its Fall to be four Seconds: Now to find, from this Measure, what is the *Impetus* of the falling Body from any greater or less Height, we must not argue and conclude the Quantity of the *Impetus* acquired in falling from this second Height, from the *Ratio* the former Height had to the stated Distance, *i. e.* we must not suppose that a Moveable falling from four times that Height, must have acquired four times the Velocity; for this is false, because in a Motion naturally accelerate, the Velocity does not increase or diminish according to the *Ratio* of the Spaces; but according to that of the Times, than which that of the Spaces is greater in a duplicate Proportion, as was before demonstrated.

Wherefore if in a right Line we assign any Part for the Measure of the Velocity, and of the Time, as also of the Space run thro' in that Time, (for all these three Magnitudes, for Brevity's Sake, are frequently represented by one and the self-same Line) to find the Quantity of the Time and Degree of Velocity which the same Moveable hath acquir'd in another Distance, which we shall obtain not immediately from this second Distance, but from that Line which is a mean Proportional between those two Distances: An Example will make this clearer.

In the Line ac , perpendicular to the Horizon, suppose the Part ab to be the Space run thro' by a heavy Body naturally descending with an accelerate Motion ; the Time of which Passage, since I may represent it by any Line, for Brevity's Sake shall be represented by the same Line ab , which in like manner I take for the Measure of the *Impetus* and Velocity acquir'd by such Motion, so that the Part ab may be the Measure of all the Spaces that are in the Progress of the Discourse to be consider'd. Having now at Pleasure establish'd, under one sole Magnitude ab , the



Measures of three Quantities of very different Kinds, *i.e.* of Spaces, Times, and Forces ; let it be required to determine in the assign'd Space, and at the Height ac , what must be the Time of the Descent of the Moveable from a to c , and what the *Impetus* it shall have acquired at the Term c , in relation to the Time and to the *Impetus* which are measured by ab .

Both these Things required, will be determined by taking a mean Proportional ad , between the two Lines ac and ab , and affirming that the Time of the Fall thro' the whole Space ac , is such as is the Time ad , in relation to the Time ab ,
E c c 2
which

which at the Beginning was taken for the Quantity of the Time in the Descent ab : And likewise we will say that the *Impetus* or the Degree of Velocity which the falling Body will acquire at the Term c , in relation to the *Impetus* it had in b , is such as is the same Line ad , in relation to



ab , since the Velocity increases in the same *Ratio* the Time does ; of which Conclusion (altho' it was assumed as a *Postulatum*, yet) the Author was pleased to explain the Application above in Prop. III.

This Point being well understood and established, we pass to the Consideration of the *Impetus* arising from two compound Motions ; one whereof is compounded of a horizontal and always equable one, and of one perpendicular to the Horizon, and that also equable ; and the other compounded of an horizontal one, in like manner always equable, and of a perpendicular one naturally accelerate.

We have already seen, that when both are equable, that that which arises from their Composition, is equal in *Power* to both ; which, for Clearness Sake, we will give an Example of :

Suppose
†

Suppose a Moveable falling along the perpendicular Line ab , to have, *e. g.* three Degrees of equable *Impetus*; but if it was carried along the Line bc towards c , then the Velocity and *Impetus* to be four Degrees: So that in the same Time that falling it would pass along the Perpendicular, *v. g.* three Yards, it would in the Horizontal pass four; but in the Motion compounded of both, it will come in the same Time from the Point a to the Point c , proceeding always along the Diagonal ac , which is not seven Yards long, as it should be, if compounded of the two



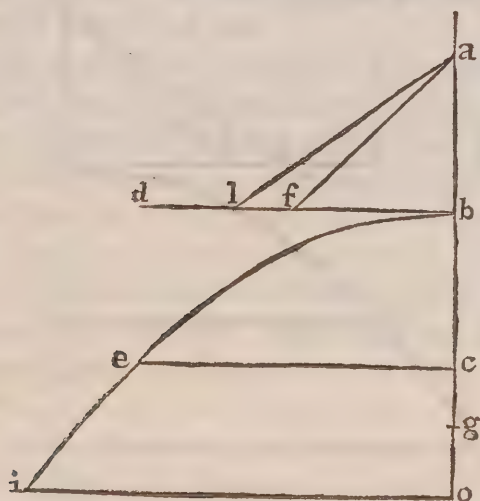
Lines ab , 3, and bc , 4; but in five Yards long, which in Power is = both 3 and 4; because the Squares of 3 and 4, which are 9 and 16, added together, make 25 = the Square of ac , which is = the two Squares of ab and bc : Whence ac shall be as much as is the Side, or, which is the same Thing, the Root of the Square 25, which is 5: For a constant and certain Rule therefore, when we would assign the *Impetus* resulting from two *Impetus*'s given, *viz.* the one Horizontal, and the other Perpendicular, but both equable, their Squares must be taken, and from their Sum the Square-Root must be extracted: And thus, in the proposed Example, that Moveable which by the Force of its perpendicular Motion strikes the Horizon with three Degrees of Power, and by its horizontal Motion shall strike

strike in *c* with four Degrees of Power. If now both Strokes are struck together, the Blow shall be like that made by a striking Moveable, which was mov'd with five Degrees of Velocity and Power : And this Percussion in all Points of the Diagonal would be the same, because the compound *Impetus*'s, without any Increase or Decrease, are always the same.

Now let us see what will happen in a Composition of an equable horizontal Motion, with a Motion perpendicular to the Horizon, which beginning its Fall from Rest, is constantly naturally accelerated : Now it is manifest that the Diagonal, which is the Line of Motion compounded of these two, is not a right Line, but a *Semi-parabolic* one, as was before demonstrated ; in which, because of the continual Increase of the Velocity of the perpendicular Motion, the *Impetus* continually increases : Wherefore to determine what the *Impetus* in any assign'd Point of that parabolic Diagonal is, we must first assign the Quantity of the uniform horizontal *Impetus*, and then investigate what is the *Impetus* of the falling Body in the Point assign'd ; which can't be determined without the Consideration of the Time spent from the Beginning of the Composition of those two Motions ; which Consideration of the Time is not required in the Composition of the equable Motions, whose Velocities and *Impetus*'s are always the same : But here, where is inserted into the Composition a Motion, which, taking its Beginning from the greatest Tardity, constantly increases its Velocity, according to the Continuation of the Time, the Quantity of the Time must of Necessity be taken in, to shew us the Quantity of the Degree of Velocity in the assign'd Point,
because

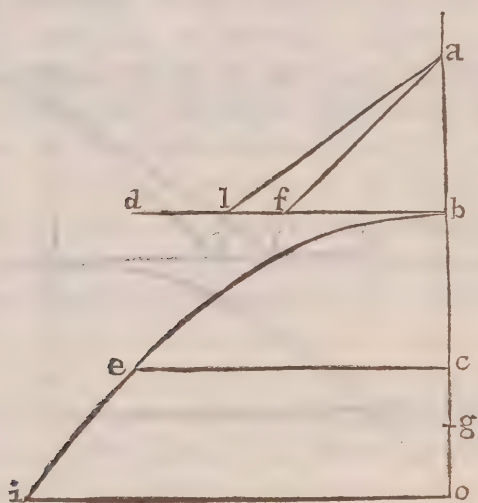
because as to the Rest, the *Impetus* compounded of these two, (as in uniform Motions) is equal in *Power* to both the others compounding.

But this also may be better explain'd by an Example : In a Line ac perpendicular to the Horizon, take any Part ab , which let represent the Measure of the Space run thro' with a natural Motion in that Perpendicular ; let the same Line ab represent likewise the Measure of the Time, as also of the Degree of Velocity, or rather of the *Impetus*.



First ; it is manifest that if the *Impetus* of the Moveable in b , from Rest in a , be turn'd along bd , parallel to the Horizon, with an equable Motion, the Quantity of the Velocity shall be such, that in the Time ab it shall pass a Space double to the Space ab , which let be the Line bd . Then supposing $bc = ba$, and drawing $ce \parallel bd$, and = it, thro'

thro' the Points b and e describe the parabolic Line be ; and because in the Time ab , with the *Impetus* ab , the horizontal Line bd or ce , double to ab , is run thro', and in another such Time the Perpendicular bc is passed with an Acquisition of *Impetus* in c = the said horizontal Line; Therefore the Moveable in a Time = ab , shall be carried from b to e , along the parabolic Line be , with an *Impetus* compounded of two, each of which is = the *Impetus* ab : And because one of them is horizontal, and the other



perpendicular, the compound *Impetus* shall be in *Power* = them both, *i. e.* shall be the Double of one of them.

Whence supposing $bf = ba$, and drawing the Diagonal af , the *Impetus* and Percussion in e of the Moveable falling from the Height a , shall be greater than the Percussion

cussion in b , or than the Percussion of the horizontal *Impetus* along the Line bd , according to the *Ratio* of af to ab .

But if still retaining ba for the Measure of the Space of the Fall from Rest in a unto b , and for the Measure of the Time and *Impetus* of the falling Body acquired in q , the Altitude bo should not be equal to, but greater than ba , take bg a mean Proportional between ab and bo ; this bg would be the Measure of the Time, and of the *Impetus* in o , which the Moveable falling thro' bo hath acquired in o ; and the Space along the horizontal Line, which is run thro' with the *Impetus* ab in the Time ab , would be the Double of ab .

Supposing then lb to be equal to bg , and drawing the Diagonal al , that will shew us the Quantity compounded of the two *Impetus*'s, the horizontal and perpendicular, by which the *Parabola* is described; of which the horizontal and equable is that acquired in b , by the Descent ab ; and the other is that acquired in o , or if you will in i , by the Descent bo , whose Time, as also the Quantity of its *Momentum*, was bg . And after the same Manner we may investigate the *Impetus* in the extreme Term of the *Parabola*, in case its Altitude were less than ab , by taking a mean Proportional between them both, and setting it off upon the horizontal Line from b to f , and drawing the Diagonal af , for this af gives us the *Impetus* in the extreme Term of the *Parabola*.

To what has been hitherto said concerning *Impetus*'s, Blows, or Percussions of these Projects, I must add one very necessary Consideration more, and that is, That it is not enough to have Regard to the Velocity only of *Projects*, in order justly to determine the Power and Force of

the Blow ; but we must also consider apart the State and Condition of the Subject which receives the Blow ; in the Efficacy of which on many Accounts it is greatly concerned.

And first ; there is no one but knows, that the Thing struck doth so much suffer by the Velocity of the Thing striking, by how much it opposes itself to it, or resists it, and either totally or partially stops its Motion : That if the Blow happen upon such a Thing as gives Way without any Resistance to the Velocity of the Thing striking ; such a Blow will be of no Moment : And in like Manner that if any one hastes to run his Enemy thro' with a Spear, if it happens that at the Moment the Spear touches him, he gives back with the same Swiftness the other pursues, there will be no Thrust, but the Action will be only a mere or simple Touch, without doing any Harm.

But if the Subject which receives the Blow be such as does not wholly, but partly only give Way to the Percutient, the Blow shall hurt, but not with its whole *Impetus*, but only with the Excess by which the Velocity of the Percutient exceeds the Velocity of the Retrogression and giving Way of the Subject that is struck : So that if, e. g. the Thing striking falls upon the Thing struck with ten Degrees of Velocity, and this gives Way with four Degrees ; the *Impetus* and Percussion shall be the same as if it had been made with six Degrees. And, lastly, the Percussion will be entire, and the greatest, on the Part of the Percutient, when the Thing struck gives no Way at all, but wholly opposes and stops the whole Motion of the Percutient, if such a Case can happen : I said *on the Part of the Percutient*, because when the Thing struck is

mov'd

†

mov'd with a contrary Motion towards the Percutient, the Blow and Shock at meeting shall be so much the greater, as those two contrary Velocities taken together are greater than the single Velocity of the Percutient.

'Tis necessary to remark further, that that greater or lesser Cession may arise, not only from the Quality of the Matter, more or less hard, as if it be of Iron, of Lead, or of Wooll, &c. but also from the Position of the Body which receives the Blow; if the Body be so placed that the Percutient hits it at right Angles, the Blow will be the greatest of all: but if the Percutient strikes it obliquely, the Blow will be weaker, and that still more and more weak, according as it is struck more and more obliquely; because an Object, altho' of the solideest Matter, so situate, doth not stop or destroy the whole *Impetus* and Motion of the Percutient, which slanting passes on after having continued its Motion along, at least, some Part of the Surface of the resisting opposed Body. When therefore, the Magnitude of the *Impetus* of the *Project* at the Extremity of the parabolic Line was above determined, it ought to be understood of the Percussion received upon a Line at right Angles to the parabolic one, or to its Tangent in the aforesaid Point: For altho' that Motion is compounded of an horizontal and a perpendicular one, yet the *Impetus* is not the greatest, either upon the horizontal Plane, or upon that perpendicular to the Horizon, being received upon them both obliquely.

SAGR. Your mentioning these Blows and Percussions puts me in mind of a Problem, or, if you will, Question in Mechanics, a Solution of which I have never met in

any Author, nor any thing to lessen my Admiration, or in the least satisfy my Judgment about it.—The Thing is this ; I cannot conceive whence it happens, and on what Principles that Energy and immense Force depends, which is found in Percussion, whilst with a simple Blow of a Hammer of not above eight or ten Pounds Weight, we see such Resistances overcome, as would not yield to the Weight of any heavy Body that, without Percussion, by only bearing upon it and pressing it, hath an *Impetus*, altho' this Body be heavier than that many hundred times. I would likewise find a Way to measure the Force of this Percussion, which I do not think to be infinite, but believe it is limited, and thence may be compared with, and at length brought under the Laws relating to other Powers of pressing Gravities, either of Leavers, or Screws, or other Mechanic Instruments, of the Multiplication of whose Force I am fully satisfied.

SALV. You are not the only one that does not see the Reason of this wonderful Effect and stupendous Accident. I consider'd of it for some Time, but in vain, my Confusion constantly increasing ; 'till at length meeting with our Academic, I received double Pleasure ; first in finding that he himself was a long Time at the same Loss ; and next in hearing him say, that after he had spent some thousands of Hours in studying and contemplating thereon, he had hit upon some Things very different from our first Conceptions relating to that Subject ; Things wholly new, and on that Account worthy our Esteem.

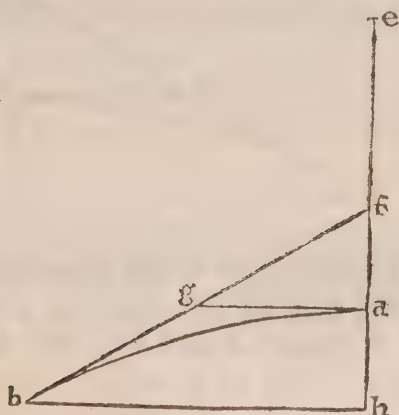
And

And because I already perceive your Curiosity is mov'd, to hear what these Things are, which are so far beyond what I could have imagined, I'll save you the Trouble of asking; and promise, after we have gone thro' this Tract concerning Projects, to explain to you all those pleasant but wonderful Things I can remember from the Discourses of our Academic. In the mean time let us go on with the Propositions of our Author.

PROP. V. PROB. II.

In the Axis of a given *Parabola* continued, to find a Point above, from which the Moveable falling, shall describe the said *Parabola*.

Let the *Parabola* be ab , whose Amplitude is hb , and its Axis continued he ; in which we are to find a Point



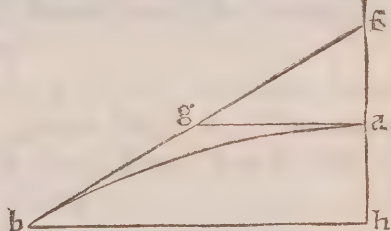
on high, from which the Moveable falling, and turning the *Impetus* gained in a , along the horizontal Line, may describe the *Parabola* ab .

Draw

Draw the horizontal Line ag , which shall be $\parallel bh$, and supposing $af = ab$, draw the right Line fb , which will touch the *Parabola* in b , and cut the horizontal Line ag in g ; to fa and ag take ae a third Proportional, i. e. let ae be made of such a Length as that ag may be a mean Proportional between it and af : Then I say, that e is the Point above sought, from which a Movable falling from Rest in e , and turning the *Impetus* gain'd in a along the horizontal Line, the *Impetus* of the Descent in b from Rest in a joining it, shall describe the *Parabola* ab .

Let of semicircle ab a given a be the Axis of the *Parabola* continued to find from which the Movable falling the

the *Parabola* in b shall describe the *Parabola* ab : in which we are to find a Point



For if we suppose ea to be the Measure of the Descent from e to a , and also of the *Impetus* acquired in a , ag (*viz.* a Mean between ea and af) will be its Time and *Impetus* coming from f to a , or from a to b : And because falling from e , in the Time ea , with the *Impetus* acquired in a , it moves in its horizontal Passage with an equable Motion the double of ea ; being carried, therefore, with the same *Impetus*, it will pass in the Time ag , the

the double the Mean ag , viz. bb : For the Spaces passed with the same equable Motion, are one to another as the Times of the said Motions; and along the Perpendicular, with a Motion from Rest in a , ab is passed in the same Time ga , therefore the Amplitude bb , and Altitude ba , are passed by the Moveable in the same Time; therefore the *Parabola* ab is described by the Descent of the falling Body from e , which was required.

COROLLARY.

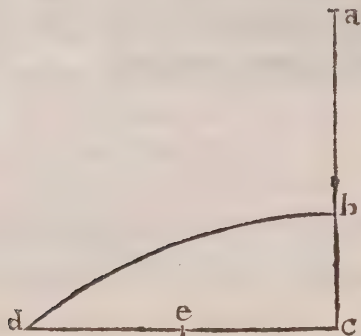
Hence it is manifest, that the half Base or Amplitude of the *Semi-parabola*, (which is the fourth Part of the Amplitude of the whole *Parabola*) is a mean Proportional between its Amplitude, and Sublimity; from which Sublimity the Moveable falling, describes that *Semi-parabola*.

PROP. VI. PROB. III.

The Sublimity and Altitude of a *Semi-parabola* being given, to find its Amplitude.

To

To the horizontal Line dc , let ac be perpendicular, in which let the Altitude cb be given, and the Sublimity ba : Now we are to find in the horizontal Line cd , the Amplitude of the *Semi-parabola*, which is described from the Sublimity ba , and the Altitude bc .



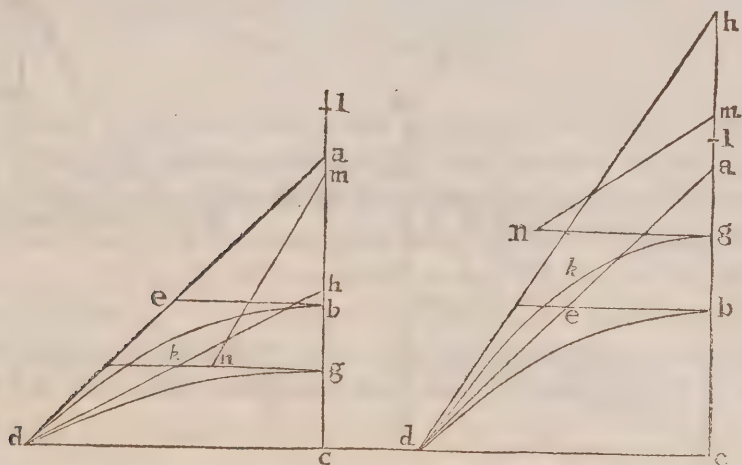
Take a mean Proportional between bc and ba , the Double of which Mean let be cd : Then I say, cd is the Amplitude sought. This is manifest from the foregoing Proposition.

PROP. VII. THEOR. IV.

In Projects by which *Semi-parabola's* of the same Amplitude are described; there is less *Impetus* required in that Project which describes that whose Amplitude is double its Altitude, than in any other.

Let

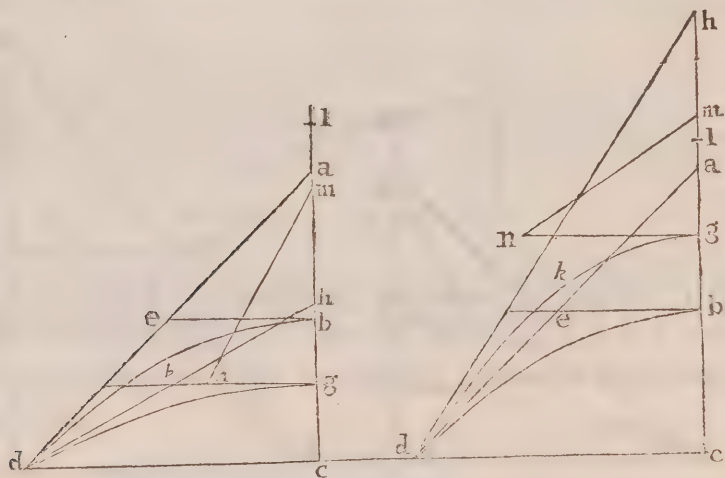
Let the *Semi-parabola* be bd , whose Amplitude cd is double its Altitude cb , and in its Axis continu'd upwards let ba be supposed = the Altitude be , and draw the Line ad , which will touch the *Semi-parabola* in d , and cut the horizontal Line be in e ; and be will be = bc , or to ba . It is manifest this *Semi-parabola* is describ'd by a Project, whose equable horizontal *Impetus* is such as is in b of a Body falling from Rest in a , and the *Impetus* of it



descending naturally downwards, such as is that of a Thing coming to c , from Rest in b . Whence it is manifest, that the *Impetus* compounded of these, and that strikes in the Term d , is as the Diagonal ae , i. e. in Power = them both. Now let there be some other *Semi-parabola*, as gd , whose Amplitude cd is the same as before, but its Altitude cg less, or greater than the Altitude bc , and let bd touch the same, cutting the horizontal Line drawn thro' g in the Point k , and make $hg : gk :: gk : gl$;
G g g then

then from what has been before demonstrated, gl will be the Sublimity, from which the Project falling shall describe the *Parabola* gd .

Let gm be a mean Proportional between ab and gl , gm shall be the Time and *Momentum* or *Impetus* in g , of the Body falling from l , (for it has been supposed that ab is the Measure of both Time and *Impetus*). Again, let gn be a mean Proportional between bc and cg , this



gn shall be the Measure of the Time and the *Impetus* of the Body falling from b to c . If then the Line mn be drawn, it shall be the Measure of the *Impetus* of the Project along the *Parabola* bd , striking in the Term d , which *Impetus*, I say, is greater than the *Impetus* of the Project along the *Parabola* bd , whose Quantity was as ae : For because gn is supposed a Mean between bc and cg , and $bc = be$, that is to bg : (for they are each of them Subduple of dc) it will be $cg : gn :: gn : gk$, and cg or bg :

$hg : gk :: ng_q : gk_q$. But as hg to gk , so gk is made to gl ; therefore $ng : gk_q :: gk : gl$: But $gk : gl :: gk_q : gm_q$; for gm is a Mean between gk and gl ; therefore the three Squares ng , gk , and gm , are continual Proportionals; and the two Extremes ng and gm taken together, *i.e.* the Square of mn , is greater than the double of the Square of gk , to which the Square of ae is double; therefore the Square of mn is greater than the Square of ae ; and the Line mn greater than the Line ae . Q.E.D.

COROLLARY.

Hence it appears, that, *vice versa*, in the Project out of d , along the *Semi-parabola* db , less *Impetus* is required than along any other of an Elevation greater or less than the Elevation of the *Semi-parabola* bd , which is according to the Tangent ad , making an half Right Angle above the Horizon: Which since it is so, it is manifest, that if Projections are made with the same *Impetus* from the Term d at different Elevations, the greatest Projection or Amplitude of the half or whole *Parabola*, will be that which is made at the Elevation of half a Right Angle or 45° and the rest made at greater or less Elevations will be less.

SAGR. The Force of necessary Demonstrations is extremely both wonderful and delightful, and such are Mathematical ones only: Before, I took it upon Trust, from the Accounts I had from several Gunners, that the greatest of all the Ranges made by a Cannon or Mortar, *i.e.* that where the Ball is flung farthest, is that which is made at

an Elevation of half a Right Angle, or of 45 Degrees, or, as they say, of the sixth Point of the Square: But the understanding the Reason of this, is far beyond the simple Knowledge drawn from the Testimony of others, or even from often repeated Experiments.

SALV. You say very true; to which I add, that the Knowledge of one single Effect acquir'd by its Causes, opens our Intellects, and helps us to understand other Effects, without the Necessity of having Recourse to Experiments: just as it happens in the present Case; where, after we were assur'd by Demonstration, that of all Ranges, that is the greatest which is made at the Elevation of 45° , the Author demonstrates to us, that which perhaps was never experienc'd: It is this; That of the other Ranges, those are equal among themselves whose Elevations exceed or fall short by equal Angles of 45° ; so that the Balls shot upon a Plane sufficiently large, one at the Elevation of seven Points above the Horizon, and the other of five, shall light upon the Ground at equal Distances from the Mortar that flung them: Thus the Ranges at the Elevations of 8 and 4 Points, of 9 and 3, &c. shall be equal. Take the Demonstration.

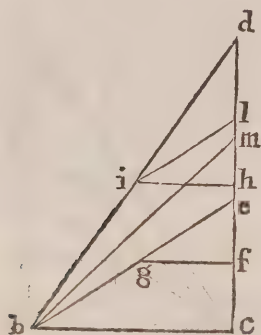
PROP. VIII. THEOR. V.

The Amplitudes of *Parabola's* describ'd by Projects flung with the same Force at Elevations distant by equal Angles above and below 45° , are = each other. Let the horizontal Line bc , and the Perpendicular cm , about the Right Angle c of $\triangle mcb$ be = each other, then will $\angle$

mbc

†

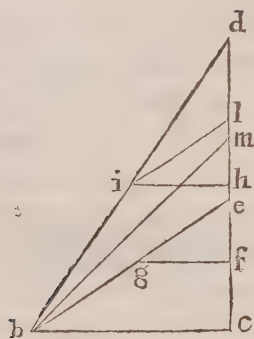
$m\bar{b}c$ be half a right one; and producing cm to d , make at the Point b two equal Angles, $m\bar{b}e$ and $m\bar{b}d$, above and below the Diagonal bm . 'Tis to be demonstrated, that the Amplitudes of the *Parabola's* describ'd by the Projects thrown with the same Force from the Term b , at



the Elevations of the Angles $e\bar{b}c$ and $d\bar{b}c$, are equal to each other. First then, because the external Angle bmc is = the internal ones $m\bar{b}d$ and dbm , $\angle m\bar{b}c$ will also be = them. But if instead of $\angle dbm$, we put $m\bar{b}e$, the same Angle $m\bar{b}c$ will be = the two, $m\bar{b}e$ and $b\bar{d}c$; and taking away the common Angle $m\bar{b}e$, the remaining one $b\bar{d}c$ will be = the remaining one $e\bar{b}c$: Therefore $\triangle s\ dcb$ and bce are similar: Bisect dc and ec equally in h and f , and draw hi and $fg \parallel$ the Horizon cb , and make $dh : bi :: bi : bl$, then $\triangle ihl$ will be similar to $\triangle ibd$, to which also is similar $\triangle egf$: And since ib and gf are equal (each being the half of bc) fe , i. e. fc , will be = bl ; adding the common Line fb , cb will be = fl . If therefore we imagine the *Semi-parabola* to be describ'd by or thro' b and b , whose Altitude is bc , and Sublimity hl , its

its Amplitude will be cb , which is double to bi , the Mean between db or cb and bl ; and db shall be a Tangent to it, the Lines cb and bd being equal.

And if again we conceive the *Parabola* to be describ'd by or thro' f and b , from the Sublimity fl , with the Alti-



tude fc , a mean Proportional between which two is fg , whose Double is the horizontal Line cb ; cb , as before, shall be its Amplitude, and eb a Tangent to it, since ef and fc are equal; but $\angle$'s dbc and ebc (their Elevations) are equally distant from a half Right Angle, or 45° . Therefore, &c. Q. E. D.

PROP. IX. THEOR. VI.

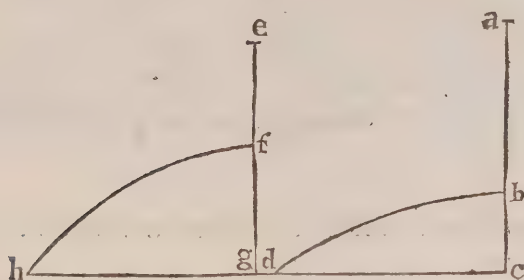
The Amplitudes of *Parabola's*, whose Altitudes and Sublimities are reciprocally proportional, are equal.

Let the Height gf of the *Parabola* fb have the same Proportion to the Height cb of the *Parabola* bd , as the Sublimity ba has to the Sublimity fe : Then I say, the Amplitude bg is = the Amplitude dc ; for since the first

gf

†

gf has the same *Ratio* to the second cb , as the third ba has to the fourth fe , the Rectangle gfe of the first and fourth will be $= \square cba$ of the second and third; therefore the Squares which are $=$ these Rectangles will be $=$ each other; but the Square of half gb is $= \square gfe$; and



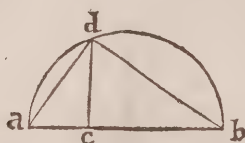
the Square of half cd is $= \square cba$; therefore these Squares, and their Sides, and the Doubles of their Sides: But these are the Amplitudes gb and cd ; therefore the Proposition is manifest.

LEMMA.

If a right Line be cut at Pleasure, the Squares of the Means between the whole and two Parts are $=$ the Square of the whole.

Let

Let ab be cut at Pleasure in c : I say the Squares of the Lines, which are mean Proportionals between the Whole ab , and the Parts ac and cb taken together, are equal to the Square of the Whole ab . Upon the whole Line ba , describe a Semicircle, and from c erect a Per-



pendicular cd , and draw the Lines ad and bd , and the Thing is plain.

For da is a mean Proportional between ab and ac , and db a Mean between ab and bc ; and the Squares of the Lines da and db taken together, are = the Square of the whole Line ab , $\angle adb$ in the Semicircle, being a right one : therefore, &c.

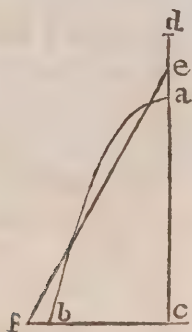
PROP. X. THEOR. VII.

The *Impetus* or *Momentum* of any *Semi-parabola* is equal to the *Momentum* of any Moveable falling naturally along such a Perpendicular to the Horizon, as is = the Line compounded of the Sublimity, and of the Altitude of the *Semi-parabola*.

Let

Let the *Semi-Parabola* be ab , whose Sublimity is da , and its Altitude ac ; by which two is made the Perpendicular dc ; then, I say, the *Impetus* of the *Semi-parabola* in b is = the *Momentum* of a Moveable descending naturally from d to e .

Suppose dc to be the Measure of both Time and *Impetus*; and take a mean Proportional between cd and da ; to which let ef be equal. Again, let ce be a Mean between dc and ca : Now fc shall be the Measure of the Time and *Momentum* of the Moveable falling thro' da ,



from Rest in d , but ce will be the Time and *Momentum* of the Moveable falling thro' ac , from Rest in a . And the Diagonal ef will be the *Momentum* compounded of both these; that is, that of the *Semi-parabola* in b ; and because dc was cut at Pleasure in a , and cf and ce are Means between the whole cd and the Parts da and ac , their Squares taken together will be = the Square of the whole; but from the foregoing *Lemma*, the Square of ef is also = the same Squares; therefore the Line ef is = dc . Whence it is manifest, that the *Momenta* along dc and along the *Semi-parabola* ab , in c and b are equal. *Q. E. D.*

H h h

COROLLARY.

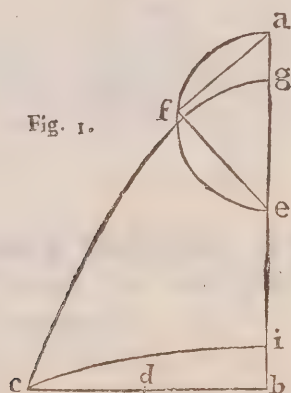
COROLLARY.

Hence it is manifest, that of all *Parabolas* whose *Altitudes* join'd with their *Sublimities* are equal, the *Impetus's* are also equal.

PROP XI. PROB. IV.

The *Impetus* and *Amplitude* of a *Semi-parabola*, being given, to find its *Altitude*.

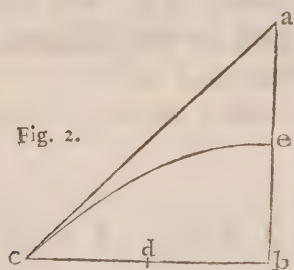
Let the *Impetus* given be defined by the Perpendicular to the Horizon ab (Fig. 1.) and let the *Amplitude* in the



horizontal Line be bc . It is required to find the *Altitude* of the *Semi-parabola*, whose *Impetus* is ab and *Amplitude* bc .

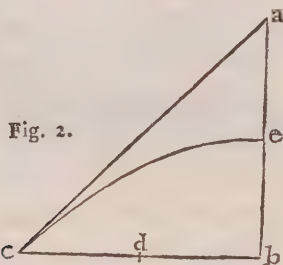
It

It is manifest, from what has been already proved, that half the Amplitude bc will be a mean Proportional between the Altitude and Sublimity of the same *Semi-parabola*, whose *Impetus*, by the preceding Proposition, is the same with the *Impetus* of the Moveable falling, from Rest in a , thro' the whole ab . Wherefore ab is to be so cut that the Rectangle contained by its Parts may be $= \square$ of half bc , which let be bd . Hence it appears to be necessary, that db do not exceed the half of ab : For of Rectangles contained by the Parts, the greatest is when the whole Line is bisected equally. Divide then ba equally in e , and if bd be $= be$ (see Fig. 2) the Business is done; and the Altitude of the *Semi-parabola* shall be be , and its Sublimity ea : (here you may observe by the Way) that the Amplitude of a *Parabola* at the Elevation of a half Right Angle or 45° , as before demonstrated, is the greatest of all those described with the same *Impetus*). But now suppose bd



to be less than the half of ba , which ba is to be so cut that the Rectangle under its Parts be $= \square$ of bd : In order to do this, bisect ab equally in e , and describe the Semi-circle efa (see Fig. 1.) in the Periphery of which from a

lay off $af = bd$ and draw fe , to which make eg equal. Then the Rectangle bga with the Square of eg will be = the Square of ea , by the 6th Prop. of the 2d Book of *Euclid*; to which also the two Squares of af and fe are equal: Taking away then the equal Squares of ge and fe , there remains the Rectangle $bga = \square$ of af , i. e. to $\square$ of bd ; and the Line bd is a mean Proportional between bg and ga ; whence it is manifest, that of the *Semi-para-*



bola, whose Amplitude is bc , and *Impetus* ab , the Altitude is bg and Sublimity ga . But now if bi be set off below $= ga$; bi shall be the Altitude, and ia the Sublimity of the *Semi-parabola ic*.

From what has been already demonstrated, we are taught,

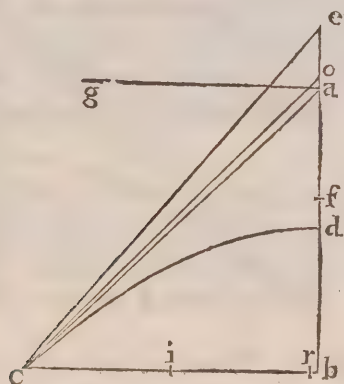
(PROP. XII. PROB. V.)

To calculate the Amplitudes of all *Semi-parabolas* described by *Projects* thrown with the same *Impetus*, and to make Tables thereof.

From

From what has been before demonstrated, it is manifest that *Parabola's* are then describ'd by *Projects* with the same *Impetus*, when their Sublimities together with their Altitudes, do make equal Perpendiculars above the Horizon. These Perpendiculars therefore are to be comprehended between the same horizontal Parallels.

Suppose therefore the Perpendicular ba to be = the horizontal Line cb , and draw the Diagonal ac . Then $\angle acb$ will be half a right one or 45° . And the Perpendicular ba being divided equally in d , the *Semi-parabola* dc will be that which is describ'd from the Sublimity ad together with the Altitude db : and its *Impetus*



in c shall be as great as that of the Moveable, falling from Rest in a thro' the Line ab , is in b : and if ag be drawn $\parallel bc$; the Altitudes and Sublimities united of all the remaining *Semi-parabola's*, whose *Impetus* shall be the same with that now mention'd, must be bounded by the Parallels ag and bc . Again, since it has been demonstrated that the Amplitudes of *Semi-parabola's* whose Tangents are
equally

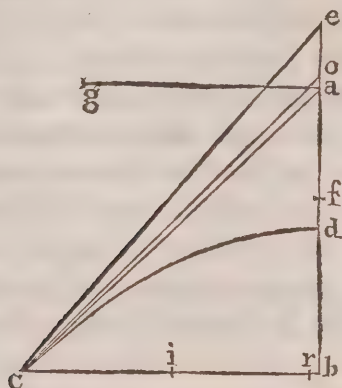
then be fo : Of the *Semi-parabola* therefore inscrib'd in $\triangle ecb$, according to the Tangent ce , whose Amplitude is cb ; the Altitude bf , and the Sublimity fo is found. But the whole Line bo rises above the Parallels ag and cb , when it was our Business to keep it within those Bounds, for so both it and the *Semi-parabola* dc will be describ'd by the Projects flung from c with the same *Impetus*. Therefore we must find another like this (For innumerable greater and smaller like one another, may be describ'd within $\angle bce$) whose united Sublimity and Altitude (homologous to ba) shall be equal to ba . Make then $ob : ba ::$ the Amplitude $bc : cr$, which cr will be the Amplitude of the *Semi-parabola* according to the Elevation or Angle bce ; whose Sublimity together with its Altitude is = the Distance between the Parallels ga and cb , which was requir'd. The Work therefore is as follows.

Take the Tangent of the given Angle bce , to the half of which add the third Proportional to it, and to bi the half of bc , which let be fo : Then make $ob : ba :: bc$ to a fourth, which let be cr , viz. the Amplitude sought.

EXAMPLE.

Let $\angle ecb$ be 50° , its Tangent will be 11918; whose half bf is = 5959; the half of bc is 5000, the third Proportional to these halves is 4195; for $5959 : 5000 :: 5000 : 4195$, which added to bf , (5959) makes 10154 for bo . Make again as ob to ba , i. e. as 10154 to 10000 so is bc , i. e. 10000 (for each of them is the Tangent of

45°) to a fourth which shall be = the required Altitude $rc = 9848$, of such Parts as bc the greatest Amplitude is 10000.



To these the Amplitudes of the whole *Parabola's* are double, viz. 19696 and 20000. And so much also is the Amplitude of the *Parabola*, at an Elevation of 40°, since this is equally distant from that of 45°. See Table A. Pag. 436.

SAGR. I am at a Loss as to some Part of this Demonstration: I don't understand how a mean Proportional between bf and bi must be necessarily greater than fa ; as our Author says it must.

SALV. That seems to follow from hence, viz.

The Square of the mean of three proportional Lines is = the Rectangle of the other two; whence the Square of bi or of bd = it, ought to be = the Rectangle of the first fb , into the third sought, which third must of Necessity

cessity be greater than fa , because the Rectangle of bf in fa , is less than the Square of bd , and the Defect is = the Square of df , as *Euclid* demonstrates in * one of his Propositions of his second Book.

It must be observed further, that the Point f , which divides the Tangent eb equally, will often fall otherwise, above the Point a , and once in the very Point a ; In which Cases it is evident of itself, that a third Proportional to the Half of the Tangent and to bi (which gives the Sublimity) is wholly above the Point a .

But our Author took a Case wherein it was not manifest that the said third Proportional is always greater than fa , and which therefore set off, above the Point f , extends beyond the Parallel ag .

Now let us go on.

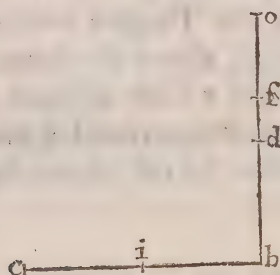
It will not be useless by Help of this Table to calculate another of the Altitudes of the same *Semi-parabola's* of Projects thrown with the same *Impetus*.

PROP. XIII. PROB. VI.

From the given Amplitudes of *Semi-parabola's* in Table A, the *Impetus* with which every one is describ'd, being the same, to find the Altitudes of those *Parabola's*.

* Prop. V.

Let the given Amplitude be bc ; and let the Measure of the *Impetus* which is suppos'd always the same, be ob , viz. the Aggregate of the Altitude and Sublimity: Now the Altitude itself is to be separately found; which will then be done, when bo shall be so divided that the Rectangle under its Parts be = the Square of half the Amplitude bc . Let such Division fall in f , and let both ob and



bc be cut equally in d and i : then is the Square of ib = $\square bfo$: But the Square of do is = the same Rectangle together with the Square of fd : If therefore from the Square of do be taken the Square of bi which is = $\square bfo$, there will remain the Square of fd , whose Side df + the Line bd , will give the Altitude sought bf .

As thus;

From half the Square of bo given, subtract the Square of bi also given; take the Square Root of the Remainder, which add to ab given, and it gives the Altitude bf sought.

EXAMPLE.

EXAMPLES

The Altitude of a *Parabola* describ'd at the Elevation of 55° is to be found.

The Amplitude from the fore-made Table A is 9396; its Half is 4698; the Square of which is 22071204; Take this from the Square of half bo , which is always the same; viz. 25000000, and the Remainder is 2928796, whose Square Root is 1710 very nearly; this added to 5000, viz. to the Half of bo , gives 67101, and so much is the Altitude bf . See Table B.

It may be of use to add a third Table, C, of the Altitudes and Sublimities of *Semi-parabola's* whose Amplitudes shall be the same.

SAGR. This will be a great Pleasure to me, since by its Help I may come to see the Difference of the *Impetus's* and Forces which are requir'd to throw a Project to the same Distance, or Range; which Difference I believe is very considerable according to the different Elevations; so that if any one at the Elevations of 3 or 4, or 87 or 88 Degrees, would fling a Ball as far as it was flung at the Elevation of 45 Degrees (at which it has been demonstrated the least *Impetus* is requir'd) I believe it would require a vastly greater Force.

SALV. You are right: and you will find that to accomplish this Business thoroughly at all Elevations, we shall be oblig'd to take large Steps towards an infinite *Impetus*.

PROP. XIV. PROB. VII.

To find the Altitudes and Sublimities of *Semi-parabola's*, whose Amplitudes shall be equal to all the Degrees of Elevation.

This is easily done. For supposing the Amplitude of the *Semi-parabola* to be always 10000 Parts, the Half of the Tangent of any Degree of Elevation will give the Altitude.

EXAMPLE.

Let the Elevation of the *Semi-parabola* be 30° , and the Amplitude, as is suppos'd, 10000, the Altitude will be 2887, for so much very nearly is the Half of the Tangent: And having found the Altitude, the Sublimity is gotten as follows; Forasmuch as it has been demonstrated that Half the Amplitude of the *Semi-parabola* is a mean Proportional between the Altitude and Sublimity; and the Altitude is already found, and Half of the Amplitude being always the same, *viz.* 5000 Parts; If the Square of this be the given Altitude, you will have the Sublimity sought. As in the Example: The Altitude found was 2887: The Square of 5000, is 25,000,000; which divided by 2887, gives 8659 very nearly for the Sublimity sought.

SALV. Here now we see first, that the aforementioned Conjecture is true, *viz.* that at different Elevations, the more they recede from the middlemost or 45° , whether

The Amplitudes of Semi-parabolas describ'd by the same Impetus		
Gr		Gr
45	10000	
40	9994	44
47	9970	43
48	9945	42
49	9902	41
50	9848	40
51	9782	39
52	9704	38
53	9612	37
54	9511	36
55	9396	35
56	9272	34
57	9130	33
58	8980	32
59	8829	31
60	8659	30
61	8481	29
62	8290	28
63	8090	27
64	7880	26
65	7660	25
66	7431	24
67	7191	23
68	6944	22
69	6692	21
70	6428	20
71	6157	19
72	5878	18
73	5592	17
74	5300	16
75	5000	15
76	4694	14
77	4383	13
78	4067	12
79	3746	11
80	3420	10
81	3090	9
82	2756	8
83	2419	7
84	2079	6
85	1736	5
86	1391	4
87	1044	3
88	698	2
89	349	1

The Altitudes of Semi-parabolas whose Impetus's are the same.		
Gr		Gr
1	3	46 5173
2	13	47 5340
3	28	48 5523
4	50	49 5608
5	76	50 5668
6	108	51 6038
7	150	52 6207
8	194	53 6370
9	245	54 6546
10	302	55 6710
11	365	56 6873
12	432	57 7033
13	506	58 7190
14	585	59 7348
15	670	60 7502
16	761	61 7640
17	855	62 7766
18	955	63 7899
19	1060	64 8028
20	1170	65 8154
21	1285	66 8286
22	1402	67 8414
23	1527	68 8507
24	1658	69 8715
25	1780	70 8830
26	1922	71 8940
27	2001	72 9045
28	2204	73 9144
29	2351	74 9240
30	2499	75 9330
31	2653	76 9415
32	2810	77 9493
33	2967	78 9507
34	3128	79 9636
35	3280	80 9698
36	3456	81 9755
37	3621	82 9806
38	3793	83 9851
39	3902	84 9890
40	4132	85 9924
41	4302	86 9951
42	4477	87 9972
43	4654	88 9987
44	4827	89 9998
45	5000	90 10000

The Altitudes & Sublimities of Semi-parabolas whose Amplitudes are the same Viz 10,000 Parts cal- culated to every Degree of Elevation.		
Gr	Altit.	Subti.
1	87	286533
2	175	142450
3	262	95802
4	349	71531
5	437	57142
6	525	47573
7	614	40716
8	702	35587
9	792	31565
10	881	28367
11	972	25720
12	1063	23518
13	1154	21701
14	1246	20056
15	1339	18663
16	1434	17405
17	1529	16355
18	1624	15389
19	1722	14522
20	1820	13736
21	1919	13024
22	2020	12376
23	2123	11778
24	2226	11230
25	2332	10722
26	2439	10253
27	2547	9814
28	2658	9404
29	2772	9020
30	2887	8659
31	3008	8336
32	3124	8001
33	3247	7650
34	3373	7413
35	3501	7141
36	3633	6882
37	3768	6635
38	3906	6395
39	4049	6174
40	4196	5959
41	4346	5752
42	4502	5553
43	4662	5362
44	4827	5177
45	5000	5000
46	5177	4828
47	5363	4662
48	5553	4502
49	5752	4345
50	5959	4196
51	6174	4048
52	6399	3906
53	6635	3768
54	6882	3632
55	7141	3500
56	7413	3372
57	7699	3247
58	8002	3123
59	8332	3004
60	8600	2887
61	9020	2771
62	9403	2658
63	9813	2547
64	10251	2438
65	10722	2331
66	11230	2226
67	11770	2122
68	12355	2020
69	13025	1919
70	13237	1819
71	14521	1721
72	15388	1624
73	16354	1528
74	17437	1433
75	18660	1339
76	20054	1240
77	21657	1154
78	23523	1062
79	25723	972
80	28350	881
81	31569	792
82	35577	702
83	40222	613
84	47572	525
85	57150	437
86	71503	349
87	95405	262
88	143181	174
89	286499	87
90	Hymin	

whether they be higher or lower, the greater *Impetus* and Violence is requir'd, to throw a Project to the same Distance : Because, since that *Impetus* lies in a Mixture of two Motions, horizontal, equable, and perpendicular naturally accelerate, and the Measure of that *Impetus* is the Aggregate of the Altitude and Sublimity ; from the Table it is seen that such an Aggregate is least at the Elevation of 45° , where also the Altitude and Sublimity are = each other ; that is each 5000, and their Aggregate 10000. But view another greater Elevation, as for Example's Sake 50° , we shall find the Altitude to be 5959, and the Sublimity 4196, which added together, make 10155. And so much also we shall find the *Impetus* to be at 40° , these Elevations being equally distant from the middle one or 45° : Where secondly we may note, that it is true that equal *Impetus's* are sought from Pairs of Elevations equally distant from the middle one, with this pleasant Variation, *viz.* that the Altitudes and Sublimities of the Elevations above 45° answer alternately to the Sublimities and Altitudes of the Elevations under 45° : So that, since in the propos'd Example at the Elevation of 50° , the Altitude is 5959 and Sublimity 4196 : at the Elevation of 40° , contrary-wise, it falls out, that the Altitude is 4196, and Sublimity 5959 : and the same happens in all the rest, without the least Difference, save only for avoiding of Tedioufness in Calculation, we have had no Regard to Fractions, which in so great Sums are of no Moment, but may be safely rejected.

SAGR. I am observing, that of the two *Impetus's* horizontal and perpendicular, in Projections, the higher they
are

are, the less they require of the Horizontal, and the more of the Perpendicular: And on the contrary, at lesser Elevations there is required a great Force of horizontal *Impetus* because the Project must be carried at a small Height. But altho' I understand very well, that at the Elevation of 90° , all the Force imaginable is not sufficient to drive the Project one single Inch out of the Perpendicular, but that it must necessarily fall in the same Place it was thrown from; yet I dare not, with the same Certainty, affirm, that also at no Elevation, *i. e.* in an horizontal Position, the Project cannot by any less than an infinite Force, be flung to any Distance. So that, *e. g.* a Culverin would not be able to fling an Iron Ball horizontally, or, as they say, from a Point blank, *i. e.* at no Elevation: I say, that in such Case, I'm in some doubt; but that I do not positively deny the Fact, another Accident no less strange forbidding me, of which I have a Demonstration necessarily conclusive. The Accident is this: 'Tis impossible to stretch a Rope right out or directly, and to keep it parallel to the Horizon; but it will always be arch'd or curv'd; nor is any Power whatsoever sufficient to keep it strait.

SALV. So then, *Sagredus*, you cease admiring in this Case concerning the wonderful Effect in the Rope, because you have a Demonstration of it. But if this Thing be well weigh'd, we may perhaps find some Agreement between the Accidents of that Project and this Rope. The Curvity of the Line of the horizontal Project seems to arise from two Powers, one whereof (which is the Power of the Projicient) forces it horizontally, but the other (which is its own Gravity, draws it perpendicularly

downwards;
†

downwards; But now in the stretching of the Rope, beside their Forces who draw it horizontally, there is moreover the Gravity of the Rope, which naturally inclines it to tend downward: Now these two Generations are very nearly alike. And if to the Gravity of that Rope you allow such Power and Energy as can resist, and overcome the Force, be it never so great, which would draw it out directly, why won't you allow it to the Gravity of the Ball or Bullet?

Besides I must tell you, that which at the same Time will both amaze and delight you, 'tis this, that the Rope thus stretch'd more or less, bends it self into Lines very nearly parabolical, and the Likeness is such, that if on a plain Superficies and perpendicular to the Horizon you describe a parabolic Line, and turn it up-side-down, and to the Extremities of the Base of the describ'd *Parabola* you hold a Chain or Cord, by slackening it more or less, you'll see it bend and fit itself to the same *Parabola*; and this Fitting shall be so much the more exact, by how much the describ'd *Parabola* is less curv'd, *i. e.* more distended; so that in *Parabola's* describ'd at Elevations less than 45° , the Chain agrees with the *Parabola* almost to an Hair.

SAGR. By Help then of such a small Chain curiously made, many parabolic Lines may in an Instant be describ'd on a plain Superficies.

SALV. They may; and that with no small Advantage: as hereafter I shall shew you.

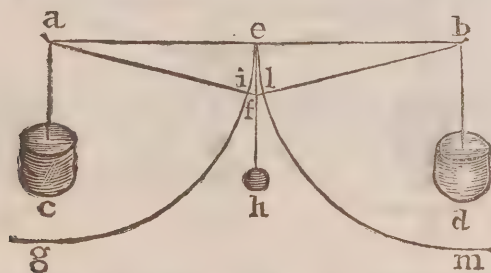
SIMP. But before we go on, I would at least have that Proposition demonstrated of which you say you have a Demonstration necessarily conclusive, *viz.* that it is impossible for a Rope to be stretch'd directly, and keep parallel to the Horizon.

SAGR. I'll try if I can recollect the Demonstration; in order to understand which, *Simplicius*, you must take for granted, that which in all Mechanical Instruments is prov'd to be true; not only by Experience, but Demonstration: and 'tis this; that the Velocity of the Mover, tho' this Mover has but little Power, may overcome the Resistance, altho' vastly great, of the Thing resisting, (which must be mov'd slowly) whenever the Velocity of the Mover hath a greater Proportion to the Tardity of the Resister, than the Resistance of that which is to be moved, has to the Power of the Mover.

SIMP. This I know well already, 'tis demonstrated by *Aristotle* in his mechanic Questions; and tis manifestly seen in the *Leaver* and *Stilliard*, in which the *Counterpoise* which weighs not above four Pounds, will raise a Weight of 400, provided the Distance of the said *Counterpoise* from the Center upon which the Beam turns, be more than 100 times greater than the Distance of the said Center from the Point, at which the great Weight hangs; And this comes to pass, because the *Counterpoise*, whilst it descends, passeth a Space above an hundred times greater than the Space which the great Weight ascends in the same Time, which is the same Thing as to say, that that little
Counterpoise

Counterpoise moveth with a Velocity above an hundred Times greater than the Velocity of the great Weight.

SAGR. You reason very justly, and make no Scruple of granting that howsoever small the Power of the Mover be, yet it will o'ercome any whatever great Resistance at all Times when that shall exceed more in Velocity than this does in Power and Gravity. But let us come to the Case of the Rope : Let the following Figure be drawn ;



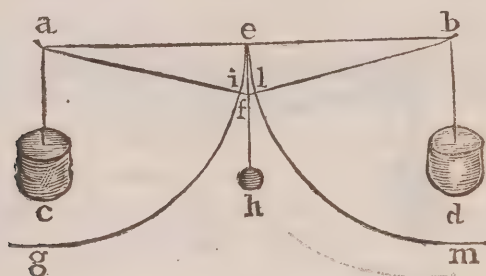
Imagine the Line ab passing thro' two fixed or immoveable Points a and b , to sustain, hang'd at its Extrems, as you see, two great Weights c and d , which drawing it with great Force keep it straight stretch'd, as a simple Line without any Gravity. Then I say if to its Middle, as the Point e , you hang a Weight never so small, as is h , the Line ab will yield and incline towards the Point f , and by consequence lengthening, will constrain the two great Weights c and d to ascend : Which I demonstrate as follows ;

About the two Points a and b , as Centers, describe two Quadrants eig , elm : and since the two Semi-diameters

K k k

a i

ai and bl are = the two ae and eb , the Excesses fi , fl will be Quantities by which the Parts af and fb exceed ae and eb , and consequently they determine the Ascent of the Weights c and d ; always therefore when the Weight b has a Power to descend to f ; which might then be when the Line ef , which is the Quantity of the Descent of that Weight b , has a greater *Ratio* to the Line fi , which determines the Ascent of the two Weights c and d , than the Gravity of both those Weights, has to the Gra-



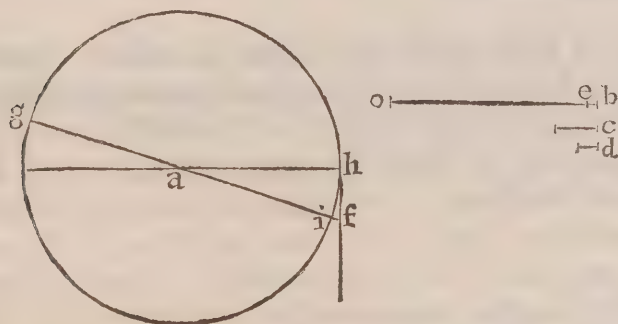
vity of the Weight b . But now this will necessarily happen, be the Gravity of the Weights c and d never so great, and that of b never so small. For the Excess of the Weights c and d , above the Weight b , is not so much, but that the Excess of the Tangent ef , above the Part of the Secant fi , is more in Proportion. Which I prove as follows.

Let there be a Circle whose Diameter is gai : and the same *Ratio* as the Gravity of the Weights c and d in the foregoing Fig. have to the Gravity of b , let the Line bo have to a fourth c , less than which let be the Line d , so that bo shall have a greater Proportion to d than it has to c . Between ob and d take a third Proportional be :

And

†

And as oe is to eb , so make the Diameter gi (by producing it) to if ; and from f draw the Tangent fn . And because it is $oe : eb :: gi : if$, it will be by Composition $ob (oe + eb) : be :: gf (gi + if) : fi$; but between ob and be , d is a mean Proportional: And between gf and fi , nf is a Mean; therefore nf has the same Proportion to fi , as ob has to d , which Proportion is greater than that which the Weights c and d have to the Weight b . Since then the Descent or Velocity of the Weight b has a greater *Ratio* to the Ascent or Velocity of the Weights c



and d ; than the Gravity of the said Weights c and d has to the Gravity of the Weight b ; it is manifest, that the Weight b must descend, that is, that the Line ab must depart from its Straitness: And this, which happens to a right Line ab having no Gravity at all, upon hanging in e a Weight b never so small; the very same will happen to the Rope, which is suppos'd to be made of ponderous heavy Matter without the Addition of any other Weight, because the Weight of the Matter the Rope ab is made on may be said to be added or hung to it.

SIMP. I am thoroughly satisfied; wherefore *Salviatus* may, according to his Promise, explain to us, what that Advantage is, which may be drawn from the fore-mention'd like Chain; and then shew us what Observations our Academic has made about *the Force of Percussion*.

SALV. We have Business enough for this Day to contemplate of what has been already said; the Time, it now growing towards Evening, would not admit us to go thro' what you propose; wherefore we must put it off till another Opportunity.

SAGR. I agree with you: And the rather, because I have often heard, from some of our Academic's intimate Acquaintance, that this Business of *Percussion* is very obscure; and that of those that have hitherto treated about it, not one has done it thoroughly and clearly. Amongst the Conclusions that I have heard of, I can't help mentioning one that is very extraordinary, namely, that the Force of *Percussion* is indeterminate if not infinite. However, we'll wait *Salviatus's* Leisure. But in the mean Time tell me what those Things are which are wrote at the End of the Treatise of *Projects*.

SALV. They are certain Propositions relating to the Center of Gravity of Solids, which our Academic found out in his Youth: Conceiving that what *Frederick Comandine* had wrote about it, to be imperfect, he thought that these Propositions following would supply what was wanting in *Comandine's* Treatise; He apply'd himself to this Study at the Request of the most Illustrious Lord
Marquiss

Marquis *Guid' Ubaldo dal Monte*, (the greatest Mathematician of his Time ; as his several printed Works do witness ;) and gave a Copy of what he had wrote on this Head to that noble Lord, and had indeed Thoughts to have pursu'd this Matter farther in other Solids untouch'd by *Comandine* : but when after some Time he met with the Book of *Signore Luca Valerio*, a most excellent Mathematician, and saw that he had solv'd all these Matters, without omitting any Thing, he left of ; tho' the Methods *Signore Valerio* and he took were very different to each other.

SAGR. I should take it as a Favour if 'till our next Meeting you would leave the Book with me to peruse.

SALV. With all my Heart ; and I hope the Propositions will highly delight you.





APPENDIX.

Wherein are Contained

PROPOSITIONS *and their* DEMONSTRATIONS,

Formerly wrote by the same AUTHOR about the

Center of Gravity of Solids.

POSTULATE.

 F equal Weights alike dispos'd at different Balances or Beams, we take for granted that if the Center of Gravity of these Weights compounded, that hang at one Balance, divides this Balance in any Proportion, that then the Center of Gravity of those at the other Balance will divide that Balance according to the same Proportion.

LEMMA.

Let the Line ab be equally divided in c ; whose half ac let be divided in e , in such Manner that what *Ratio*
 $h\ e$

be has to ea , the same ae may have to ec . Then I say be is the double of ea . For because $be : ea :: ea : ec$; it will be by Composition and Permutation $ba : ac :: ae : ec$.



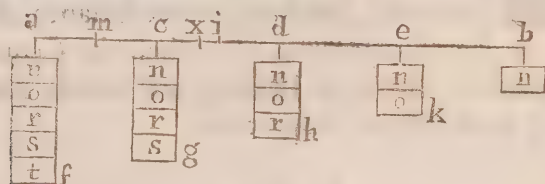
But it is as ae to ec , i. e. as ba to ac , so is be to ea . Wherefore be is the double of ea .

These suppos'd, we may demonstrate the following

PROPOSITION.

If Magnitudes equally exceeding one another at any Rate of Excess, and whose common Excess is = the least of those Magnitudes, are so dispos'd as to hang at equal Distances on a Balance (or Beam); the Center of Gravity of all of them will so divide the Beam, that that Part of it towards the lesser Magnitudes will be double to the remaining Part of it.

On the Beam ab then let there hang at equal Distances any Number of Magnitudes (at has been said) as f, g, h, k, n : the least of which let be n . And let the Center of



Gravity of all the Magnitudes so dispos'd, be κ . Then we are to shew, that the Part of the Beam bx towards the lesser Magnitudes is double to the remaining Part of it, κa .

Let

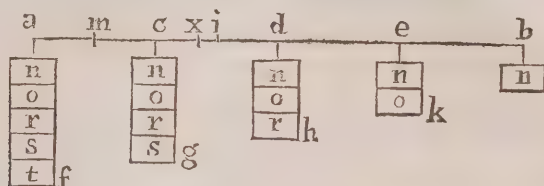
†

Let the Beam be divided equally in the Point d , which Point will, of Course, fall on some Point of the Suspensions, or in the midst of two of them; And let the remaining Distances of Suspensions, which are between a and d , be equally divided in the Points m and i . And let all the Magnitudes be divided into Parts $= n$: Now the Parts of f will be so many in Number as there are Magnitudes hanging from the Beam: but the Parts of g will be fewer by one, and so of the rest. Let the Parts of f therefore be n, o, r, s, t ; Those of g ; n, o, r, s ; Those of h ; n, o, r ; And those of k ; n, o ; And all the Magnitudes, mark'd n shall be $=$ the Magnitude f ; And the Magnitudes mark'd o shall be $= g$; Those mark'd r are $= h$; and those mark'd s , to k ; and the Magnitude t is $= n$. Therefore because all the Magnitudes mark'd n , are $=$ one another, they will weigh equally in the Point d , which divides equally the Balance or Beam ab ; And for the same Reason, all the Magnitudes mark'd o , equiponderate in i ; Those mark'd r in c ; And those mark'd s equiponderate in m . But t is suspended at a . Therefore in the Balance ad , at the equal Distances d, i, c, m, a , there are Magnitudes suspended, exceeding one another equally, and whose Excess is $=$ the least Magnitude: And the greatest which is compounded of all the n 's hangs at d ; the least which is t , hangs at a ; and the rest are orderly dispos'd. And again there is another Balance ab ; in which the other Magnitudes equal to the foremention'd in Number and Magnitude are dispos'd in the same Order. Wherefore the Balances ab and ad are divided by the Center of all the Magnitudes in the same Ratio, one as the other. But the Center of Gravity of the said Magni-

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tudes

tudes is x ; wherefore x divides the Balances ba and ad in the same *Ratio* one as the other: So that $bx : xa :: xa : xd$



(by the foregoing Postulate) wherefore bx is the Double of xa , by the above mention'd *Lemma*. Q. E. D.

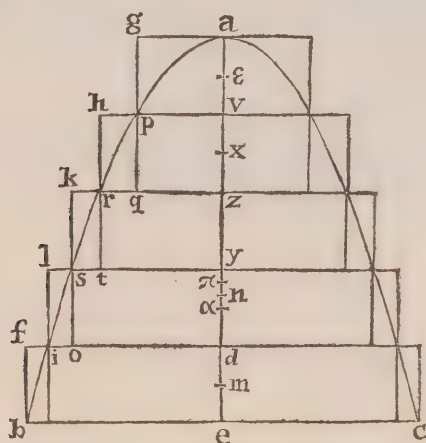
PROPOSITION.

If in a *Parabolic Conoid* a Figure be inscrib'd, and another circumscrib'd by Cylinders of equal Height; and the *Axis* of the said *Conoid* be divided, so that the Part towards the *Vertex* be double the Part towards the Base; the Center of Gravity of the inscrib'd Figure will be nearer to the Base than that Point of Division will be, and the Center of Gravity of the circumscrib'd shall be farther than that Point from the Base of the *Conoid*; and the Distance of either Centers from such Point will be = a Line which is a Sixth Part of the Height of one of those Cylinders of which the Figures consist.

DEMONSTRATION.

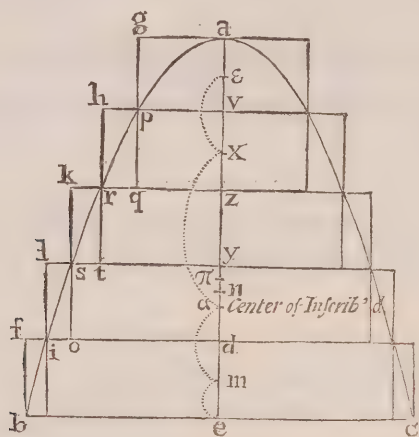
Let there be a *Parabolic Conoid*, and the Figures that have been mention'd, the one inscrib'd, the other circumscrib'd: And let the *Axis*, which is ae , be divided in n , so that an be the Double of ne . We must shew the Center of Gravity of the inscrib'd Figure to be in the Line ne , but the

the Center of the circumscrib'd to be in an . Let the Figures so dispos'd be cut by a Plane thro' the *Axis*, and let the Section of the *Parabola* be bac ; and let the Section of the Cutting Plane and Base of the Conoid be the Line bc ; and let the Sections of the Cylinders be the rect-angular Figures as describ'd in the Figure before us: Then the first Cylinder of the Figure inscrib'd, whose *Axis* is de , has the same *Ratio* to the Cylinder whose *Axis* is dy , as the Square of id has to the Square of sy , id *est*, as da to ay : But the



Cylinder whose *Axis* is dy to the Cylinder yz , is as the Square of sy to the Square of rz ; *i. e.* as ya to az ; and for the same Reason, the Cylinder whose *Axis* is zy , to that whose *Axis* is zv , is as za to av . Therefore the said Cylinders are one to another, as the Lines da , ay ; za , av : but they exceed one another equally, and the Excess is = the least of them, so that az is double to av ; and ay is triple to the same; and da quadruple; therefore the said Cylinders are certain Magnitudes equally exceeding one

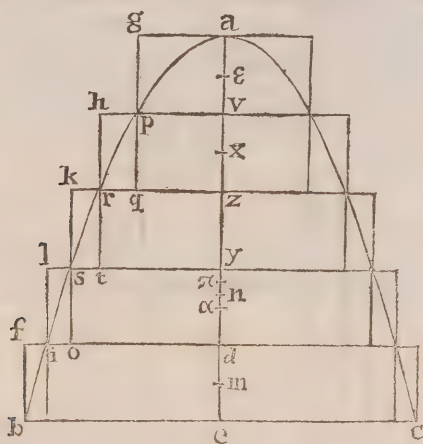
another, whose common Excess is = the least of them, and the Line xm is that in which they are suspended at equal Distances: For each Cylinder has its Center of Gravity in the Midst of its Axis. Wherefore by what has been above demonstrated, the Center of Gravity of the Magnitude compounded of them all, divides the Line xm , so that the Part towards x is double to the remaining one (by the last Prop.) Divide it therefore, and let xa be double am ; The Point a then, is the Center of Gravity of the inscrib'd Figure.



Divide av equally in ϵ ; ϵx shall be double of me ; but xa is double am (by Construction), wherefore ϵa will be the double of ae ; wherefore ϵe will be three times ea , but ae is triple en ; it's manifest therefore that en is greater than ea , and therefore a , which is the Center of the inscrib'd Figure, happens nearer the Base of the *Conoid* than n ; And because it is as ae to en , so the Part taken away ϵe , to the Part taken away ea ; And the Remainder

der to the Remainder, *id est*, $a\varepsilon$ to $n\varepsilon$, will be as $a\varepsilon$ to $e\varepsilon$. Therefore $a\varepsilon$ is the third Part of $a\varepsilon$, and the sixth Part of $a\varepsilon$.

After the same Manner the Cylinders of the circumscrib'd Figure will be demonstrated to be equally exceeding one another, and their common Excess to be = the least of them; And to have their Centers of Gravity at equal Distances in the Line εm . If therefore εm be divided in π , so that $\varepsilon\pi$ be double of the Remainder πm ; π will be the Center of Gravity of the whole circumscrib'd Magnitude, and since $\varepsilon\pi$ is double πm ; and $a\varepsilon$ less than double εm ;



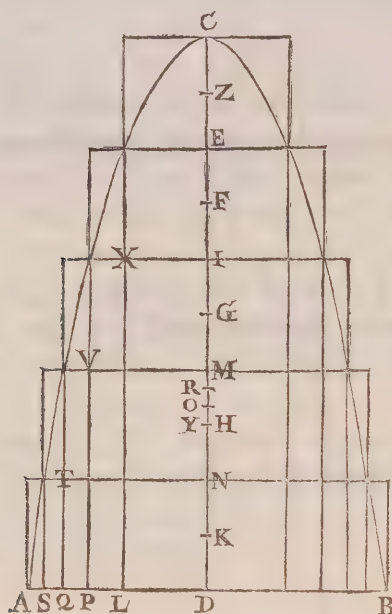
(they being = one another); the whole $a\varepsilon$ shall be less than three times $e\varepsilon$; wherefore $e\varepsilon$ will be greater than $e\varepsilon$; and since εm is triple $m\varepsilon$, and $m\varepsilon$ with twice εa is likewise three times $m\varepsilon$; the whole $a\varepsilon$ with $a\varepsilon$ shall be three times $e\varepsilon$; but $a\varepsilon$ is three times $e\varepsilon$; wherefore the remaining Part $a\varepsilon$ will be three times the Remainder πn . Therefore $n\varepsilon$ is a sixth Part of $a\varepsilon$. Which were the Things to be prov'd.

COROL-

Center of Gravity of Solids. 455

we are to prove that the Center of Gravity of the inscrib'd Figure is in the Line OD; and that the Center of the circumscrib'd is in CO. Let the Figures be cut by the Plane thro' the *Axis* and C, as has been said. Then because the Cylinders SN, TM, VI, and XE, are one to another as the Squares of the Lines SD, TN, VM, and XI, and these one to another as the Lines NC, CM, CI, CE; And because these exceed each other equally, and the Excess is = the least, *viz.* to CE, and the Cylinder TM = the Cylinder QN; And the Cylinder VI to PN; and XE to LN. therefore the Cylinders SN, QN, PN, LN exceed one another equally, and the common Excess is = the least of them, *viz.* to the Cylinder LN. But the Excess of the Cylinder SN, above the Cylinder QN, is the Ring whose Height is QT; that is ND; and Breadth, SQ. And the Excess of the Cylinder QN, above PN, is the Ring, whose Breadth is QP; but the Excess of the Cylinder PN, above LN, is the Ring, whose Breadth is PL. Wherefore the said Rings SQ, QP, PL, are = each other, and to the Cylinder LN; therefore the Ring ST is = the Cylinder XE: The Ring QV, which is the Double of ST, is = the Cylinder VI; which also is the Double of the Cylinder XE: And for the same Reason the Ring IX will be = the Cylinder TM; and the Cylinder LE = the Cylinder SN. In the Beam or Balance therefore KF, connecting the middle Points of the Right Lines EI and DN, and cut into equal Parts by the Points H and G, there are certain Magnitudes, *viz.* the Cylinders SN, TM, VI, XE; and the Center of Gravity of the first Cylinder is K, and of the second H; of the third G; and of the fourth F. And we have another Beam MK, which

which is the Half of FK, and divided by as many Points into equal Parts, *viz.* MH, HN, NK, and on it, other Magnitudes, equal in Number and in Magnitude to those on the Beam FK, and having the Centers of Gravity in the Points M, H, N, K, and dispos'd in the same Order. For the Cylinder LE has its Center of Gravity in M; and is = the Cylinder SN, having its Center in K: But the



Ring PX has its Center, H; and is = the Cylinder TM; whose Center is H: And the Ring QV, having its Center N, is = the Cylinder VI; whose Center is G: And lastly the Ring ST, having its Center K, is = the Cylinder XE, whose Center is F. Therefore the Center of Gravity

of
†

of the said Magnitudes will divide the Beam in the same Proportion. But the Center of them is one, and therefore some Point common to both Beams, which let be Y. Therefore FY will be to YK as YK to YM. Therefore FY is the Double of YK; and CE being divided equally in Z, ZF will be the Double of KD; and therefore ZD the Triple of DY.

But CD is the Triple of the right Line DO: therefore the right Line DO is greater than DY; and therefore Y, the Center of the inscrib'd Figure, is nearer to the Base than the Point O. And because, as CD is to DO, so is the Part taken away ZD, to the Part taken away DY; the Remainder CZ will be to the Remainder YO, as CD to DO; to wit, YO will be a third Part of CZ; that is, a sixth Part of CE.

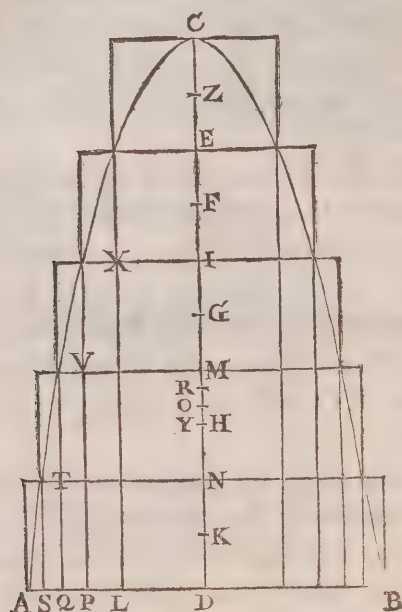
By the same way of Reasoning we will demonstrate, that the Cylinders of the circumscrib'd Figure exceed one another equally, and that the common Excess is = the least of them, and that their Centers of Gravity are rang'd at equal Distances on the Beam KZ. And likewise, that the Rings = those same Cylinders, are in like manner dispos'd on another Beam KG, the Half of the said KZ, and that therefore the Center of Gravity, which let be R, of the Figure circumscrib'd, so divides the Beam, as that ZR is to RK, as RK is to RG. Therefore ZR shall be double of RK: But CZ is = the right Line KD, and not double to it. The Whole CD shall be less than triple of DR: Wherefore the right Line DR is greater than DO. That is, the Center of the circumscrib'd Figure is farther off the Base, than the Point O. And because ZK is triple of KR; and KD with twice ZC is triple to KD; the Whole CD

M m m

with

458 APPENDIX relating to the

with CZ will be the Triple of DR. But CD is triple to DO. Wherefore the remaining Part CZ will be triple to



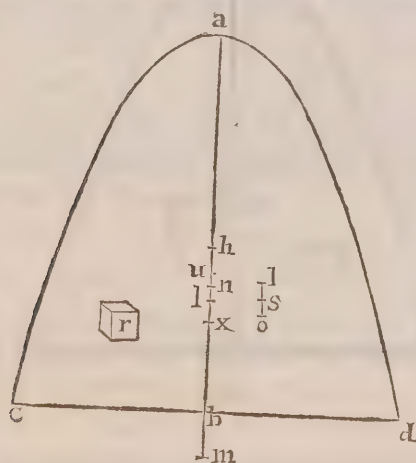
the remaining Part RO; *i. e.* OR is the sixth Part of EC: Which is the Proposition.

This being demonstrated, we may prove that *The Center of Gravity of the Parabolic Conoid divides the Axis in such Manner, that the Part towards the Vertex is the Double of the remaining Part towards the Base.*

Let

Center of Gravity of Solids. 459

Let there be a *Parabolic Conoid*, whose *Axis* let be ab , divided in n , so that an is the Double of nb . We are to shew that the Point n is the Center of Gravity of the *Conoid*. For if n be not so, this Center must be either below it or above it; Suppose it first, to be below: and let it be x ; and let the Line lo be drawn $= nx$; and let lo be divided at Pleasure in s : and what Proportion lx and os both together have to os , let the *Conoid* have the same to the Solid r : And let a Figure of *Cylinders* having



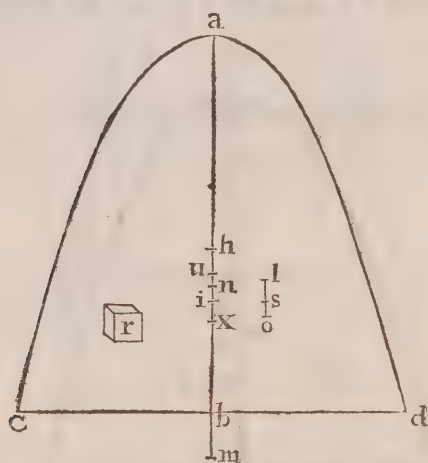
equal Heights, be inscrib'd in the *Conoid*, so that that which is intercepted between the Center of Gravity and the Point n , be less than ls ; and let the Excess of the *Conoid* above it be less than the Solid r .

And that this can be done, is manifest. Let the inscrib'd Figure therefore be that whose Center of Gravity is i ; now ix will be greater than so : And, because it is

M m m 2

as

as $\propto b$ with so is to os , so is the *Conoid* to r ; (and r is greater than the Excess, whereby the *Conoid* exceeds the inscrib'd Figure); the Proportion of the *Conoid* to the said Excess will be greater than that of both $b\propto$ and os to so : And by Division, the inscrib'd Figure will have a greater *Ratio* to the said Excess than $b\propto$ has to so . But $b\propto$ has to $\propto i$ a *Ratio* still less than it has to so . Therefore the in-



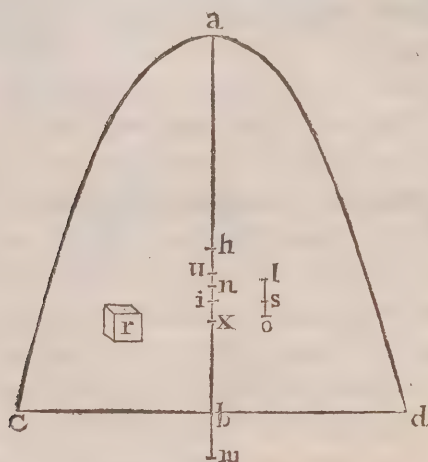
scrib'd Figure will have a much greater Proportion to the remaining Parts than $b\propto$ has to $\propto i$. What Proportion therefore the inscrib'd Figure has to the remaining Parts, the same some other Line will have to $\propto i$; which will necessarily be greater than $b\propto$. Let it therefore be $m\propto$. We therefore have $\propto$ the Center of Gravity of the *Conoid*: But the Center of Gravity of the Figure inscrib'd therein is i . Therefore the Center of Gravity of the remaining Portions by which the *Conoid* exceeds the inscrib'd Figure, will

will be in the Line αm , and in that Point of it at which it shall be so terminated, that that Proportion which the inscrib'd Figure has to the Excess of the *Conoid* above it, the very same it may have to αi . But it has been prov'd that this Proportion is that which $m\alpha$ has to αi . Therefore m will be the Center of Gravity of those Portions whereby the *Conoid* exceeds the Figure inscrib'd. Which Certainty cannot be. For if thro' m a Plane be drawn parallel to the Base of the *Conoid*, the said Portions will be all on one and the same Side, and will not be divided by it. Therefore the Center of Gravity of the said *Conoid* is not below the Point n . Neither is it above it. For, if it can be, let it be h . And again, as before, let the Line lo be drawn = the said hn , and let it be divided at Pleasure in s . And what Proportion bn and so both together have to sl , the same let the *Conoid* have to r : and let a Figure of Cylinders, as aforesaid, be circumscrib'd about the *Conoid*, exceeding the *Conoid* by a Quantity less than the Solid r : And let the Line between the Center of Gravity of the circumscrib'd Figure and the Point n be less than so , the remaining Part ub will be greater than ls . And, because it is, as bn and os both together is to sl , so is the *Conoid* to r ; (and r is greater than the Excess by which the circumscrib'd Figure exceeds the *Conoid*); therefore bn and so together, have a less Ratio to sl than the *Conoid* has to the said Excess. But bu is less than bn and so both together; And ub is greater than sl : Therefore the *Conoid* has a much greater Ratio to the said Portions than bu has to ub .

Therefore the same Ratio which the *Conoid* has to those Portions, will a Line greater than the said bu have to ub .

Let

Let it have it; and let the same be mu ; and because the Center of Gravity of the circumscrib'd Figure is u ; and that of the *Conoid* is h , and because it is, as the *Conoid* to the remaining Portions, so is mu to uh , m will be the Center of Gravity of the remaining Portions: Which is



likewise impossible. Therefore the Center of Gravity of the *Conoid* is not above the Point n . But it has been demonstrated that it is not below it. It remaineth therefore, that it must necessarily be in n it self. And this may by the same arguing be demonstrated of a *Conoid* cut by a Plane not perpendicular to the *Axis*.

The same Thing in another Manner, as appears by the following Proposition.

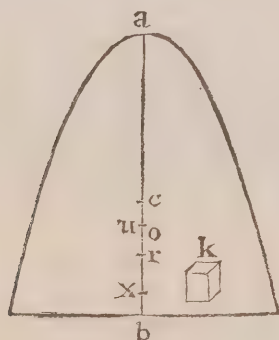
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Center of Gravity of Solids. 463

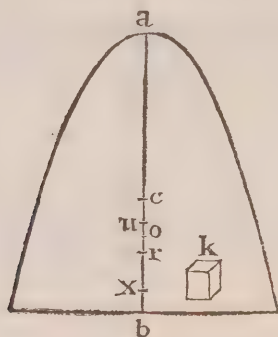
The Center of Gravity of the Parabolic Conoid falls between the Center of the circumscrib'd Figure and the Center of the inscrib'd one.

Let there be a *Conoid*, whose *Axis* is ab , and let the Center of the circumscrib'd Figure be c , and that of the inscrib'd one be o . I say, that the Center of the *Conoid* is between the Point c and o . For, if not, it will be below them, above them, or in one of them. Let it be below them, as in r . And, because r is the Center of



Gravity of the whole *Conoid*; and the Center of Gravity of the inscrib'd Figure is o : therefore the Center of Gravity of the remaining Portions, whereby the *Conoid* exceeds the inscrib'd Figure, will be in the Line or extended on the Side of r , and in that Point of it in which it is so limited, that, the same *Ratio*, which the said Portions have to the inscrib'd Figure, or may have to the Line falling between r and that Point. Let this *Ratio* be that which or has to rx : Then x will fall either without or within the *Conoid*, or in its Base. To suppose it to fall
either

either without or in the Base, it is already manifest to be absurd. Let it fall within : And because xr is to ro as the inscrib'd Figure is to the Excess of the *Conoid* above it ; the same *Ratio*, which br has to ro , let the inscrib'd Figure have to the Solid k , which will necessarily be less than the said Excess. And let another Figure be inscrib'd, which may be exceeded by the *Conoid* by a Quantity less than that of the Solid k ; whose Center of Gravity will fall between o and c . Let it be u . And, because the first Figure is to k , as br to ro ; and the second Figure, whose Cen-



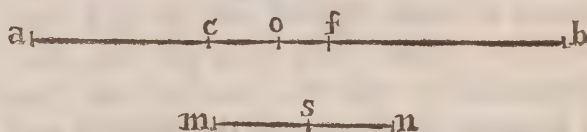
ter is u , is greater than the first, and is exceeded by the *Conoid* by a Quantity less than k : the same *Ratio*, which the second Figure has to the Excess of the *Conoid* above it, will a Line greater than the said br have to ru . But r is the Center of Gravity of the *Conoid* : and u of the second inscrib'd Figure. Therefore the Center of the remaining Portions will be without the *Conoid*, below b ; which is impossible. And after the same Manner it may be demonstrated, that the Center of Gravity of the same *Conoid* is not in the Line ca . And that it is neither of the Points c and

c and o , is manifest. For if you say it is, then, other Figures being describ'd, *viz.* one inscrib'd something greater than that whose Center is o , and one circumscrib'd, but less than that whose Center is c , the Center of the *Conoid* would fall without the Center of these Figures. Which was just now concluded to be impossible. It remaineth therefore, that it be between the Centers of the circumscrib'd and inscrib'd Figures. And, if it be so, it will necessarily be in that Point which divides the *Axis*, that the Part towards the *Vertex* be the Double of the remaining Part, since Figures may be inscrib'd and circumscrib'd about n , so that the Lines which fall between the Centers of those Figures and the said Point, be less than any given Line. We have led him that denies it to an Impossibility, that the Center of the *Conoid* should not fall between the Centers of the inscrib'd and circumscrib'd Figures.

If there be three proportional Lines, and the same Proportion, which the least has to the Excess of the greatest above the least, a certain assumed Line has to two Thirds of the Excess, whereby the greatest exceeds the mean: And also if the same Proportion, which that compounded of the greatest and of double the mean has to that compounded of triple the greatest and of triple the mean, another assum'd Line has to the Excess of the greatest above the mean; then the assum'd Lines both together will be a third Part of the greatest of the three Proportionals.

Let the three proportional Lines be ab , bc , and bf . And the same Proportion, which bf has to fa , let ms have to two thirds of ca . And the same Proportion which that compounded of ab and of double bc has to that compounded of triple of both ab and bc , let another Line, to wit, sn , have to ac . We are to demonstrate that mn is a third Part of ab .

Because ab , bc and bf , are Proportionals, therefore ac and cf will be also in the same Proportion. Therefore it is, $ab : bc :: ac : cf$; and $3ab : 3bc :: ac : cf$. Therefore the same Proportion, which the Triple of ab with the



Triple of bc has to the Triple of bc , ac will have to a Line less than cf . Let that be co . Wherefore by Composition, and by converse *Ratio*, oa will have the same Proportion to ac , as the Triple of ab with the Sextuple of bc has to the Triple of ab with the Triple of bc . But ac has the same Proportion to sn , as three times ab with three times bc , has to ab with double bc . Therefore by Equality, oa will have the same Proportion to ns , as three times ab with six times bc , has to ab with twice bc . But three times ab with six times bc , is the Triple of ab with twice bc . Therefore ao is the Triple of sn .

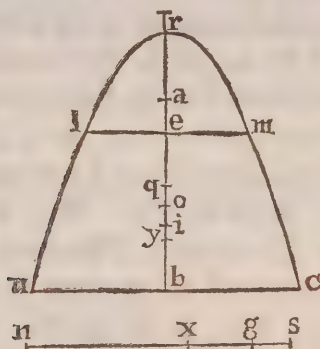
Again, because oc is to ca , as three times cb is to three times ab with three times cb : And because it is, as ca to cf , so is three times ab to three times bc : By Equality therefore,

therefore, in inordinate Proportion, as oc to cf , so will three times ab be to three times ab with three times bc : And, by converse *Ratio*, as of to fc , so is three times bc to three times ab with three times bc ; and, $cf : fb :: ac : cb :: 3 ac : 3 cb$. Therefore by Equality, in inordinate Proportion, as of is to fb , so is three times ac to three times ab and bc both together. Therefore the Whole ob will be to fb , as six times ab to three times ab and bc both together. And because fc and ca , and also cb and ba , are in the same Proportion, it will be, $fc : ca :: bc : ba$; and by Composition, as fa is to ac , so is ba and bc both together, to ba ; and so is the Triple to the Triple: Therefore, as fa is to ac , so is that compounded of three times ba and three times bc , to three times ba . Wherefore, as fa is to two thirds of ac , so is that compounded of three times ba and three times bc to two thirds of three times ba ; that is, to twice ba . But, as fa is to two thirds of ac , so is fb to ms . Therefore as fb is to ms , so is that compounded of three times ba and three times bc , to double ba . But, as ob to fb , so was six times ab to three times ab and bc both together. Therefore by Equality, ob will have the same Proportion to ms , as six times ab has to double ba . Wherefore ms will be the third Part of ob . And it has been demonstrated, that sn is a third Part of ao . Therefore 'tis evident, that mn is likewise a third Part of ab . Q. E. D.

The Center of Gravity of any Frustum of a Parabolic Conoid is in the Right Line, which is the Axis of the Frustum; which being divided into three equal Parts, the Center of Gravity is in the middle Part, and so divides

it, that the Part towards the lesser Base has the same Proportion to the Part towards the greater Base, as the greater Base has to the lesser.

Let the Solid, whose Axis is be , be cut off from the Conoid, whose Axis is rb ; and let the cutting Plane be parallel to the Base. And let it be cut by another Plane thro' an Axis perpendicular to the Base, and let it be the Section of the Parabola urc . And let the Sections of this, both of the cutting Plane and of the Base, be the

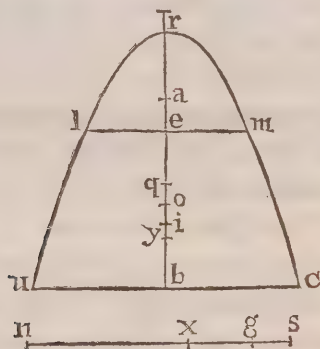


Right Lines lm and uc ; rb will be the Diameter of Proportion, or parallel to the Diameter: lm and uc will be [Ordinates, or] ordinately apply'd. Let eb therefore be divided into three equal Parts, of which let qy be the middle one. And let this be so divided in the Point i ; that, qi may have the same Proportion to iy , which the Base whose Diameter is uc , has to the Base whose Diameter is lm ; that is, which the Square of uc has to the Square of lm . It is to be demonstrated, that i is the Center of Gravity of the Frustrum $ulmc$.

Let

Let the Line ns be drawn $= br$, and let sx be $= er$. And to the said ns and sx let a third Proportional sg be taken. And, that Proportion, which ng has to gs , let the Line bq have to io . And it is no matter whether the Point o falls above or below lm . And because in the Section urc the Lines lm and uc are Ordinates rightly apply'd, it will be, as the Square of uc to the Square of lm , so is the Line br to re . But it is, as the Square of uc to the Square of lm , so is qi to iy ; and as br to re , so is ns to sx . Therefore qi is to iy , as ns to sx . Wherefore, as qy is to yi , so will both ns and sx be to sx ; and as eb is to yi , so will that compounded of three times ns and three times sx be to sx . But as eb is to by , so is that compounded of three times ns and sx both together, to that compounded of ns and sx . Therefore, as eb is to bi , so is that compounded of three times ns and three times sx , to that compounded of ns and twice sx . There are therefore three proportional Lines ns , sx , and gs ; and, the same Proportion which sg has to gn , a certain assum'd Line oi has to two thirds of eb , that is, of nx ; and the same Proportion, which that compounded of ns and twice sx , has to that compounded of three times ns and three times sx , another assum'd Line ib has to be , that is, to nx . Therefore, by what has been before demonstrated, those assum'd Lines together will be a third Part of ns , that is, of rb . Therefore rb is the Triple of bo . Wherefore o will be the Center of Gravity of the *Conoid* urc . And let a be the Center of Gravity of the *Conoid* lrm . Therefore the Center of Gravity of the Frustum $ulmc$ is in the Line ob , and in that Point which so terminates it, that the Line ao may have the same *Ratio* to the Line which

is intercepted between o and the said Point, as the Frustrum $ulmc$ has to the Portion lrm . And, because ro is two thirds of rb ; and ra two thirds of re , the remaining Line ao will be two thirds of the remaining Line eb . And, because as the Frustrum $ulmc$ is to the Portion lrm , so is ng to gs ; and as ng is to gs , so is two thirds of eb to oi ; and that the Line ao is equal to two thirds of eb : it will be, as the Frustrum $ulmc$ is to the Portion lrm , so



is ao to oi . Therefore 'tis manifest that the Point i is the Center of Gravity of the Frustrum $ulmc$, and so divides the *Axis*, that the Part towards the lesser Base, is to the Part towards the greater, as the Double of the greater Base together with the lesser, is to the Double of the lesser together with the greater. Which is the Proposition more elegantly declar'd.

If any Number of Magnitudes soever, so dispos'd one with another, that the second surpasses the first by double the first, the third surpasses the second three times the first, and the fourth surpasses the third four times the first, and

so
†

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so every following Magnitude surpasses the next before it so many times the first, according to the Number which it holds in Order: I say, if these Magnitudes be hung in Order on a Beam at equal Distances; the Center of Equilibrium of them all compounded, will so divide the Beam that the Part towards the lesser Magnitudes be triple the remaining Part.

Let the Beam be LT; and let such Magnitudes, as abovesaid, hang thereon; and let them be A, F, G, H, K; of which let the first A be hung at T. I say, the Center of Equilibrium so divides the Beam TL, that the Part towards T is the Triple of the other Part. Let TL

P				
L	O	N	X	I
a	a	a	a	a
a	a	a	a	a
a	a	a	a	b
a	a	b		F
a	b	b		
b	b	c		
b	b			G
b	c			
b	c			
c	d			
c				
c				H
d				
d				
e				
K				

be triple to LI; and SL triple LP; and QL triple to LN; and LP to LO: IP, PN, NO, and OL will be equal. And let a Magnitude in F be taken the Double of A; but in G let another be taken the Triple of the same; in H the Quadruple of the same; and so on in Order: And let those Magnitudes, wherein *a* is, be taken: And let

let the same be done in the Magnitudes F, G, H, K. For since the Magnitude remaining in F, *viz.* b , is equal to a ; let the Double of it be taken in G, the Triple in H, &c. And let those Magnitudes, wherein b is, be taken; and in the same Manner let those, wherein c is, and wherein d and e also are, be taken. Now all those, wherein a is, will be equal to K; but that compounded of all the b 's will be equal to H; that compounded of the c 's will be equal to G; but that compounded of all the d 's will be equal to F; and e will be equal to a . And be-

P				
L	O	N	X	I
a	a	a	a	a
a	a	a	a	A
a	a	a	b	
a	a	b		F
a	b	b		
b	b	b		
b	b		G	
b	c			
b	c			
c	d			
c		H		
c				
d				
d				
e				
K				

cause TI is double LI, I will be the Point of Equilibrium of the Magnitude compounded of all the a 's. And, likewise, since SP is the Double of PL, P will be the Point of Equilibrium of that compounded of all the b 's: And for the same Reason N will be the Point of Equilibrium of the Magnitude compounded of all the c 's; but o that of the Compound of all the d 's; and L that of the Magnitude e . There is therefore a certain Beam TL, whereon certain Magnitudes K, H, G, F, A, do hang at equal Distance

Distances. And again, there is another Beam LI, on which at equal Distances likewise do hang just the same Number of Magnitudes, and in the same Order as the former. For, the Compound of all the *a*'s, which hangs at I, is = K hanging at L; and the Compound of all the *b*'s, which hangs at P, is = H hanging at P; and likewise the Compound of the *c*'s, which hangs at N, is = G; and the Compound of the *d*'s, which hangs at O, is = F; and *e* hanging at L is = A. Wherefore the Beams are divided in the same *Ratio* by the Center of the compounded Magnitudes. But the Center of the Magnitude compounded of those Magnitudes is one alone. Therefore a Point common to the Right Lines TL and LI will be the Center; which let be X. Therefore as TX is to XL, so will LX be to XI; and so will the whole Line TL be to LI. But TL is the triple of LI: Wherefore TX will also be the triple of XL. Q. E. D.

If any Number of Magnitudes soever be so taken, that the second surpasses the first three times the first, but that the third surpasses the second five times the first, and the fourth surpasses the third seven times the first, and so on that the Increase of every one proceeds in that next to it multiple of the first Magnitude according to the odd Numbers following in Order; like as the Squares of Lines equally exceeding each other, the common Excess being equal to the least, do proceed; and if they be hung on a Beam at equal Distances; the Center of Equilibrium of all the Magnitudes compounded will divide the Beam, that the Part towards the lesser Magnitudes is more than triple the remaining

O o o

Part,

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Part, but, when one Distance is taken from it, is less than triple the said remaining Part.

Let there be such Magnitudes, as abovesaid, on the Beam BE. From which let some Magnitudes, so dispos'd one with another as those were in the preceding Proposition, be taken away; and let them be those compounded of all the *a*'s. The remaining Magnitudes, wherein the *c*'s are, will be rang'd in the same Order, but wanting the

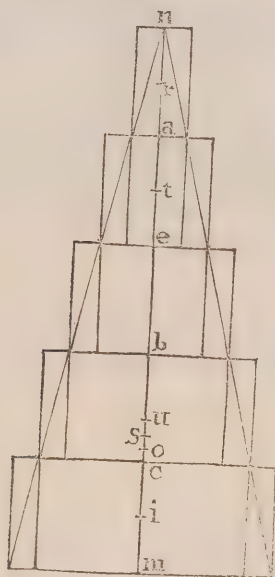
B	FOD		G	E
a	a	a	a	a
a	a	a	a	
a	a	a	a	
a	a	a	c	
a	a	a		
a	a	a		
a	a	c		
a	a	c		
a	a	c		
a	a			
a	c			
a	c			
a	c			
a	c			
a	c			
c	c			
c				
c				
c				
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c				
c				

greatest. Let ED be triple DB; and GF triple FB. D will be the Center of Equilibrium of the Compound of all the *a*'s; but F that of the Compound of all the *c*'s. Wherefore the Center of the Compound of all the *a*'s and *c*'s will fall between D and F. Let it be O. Therefore 'tis manifest that EO is more than the Triple of OB; but that GO is less than the Triple of the said OB. Q. E. D.

If
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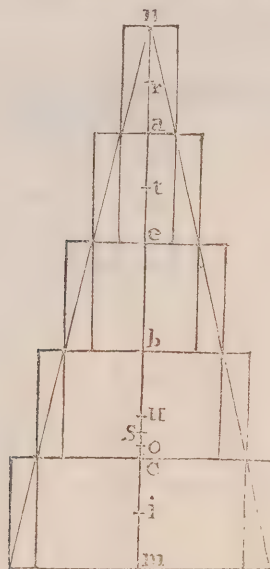
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If to any Cone or Portion of a Cone, one Figure consisting of Cylinders having equal Heights, be inscrib'd, and another circumscrib'd; and also if its Axis be so divided, that the Part which is intercepted between the Point of Division and the Vertex, be triple the remaining Part: the Center of Gravity of the inscrib'd Figure will be nearer the Base of the Cone than that Point of Division; but the Center of Gravity of the circumscrib'd Figure will be nearer the Vertex than the same Point.



Let the Cone then be that, whose Axis is nm . Let it be divided in s , so that ns be triple the remaining Part sm . I say, that the Center of Gravity of any Figure inscrib'd in the Cone, as aforesaid, is in the Axis nm , and comes nearer to the Base of the Cone than the Point s does; but that the Center of Gravity of the circumscrib'd Figure is likewise

in the Axis nm , and nearer the Vertex than s is. Let therefore a Figure of Cylinders, whose Axes mc , cb , be , ea , are equal, be apprehended to be inscrib'd. Therefore the first Cylinder, whose Axis is mc , has the same *Ratio* to the Cylinder whose Axis is cb , as the Base of the first is to the Base of the other (for their Heights are equal). But this *Ratio* is the same with that which the Square of cn has to the Square of nb . And in like Manner it may be prov'd that the Cylin-



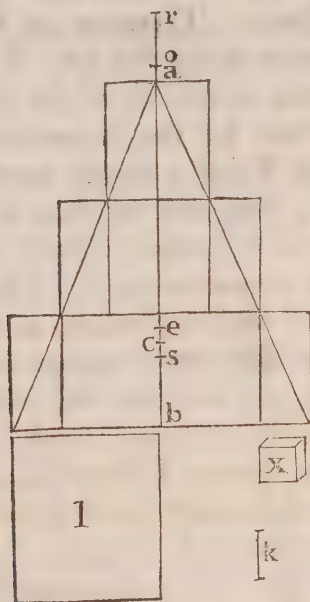
der, whose Axis is cb , has the same *Ratio* to the Cylinder, whose Axis is be , as the Square of bn has to the Square of ne ; and that the Cylinder, whose Axis is be , has that *Ratio* to the Cylinder about the Axis ea , which the Square of en has to the Square of na . But nc , nb , en , na , are Lines equally exceeding each other, and their common

Excess

Excess is equal to the least, *viz.* to na . There are therefore certain Magnitudes, *viz.* inscrib'd Cylinders, having that *Ratio* to each other in Order, which the Squares of Lines equally exceeding each other, and whose common Excess is equal to the least Line, have to one another: And they are so dispos'd on the Beam ti , that the Centers of the Gravities of every one of them are in it, and at equal Distances. Therefore by what has been before demonstrated, it is evident, that the Center of Gravity of them all thus compounded so divides the Beam ti , that the Part towards t is more than triple the remaining Part. Let this Center be o . Therefore to is more than the Triple of oi . But tn is triple to im . Therefore the whole mo will be less than a fourth Part of the whole mn , whereof ms is a fourth Part by the Hypothesis. 'Tis manifest therefore, that the Point o comes nearer the Base of the Cone than s does; (but now let there be a Figure circumscrib'd consisting of Cylinders, whose Axes mc , cb , be , ea , an , are equal to one another); in like Manner it may be demonstrated of these Cylinders, as it has been of the inscrib'd ones, that they are to one another as the Squares of the Lines mn , nc , bn , ne , an ; which exceed each other equally, and the common Excess is = the least, an . Wherefore, by the premis'd Proposition, the Center of Gravity of all the Cylinders thus dispos'd, which let be u , will so divide the Beam ri , that the Part towards r , to wit ru , is more than triple the remaining Part ui ; but that tu will be less than the Triple of the same. But nt is the Triple of im . Therefore the whole um is more than a fourth Part of the whole mn , whereof ms is a fourth Part by the Hypothesis. Therefore the Point u is nearer the Vertex than the Point s . *Q. E. D.* A

A Figure consisting of Cylinders of equal Altitude may be circumscrib'd about, and another inscrib'd in a given Cone, so that the Line which is intercepted between the Center of Gravity of the circumscrib'd Figure and the Center of Gravity of the inscrib'd Figure, be less than any given Line.

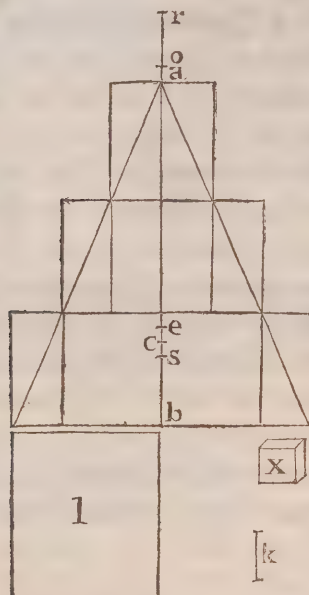
Let the given Cone be that, whose *Axis* is ab . And let the given Right Line be k . I say; Let a Cylinder l ,



equal to that which is inscrib'd in the Cone, be drawn by it self, having its Altitude = half the *Axis* ab : And let ab be divided in c , so that ac be the Triple of cb : And, the

the *Ratio* that ac has to k , let the Cylinder l have to the Solid α . But let a Figure of Cylinders having equal Altitude be circumscrib'd about the Cone, and let another be inscrib'd, so that the circumscrib'd Figure may exceed the inscrib'd one by a Quantity less than the Solid α . And let e be the Center of Gravity of the circumscrib'd Figure; which will fall above c : but let the Center of Gravity of the inscrib'd one be s , falling under c . I say now, that the Line es is less than k . For if it be not; let eo be suppos'd equal to ca . Because oe therefore has the same *Ratio* to k , that l has to α ; and the inscrib'd Figure is not less than the Cylinder l ; and the Excess, whereby the circumscrib'd Figure exceeds the said inscrib'd one, is less than the Solid α : Therefore the inscrib'd Figure will have a greater *Ratio* to the said Excess, than oe has to k : But the *Ratio* of oe to k is not less than that which oe has to es with es . Let it not be suppos'd less than k ; therefore the inscrib'd Figure has a greater *Ratio* to the Excess of the circumscrib'd Figure above it, than oe has to es . Therefore the *Ratio*, that the inscrib'd Figure has to the said Excess, the same it will have to the Line es . Let some Line er be greater than the said eo . The Center of Gravity of the inscrib'd Figure indeed is s ; but the Center of the circumscrib'd one is e . Therefore 'tis manifest, that the Center of Gravity of the remaining Portions, whereby the circumscrib'd exceeds the inscrib'd one, is in the Line re , and in that Point which so terminates it, that, the same *Ratio*, which the inscrib'd Figure has to the said Portions, the Line, intercepted between e and that Point, has to the Line es . But this *Ratio* re has to es . Therefore the Center of Gravity of the remaining Portions, whereby
the

the circumscrib'd exceeds the inscrib'd Figure, will be r . Which is impossible. For a Plane drawn thro' r , parallel to the Base of the Cone, does not cut the said Portions.



'Tis false then, that the Line es is not less than k . Therefore it will be less. And after a Manner not unlike this, the same may be demonstrated to come to pass in any Pyramid.

C O R O L L A R Y.

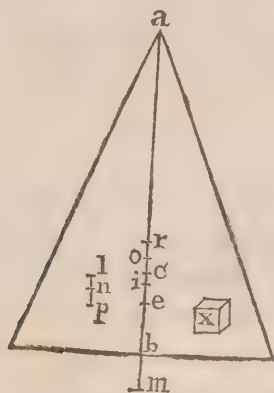
Hence it is manifest, that *One Figure of Cylinders of equal Altitude may be circumscrib'd about, and another inscrib'd*
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inscrib'd in a given Cone, so that the Lines, which are intercepted between their Centers of Gravity, and the Point, which divides the Axis of the Cone so that the Part towards the Vertex be triple the remaining Part, are less than any given Line.

For whereas the said Point, which divides the Axis so as abovesaid, is always found between the Centers of Gravity of the circumscrib'd and inscrib'd Figures, as has been demonstrated; and since it is possible, that the Line, which is in the midst between the same Centers, be less than any given Line; much less than the same given Line must that Line be, which is intercepted between either of those Centers and the said Point which divides the Axis.

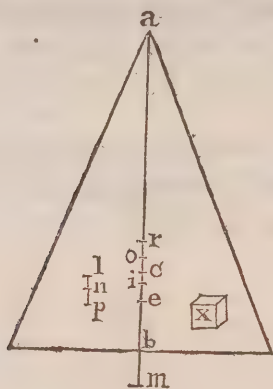
The Center of Gravity of any Cone or Pyramid divides the Axis, that the Part towards the Vertex is triple the other Part towards the Base.



Let there be a Cone, whose Axis is ab . And let it be so divided in c , that ac be the Triple of the remaining
P p p
Part

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Part *cb*. It is to be demonstrated, that *c* is the Center of Gravity of the Cone. For if it is not, the Center of the Cone will be either above, or below the Point *c*. First let it be below; and let it be *e*: And let the Line *lp* be drawn equal to *ce*; which let be divided at Pleasure in *n*. And that Ratio, which *be* and *pn* both together have to *pn*, let the Cone have to the Solid α . And let a solid Figure of Cylinders, having equal Altitude, be inscrib'd in the Cone, the Distance of whose Center of Gravity from the Point *c* is less than the Line *ln*; and let the Ex-



cess of the Cone above it, be less than the Solid α . For that these Things may be done, is manifest from what has been demonstrated. Now let such a Figure be inscrib'd as was requir'd, whose Center of Gravity let *i* be. Therefore, since *lp* is equal to *ce*, and *ic* is less than *ln*, the Line *ie* will be greater than *np*: And, because *be* and *np* both together is to *np*, as the Cone to α ; and the Excess of the Cone above the inscrib'd Figure is less than the Solid

Center of Gravity of Solids. 483

Solid κ ; therefore the Cone will have a greater *Ratio* to the said Excess than both be and np to np : And by Division, the inscrib'd Figure will have a greater *Ratio* to the Excess of the Cone above it, than be to np : But be has still a less *Ratio* to ei , than it has to np ; since ie is greater than np ; therefore the inscrib'd Figure has a much greater *Ratio* to the Excess of the Cone above it, than be has to ei . Therefore that *Ratio*, which the inscrib'd Figure has to the said Excess, some Line, greater than be , will have to ei . Let that be me . Then because me is to ei , as the inscrib'd Figure is to the Excess of the Cone above it, and because e is the Center of Gravity of the Cone, and i is the Center of Gravity of the inscrib'd Figure; therefore m will be the Center of Gravity of the remaining Portions, whereby the Cone exceeds the Figure inscrib'd in it. Which is impossible. Therefore the Center of Gravity of the Cone is not below the Point c . Nor is it above it. For, if it can be, let it be r ; and again let the Line lp , cut at pleasure in n , be assum'd: And, that *Ratio*, which bc and np both together has to nl , let the Cone have to κ ; And let a Figure be in like manner circumscrib'd about the Cone, which exceeds the Cone by a Quantity that's less than the Solid κ : And let the Line, that is intercepted between the Center of Gravity of that Figure and c , be less than np : Now let a Figure be circumscrib'd, whose Center let o be: the remaining Part or will be greater than nl . And, because as bc and pn both together is to nl , so is the Cone to κ ; and the Excess, whereby the circumscrib'd Figure exceeds the Cone, is less than κ ; and bo is less than bc and pn both together; and or greater than ln : The Cone therefore will have a much greater *Ratio* to the remaining Portions, by which the cir-

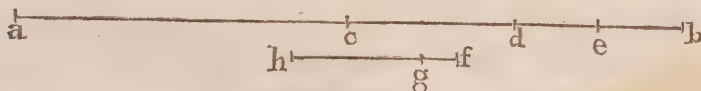
P p p 2

cumscrib'd

cumscrib'd Figure exceeds it, than *bo* has to *or*. Let *mo* have that *Ratio* to *or*: *mo* will be greater than *bc*: And *m* will be the Center of Gravity of the Portions by which the circumscrib'd Figure exceeds the Cone. Which is absurd. Therefore the Center of Gravity of the Cone is not above the Point *c*: But it is not below it; as has been prov'd. Therefore it will be *c* it self. And intirely after the same Manner the same may be demonstrated in any Pyramid whatsoever.

If four Lines are continual Proportionals; and the same Ratio, which the least of them has to the Excess of the greatest above it, some assum'd Line has to $\frac{1}{4}$ of the Excess of the greatest above the second; and the same Ratio, which a Line equal to all these (viz. the greatest, the Double of the second, and the Triple of the third) has to a Line equal to the Quadruple of the greatest, the Quadruple of the second, and the Quadruple of the third, some other assum'd Line has to the Excess of the greatest above the second; these two assum'd Lines together will be a fourth Part of the greatest of the Proportionals.

For let the four proportional Lines be *ab*, *bc*, *bd*, *be*. And, the same *Ratio*, which *be* has to *ea*, let *fg* have to $\frac{1}{4}$ of *ac*. And the *Ratio*, which a Line equal to *ab*, and



twice *bc*, and thrice *bd* has to a Line equal to the Quadruple of the Lines *ab*, *bc*, *bd*, let *hg* have to *ac*: It is to be demonstrated, that *hf* is a fourth Part of *ab*.

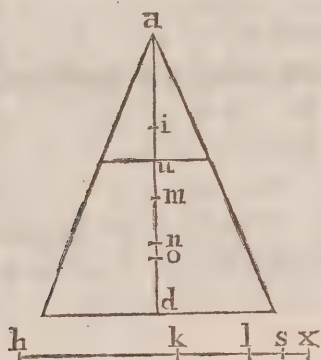
Because

Because then ab , bc , bd , be , are Proportionals; ac , cd , de , will be also in the same Proportion: And as the Quadruple of the Lines ab , bc , bd , is to ab with twice bc , and thrice bd ; so is the Quadruple of ac , cd , de ; that is, the Quadruple of ae to ac with twice cd and thrice de ; and so is ac to hg . Therefore as the Triple of ae is to ac with twice cd and thrice de , so is $\frac{3}{4}$ of ac to hg : But, as the Triple of ae is to the Triple of eb , so is $\frac{3}{4}$ of ac to gf ; therefore, by the Converse of the 24th of the 5th of *Euclid*, as the Triple of ae is to ac with twice cd and thrice db , so is $\frac{3}{4}$ of ac to hf . And, as the Quadruple of ae is to ac with twice cd and thrice db , that is, to ab with cb and bd ; so is ac to hf . And, by Permutation, as the Quadruple of ae is to ac ; so is ab with cb and bd to hf . But, as ac is to ae , so is ab to ab with cb and bd . Therefore by Equality, in inordinate Proportion, as the Quadruple of ae is to ae , so is ab to hf . Wherefore 'tis evident, that hf is a fourth Part of ab .
Q. E. D.

The Center of Gravity of any Frustum of a Pyramid or Cone whatsoever, cut by a Plane parallel to the Base, is in the Axis, and so divides it, that the Part towards the lesser Base is to the remaining Part, as the Triple of the greater Base, with the Double of the Space of the middle between the greater and the lesser Base, together with the lesser Base, is to the Triple of the lesser Base, with the same Double of the middle Space, and with the greater Base.

Let

Let the Frustum, whose Axis is ud , be cut from the Cone or Pyramid, whose Axis is ad , by a Plane parallel to the Base. And that *Ratio*, which the Triple of the greatest Base, with the Double of the Middle, and with the least Base, has to the Triple of the least, with the Double of the Middle, and with the greatest, let uo have to od . It is to be prov'd, that o is the Center of Gravity of the Frustum. Let um be a fourth Part of ud .



Let a Line bx be drawn equal to ad . And let kx be equal to au . And let xl be a third, and xs a fourth Proportional to bx and xk . And that *Ratio* which bs has to sx , let md have to a Line assum'd from o towards a ; which let be on . And because the greater Base bears the same Proportion to that, which is in the Middle between the greater and lesser Base, as da does to au ; that is, as bx does to xk : And because the said middle Base is to the lesser, as kx is to xl : the greater, middle, and lesser Bases, will be in the same Proportion with the Lines bx , xk , and xl .

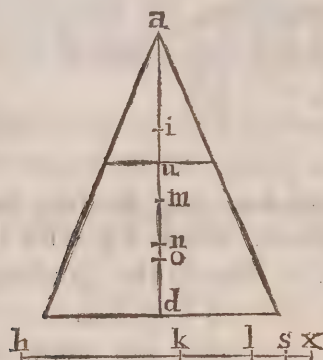
Wherefore

Wherefore as the Triple of the greater Base with the Double of the middle one and with the least, is to the Triple of the least with the Double of the middle and with the greatest; that is, as uo is to od ; so is the Triple of bx with the Double of xk and with xl , to the Triple of xl with the Double of xk and with xb : And by Composition, and Conversion, od will be to du , as bx with the Double of xk and the Triple of xl , is to the Quadruple of these, bx , xk , xl .

There are therefore four proportional Lines, bx , xk , xl , xs : And that *Ratio*, which xs has to sb , the assum'd Line no has to $\frac{3}{4}$ of du , viz. to dm ; that is, to $\frac{3}{4}$ of bk : And the same *Ratio*, which bx with the Double of xk and the Triple of xl has to the Quadruple of these, bx , xk , xl ; another assum'd Line od has to du ; that is, to bk . Therefore (by what has been demonstrated) dn will be a fourth Part of bx ; that is, of ad . Wherefore the Point n will be the Center of Gravity of the Cone or Pyramid whose Axis is ad . Let the Center of Gravity of the Pyramid or Cone, whose Axis is au , be i . It is then manifest, that the Center of Gravity of the Frustum is in the Line in prolonged from n , and in that Point of it, which with the Point n intercepts a Line, to which in has that *Ratio*, which the Frustum cut off has to the Pyramid or Cone, whose Axis is au . It remains therefore to be prov'd, that in has the same *Ratio* to no , that the Frustum has to the Cone, whose Axis is au . But as the Cone, whose Axis is da , is to the Cone, whose Axis is au ; so is the Cube of da to the Cube of au ; that is, the Cube of bx to the Cube of xk . But this is the same Proportion that bx has to xs . Wherefore by Division,

as

as bs is to sx , so will the Frustum, whose Axis is du , be to the Cone or Pyramid, whose Axis is ua . But, as bs is to sx , so also is md to on . Wherefore the Frustum is to the Pyramid, whose Axis is au , as md is to no . And because an is $\frac{3}{4}$ of ad ; and ai is $\frac{3}{4}$ of au : the remaining



Part in will be $\frac{3}{4}$ of the remaining Part ud . Wherefore in will be equal to md . And it has been demonstrated, that md is to no , as the Frustum is to the Cone au . 'Tis manifest therefore, that in has also this same Proportion to no . Wherefore the Proposition is evident.

E I N I S.



A
T A B L E
O F T H E
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of them; and that by Ocular Testimony, notwithstanding their immense Distance, since, according to that known Saying, *One View of the Object is of more Service to the Learner, than Rules a thousand times repeated*, because Intuitive Knowledge (as one expresses it) goes hand in hand with Definition. But his great Parts and Industry discover themselves no where so much as in this Treatise, from which it appears that he was the Inventor of two New Sciences, which he has demonstrated Geometrically. And what is still more remarkable in this Treatise is, that one of these Sciences is employ'd about an Eternal Subject, the Principal in Nature, about which all the great Philosophers have from time to time busied themselves, I mean *Local Motion*, which is attended with many surprising Accidents, whereof none has been hitherto in the least minded, much less demonstrated. The other Science, which is also demonstrated from its proper Principles, is employ'd about that Resistance of Solids, whereby they are preserv'd from being crush'd to Pieces by outward Force: Which Knowledge is of very great Use, especially in Mechanical Arts and Sciences, as being productive of many Accidents and Propositions hitherto taken no Notice of. And indeed in this Treatise, and in this only, a first rational Entrance is made into these two Sciences, thence justly call'd New, and a regular Process carried on by a great Number of Propositions actually demonstrated, and a Passage laid open to infinite others, as the intelligent Reader will easily grant.

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E D I T O R of the Original in *Italian*.

 I N C E Civil Society is kept up by the mutual Assistances Men receive from one another; and since Arts and Sciences do largely contribute to this End, it has been always customary to respect the Inventors of them, which indeed our wise Fore-fathers did to a very great Degree; whose Veneration for them did so rise in Proportion to the Excellency and Usefulness of the Invention, that they did not scruple to make them equal with the Gods; and with one Consent by this the greatest Mark of their Esteem for such Persons, whom they look'd upon as the Authors of their Well-being, to immortalize their Name: And certainly such as by their great Penetration have discover'd and corrected the Fallacies of many Propositions laid down by eminent Persons, and for many Ages receiv'd for Truth, are no less worthy of Praise

[b] than

than the former; nay, this they deservedly claim, tho' they only detect the Error, and do not discover the Truth; which is a Task so difficult, that it forced the Prince of Orators upon this Wish, *Utinam tam facile possem vera reperire, quam falsa convincere!* And I must needs say, that whatever Praise has been merited this Way, is justly due to these latter Ages, who, by continually sifting the Works of the sagacious Ancients, have brought the Arts and Sciences, found out by them, to a greater Degree of Perfection; which Improvement goes on daily, especially in Mathematical Sciences, wherein (not to mention others that have succeeded well) I make no Doubt, but that all will readily give the Preference to our *Galileus*, who has not only, with clear Demonstrations, detected the fallacious Reasonings, and unjust Conclusions, that others had pass'd current (as abundantly appears by his Writings) but also as he was the first that discover'd and made us acquainted with *Jupiter's* four Satellites, by Help of the Telescope (which Instrument, tho' invented in these Parts, yet was improv'd by him): To him we owe also the true Discovery of the Milky Way, of the Spots in the Sun, the Inequality of the Moon's Surface, the Emulous Phases of *Venus*, the Ring about *Saturn*, and of the Quality and Disposition of Comets: All which the ancient Astronomers and Philosophers knew nothing of; so that it may be truly said, that Astronomy has deriv'd a new Lustre from him: And as the Power and Wisdom of the great Creator is no where so conspicuous as in the Firmament and heavenly Bodies, so it is no small Addition to this great Man's Praise, to have been the first that has led us into the Knowledge
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